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Designing an Effective Combined Shewhart-CUSUM Control Scheme with Exponentially Distributed Data

Dushyant Tyagi
Department of Mathematics and Statistics
Dr. Shakuntala Misra National Rehabilitation University
Lucknow-226017, India
E-mail: dtyagi@dsmnru.ac.in

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Abstract
In this paper, the Combined Shewhart-CUSUM control scheme has been proposed to monitor the production process when the quality characteristic follows exponential distribution to quickly detect the shift in the process. The simulated values of ARL are determined after the transformation of the data into approximate normal distribution by Nelson transformation method and adding Shewhart control limits to existing CUSUM Control Chart. Scheme parameters (value of k and h) and out of control ARL are calculated at various shift and at various in-control ARL. Parameters are also calculated to detect δ standard deviation shifts, which may be helpful to the quality control practitioners in designing the Combined Shewhart-CUSUM scheme when data is highly skewed.

Keywords- Combined Shewhart-CUSUM scheme, Exponential distribution, Average runs length, Monte Carlo simulation.

1. Introduction
Statistical process control is the widely used technique for controlling the quality in manufacturing and service industry. Initially Statistical Process Control (SPC) was developed by Walter Andrew Shewhart in 1924. Shewhart puts the foundation for the control chart to check that the process is statistically controlled or not. It is noted that Shewhart (3 sigma) control chart for the mean is very useful if the shift in process parameters magnitude is 1.5 sigma or larger (Montgomery, 2018), but takes a large number of runs to detect a small constant shift in the process quality characteristic. In SPC, many times a quality control engineer concerned to monitor the occurrence of events that can occur at any point within a continuous interval of time. For example, the number of false alarm per day, wrong number connection per day or product replacement calls in a given period of time. If such events occur "uniformly at random," the number of occurrences per unit time leads to the Poisson distribution. The traditional approach for monitoring Poisson counts is based on the Shewhart control chart called c chart. However, many researchers like Nelson (1994), Gan (1994) have discovered that the c chart is inadequate for controlling highly efficient processes. For highly efficient processes, numbers of occurrences are very lesser. Its result that most of the sample observations for c chart applications will have zero value. Hence, c charts will be a plot of a series of zeros with some rare nonzero observation. Really, under such situations, the Shewhart c chart becomes quite ineffective for process control. To overcome this problem with the c chart (Nelson, 1994) proposed to monitor these types of processes by different measurements, that is, the time between successive occurrences of the events and this random variable follows the exponential distribution. Due to asymmetric shape, the exponential distribution violates the normality assumption of Shewhart chart, therefore, the use of normal-based Shewhart control chart for the time between occurrences would not be relevant and cause of misleading interpretation of data, a wrong process diagnosis or an

inappropriate SPC chart can generate disastrous accidents. To overcome this problem, Nelson (1994) suggested a method to transform exponential random variable into approximate normal so normal-based Shewhart control limits can be used to monitor the time between occurs.

Page (1954) developed a Cumulative Sum (CUSUM) control chart to detect the small shift in the process in which information from past observations is also included to take the decision at any stage of the process that means Cumulative sum of deviation from target value is plotted against the sample number. Lucas and Crosier (1982), Lucas (1985), Duncan (1986), Hawkins (1992) also showed that CUSUM control chart is moreover efficient than the traditional Shewhart's control chart for detecting a small shift in the average. Chan et al. (2000) had proposed CQC chart to monitor the time between events (TBE) data that are exponentially distributed. Lucas (1985), Vardeman and Ray (1985), Alwan (2000) had proposed the exponential CUSUM chart. Kittlitz (1999) used a double square root (SQRT) transformation method for developing I-chart, exponentially weighted moving average (EWMA) and CUSUM chart. Montgomery (2018) stated that "in many cases, the CUSUM and EWMA control charts would be better alternatives because these charts are more effective in detecting small shifts in mean". Liu et al. (2006), Tyagi (2013) proposed the optimal design of the CUSUM Chart with transformed exponential distributed data and determined the average run length (ARL) properties of the proposed chart. Lucas (1982) proposed combined Shewhart-CUSUM Quality control schemes for normally distributed data and shows that its performance is better to detect a large shift in compare to the CUSUM chart. In this study, we propose a combined Shewhart-CUSUM scheme for transformed exponentially distributed data.

In this paper, a combined Shewhart-CUSUM control scheme is proposed to observe a set of exponentially distributed data after transformation to quickly detect the shift in the mean. A transformation method suggested by Nelson is used to transform exponentially distributed data to approx. normal. The ARL for the proposed scheme is calculated by simulation technique. A comparative study on the ARL is also conducted. CUSUM control charts are represented by two ways, the tabular or algorithmic CUSUM and the V- mask CUSUM. Here, we consider the tabular CUSUM as it is more preferred by the practitioners.

The break-up of the paper is ordered as follows. In Section 2, the transformation of exponentially distributed data is discussed. In Section 3, parameters for the combined Shewhart-CUSUM scheme are determined. In Section 4, a numerical example has been given for highlighting the use of the theoretical developments. In Section 5, ARL values are determined to compare shift detection properties of the proposed Combined Shewhart-CUSUM scheme. Finally, Section 6 concludes the work by discussing the results.

2. Transformation of Exponentially Distributed Variable

A popular distribution for modeling lifetime is Weibull distribution W (λ, k), has a density function:

$$f(x, \lambda, k) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(x/\lambda\right)^k}, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{1}$$

where $\lambda > 0$ is the scale parameter and $k > 0$ is the shape parameter.

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The mean and variance of X is

$$\mu_x = \lambda.\Gamma\left(1+\frac{1}{k}\right) \tag{2}$$

$$\mathrm{Var}(X) = \lambda^2\left[\Gamma\left(1+\frac{2}{k}\right)-\left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right] \tag{3}$$

Where $\Gamma(.)$ is the complete gamma function.

As we know that exponential distribution is a particular case of Weibull distribution. i.e when shape parameter k = 1, Weibull distribution is known as exponential distribution. So with the help of equation (2) and (3) mean and standard deviation for the exponential distribution can be calculated.

An important fact used in this paper is that if X follows Weibull distribution with shape parameter K and scale parameter λ, then another variable obtained by $Y = X^p$ (with p > 0) follows $W(\lambda^p, k/p)$. Hence the distribution of an exponential random variable $W(\lambda, 1)$ raised to the power p follows $W(\lambda^p, 1/p)$. According to Nelson (1994) for any $\alpha > 0$, $W(\alpha, 3.6)$ has zero skewness value and approximate three kurtosis value i.e. follows approximate normal distribution. Since (1/0.27777) is nearly equal to 3.6. So we can easily say that putting the power p = 0.27777 on exponential distributed random variable. i.e. $W(\lambda^{0.27777}, 3.6)$, follows an approximately normal distribution.

From equation (2) and (3), the mean and standard deviation of transformed variable say, ‘Y’ are

$$\mu_Y = \lambda^{0.27777}.\Gamma\left(1+\frac{1}{3.6}\right)$$
$$= 0.9011057\,\lambda^{0.27777} \tag{4}$$

$$\Sigma_Y = \lambda^{0.27777}\sqrt{\left[\Gamma\left(1+\frac{2}{3.6}\right)-\left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]}$$
$$= 0.2780203\,\lambda^{0.27777} \tag{5}$$

3. Combined Shewhart-CUSUM Scheme for Controlling the Process Mean

For the CUSUM chart, when the process is under control, Let random variable Y follow a normal distribution with mean μ_{y0} and standard deviation σ (known or estimable), μ_{y0} is assumed to be the standard value for the quality characteristic Y. The tabular CUSUM works by plotting the cumulative sum of the deviations of observation from μ_{y0} after subtracting the reference value. Two statistic is calculated, one statistic C+ for the deviation in the upper side of the standard value and another statistic C- for the deviation in the lower side of the standard value. Where C+ and C- statistic is calculated as

$$C_i^+ = \max\left[0, y_i - (\mu_{y0} + k\sigma_y) + C_{i-1}^+\right] \tag{6}$$

$$C_i- = \min\left[0, y_i - (\mu_{y0} - k\sigma_y) + C_{i-1}-\right] \tag{7}$$

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where y_i is the i[th] transformed observation of the process.

With the starting value $C_i^+ = C_i^- = 0$. Where k = $\delta/2$, is called the reference or allowable value where δ is the smallest shift, the researcher wants to detect, measured in units of process standard deviation. Thus,

$$k = \frac{|\mu_{y1} - \mu_{y0}|}{2\sigma_y} \quad (8)$$

The cumulative statistic values C_i^+ and C_i^- are the cumulative sum of deviation from the standard value μ_{y0} that is greater than $k\sigma_y$. If any of the two statistic crossed the decision interval H, the process is said to be out of control. Here H = $h\sigma_y$ is known as the decision interval of the CUSUM chart (Montgomery, 2018).

A number of authors have used different methods to determine the value of h and k by using in-control ARL of the CUSUM chart. Woodall and Adams (1993) recommended approximation method based on specified in-control ARL given by Siegmund (1985) to develop an iterative procedure for determining the value of h for a known value of k when applying a two-sided procedure.

$$h_n = h_{n-1} - \frac{e^{2k(h_{n-1}+1.166)} - 2k(h_{n-1}+1.166) - 1 - 2k^2 ARL_0}{2ke^{2k(h_{n-1}+1.166)} - 2k} \quad (9)$$

with n = 1 , 2, 3, . . ., ARL_0 is the specified in-control ARL, and h_0 is any positive initial value. As the iterations move on, h_n will reach to the desired h value which constructs CUSUM control chart whose true in-control ARL approximates equal to the desired ARL_0 value. Woodall and Adams (1993) recommend to continue the iteration until the difference between two consecutive h values is more than 0.005 (that is, $|h_n - h_{n-1}| < 0.005$) and then take the h value to the second decimal place.

Let us study a particular case. Suppose the quality characteristic follows an exponential distribution with target mean μ_0. If the quality control engineer wants to detect a shift in the lower side, then the value should be suggested '$\mu_1 -$', which is the mean value that one wishes to detect as soon as possible. If the requirement is to detect shift in the upper side, then a value should be suggested '$\mu_1 +$', which is the mean value that one wishes to detect as soon as possible. Let x_1, x_2, ……, x_n, is a sequence of exponentially distributed process observations, so equations 6 - 9 can be used to determine the parameters of the CUSUM chart as follows:

$$k = \frac{|0.9011057\, \lambda_1{}^{0.27777} - 0.9011057\, \lambda_0{}^{0.27777}|}{2 * 0.2780203\, \lambda_0{}^{0.27777}} \quad (10)$$

$$C_i + = \max\left[0, x_i{}^{0.27777} - \left(0.9011057\, \lambda_0{}^{0.27777} + k * 0.2780203\, \lambda_0{}^{0.27777}\right) + C_{i-1} +\right] \quad (11)$$

$$C_i - = \min[0, x_i{}^{0.27777} - (0.9011057\, \lambda_0{}^{0.27777} - k * 0.2780203\, \lambda_0{}^{0.27777}) + C_{i-1} -] \quad (12)$$

For the upper one-sided CUSUM control chart detect the shift if $C_i + > h\sigma_y$ whereas for the lower one-sided CUSUM control chart detect the lower side shift if $C_i - < -h\sigma_y$.

The Combined Shewhart-CUSUM scheme adds one more condition that if y_i (transformed value of observation) > Shewhart Control Limit Upper (SCLU) or y_i < Shewhart Control Limit Lower (SCLL), chart signals out of control situation. Additional Shewhart control limit modifies the standard CUSUM chart to quickly detect large shifts of the mean from the target value.

4. Example

Suppose the quality under consideration follows exponential distribution. A sample of 100 observations is generated in which first 70 observation were drawn at random from exponential distribution with mean time between failure = 1. The last 30 observations were drawn from the exponential distribution with mean time between failure =2, shown in Table 1. Thus, we can say that the last 30 observations have been drawn from the out of control process. That is after the process mean is shifted by 1 point. By the method described in this paper with a pre-specified in-control ARL of 250, parameters can be determined as below:

Reference value 'k' is given by equation (10) i.e.

$$k = \frac{|0.9011057 * (2)^{0.27777} - 0.9011057 * (1)^{0.27777}|}{2 * 0.2780203 * \ 1^{0.27777}}$$

$$k = 0.3440894 \tag{13}$$

By equation (9) we calculate value of 'h', starting with $h_0 = 10$ along with $ARL_0 = 250$ and $k = 0.3440894$ as:

$$h_1 = h_0 - \frac{e^{2*0.3440894(h_0+1.166)} - 2*0.3440894(h_0+1.166) - 1 - 2*(0.3440894)^2*250}{2*0.3440894e^{2*0.3440894(h_0+1.166)} - 2*0.3440894}$$

$= 8.591602$,

$h_2 = 7.254729$,

$h_3 = 6.089752$,

$h_4 = 5.273459$,

$h_5 = 4.933119$,

$h_6 = 4.885745$,

$$h_7 = h_6 - \frac{e^{2*0.3440894(h_6+1.166)} - 2*0.3440894(h_6 + 1.166) - 1 - 2*(0.3440894)^2 * 250}{2 * 0.3440894e^{2*0.3440894(h_6+1.166)} - 2 * 0.3440894}.$$

$=4.884944$.

Now ($|h_7 - h_6| < 0.005$), so h = 4.884944.

By equation (11), we compute successive value of $c_i +$ starting with $c_1 + = 0$

$$C_i+ = \max[0, x_i^{0.27777} - (0.9011057 * (1)^{0.27777} + 0.3440894 * 0.2780203 * (1)^{0.27777}) + C_{i-1}+],$$

$$C_i+ = \max[0, x_i^{0.27777} - 0.9967695 + C_{i-1}+].$$

Thus $if\ C_i+ > 4.884944 * 0.2780203\ (1)^{0.27777}$ = 1.358113, the Control chart will signal out of control.

A plot of the Combined Shewhart-CUSUM control scheme based on the transformed observations which approximately normal distribution by putting the power p = 0.27777 on exponential distributed random variable, with the upper CUSUM limit of 1.358113 and Shewhart limit of 1.735167, is shown in Figure 1. It can be seen, the CUSUM chart signals a change with the 87th cumulative sum breaching the control limit while the sample point lies outside the Shewhart limit at 80th point. Thus, Combined Shewhart-CUSUM control scheme is appropriate to quickly detect small as well as large shift in the process.

Table 1. Exponential distributed simulated observations in which the first 70 observations have mean of 1 and the remaining 30 observations have a mean of 2.

S. No.	Observations	S. No.	Observations	S. No.	Observations	S. No.	Observations
1	1.275012	26	0.323179	51	3.780650	76	2.740157
2	1.839189	27	1.541008	52	1.828259	77	0.031402
3	1.684099	28	0.729656	53	0.943600	78	2.594180
4	0.273658	29	0.452781	54	0.861949	79	4.766397
5	1.287355	30	4.530053	55	0.570235	80	9.284280
6	0.433465	31	0.013239	56	1.104272	81	3.565475
7	0.416178	32	0.622854	57	0.990773	82	5.576569
8	0.414192	33	0.039747	58	0.091323	83	7.845603
9	1.165529	34	0.038283	59	1.229389	84	0.648795
10	0.027285	35	0.052568	60	0.095926	85	3.326990
11	3.387107	36	1.085602	61	1.450531	86	2.047517
12	3.048128	37	1.170370	62	0.385312	87	5.995597
13	0.341408	38	1.793508	63	0.712849	88	7.175860
14	0.469469	39	0.405439	64	1.194387	89	5.914588
15	0.391657	40	0.269010	65	0.933852	90	1.697418
16	0.155340	41	2.337722	66	0.557122	91	1.100326
17	0.651664	42	5.252697	67	1.975667	92	5.967013
18	0.331198	43	1.456239	68	1.614779	93	0.491090
19	0.435458	44	0.736100	69	1.244741	94	1.240761
20	1.736889	45	1.183378	70	0.392422	95	1.710645
21	0.283764	46	0.290360	71	0.569076	96	1.559848
22	0.792379	47	0.734786	72	0.882317	97	0.346583
23	1.267773	48	0.529566	73	3.045051	98	2.259329
24	2.565313	49	0.360139	74	1.238553	99	4.476474
25	1.252512	50	1.171523	75	1.550555	100	6.689868

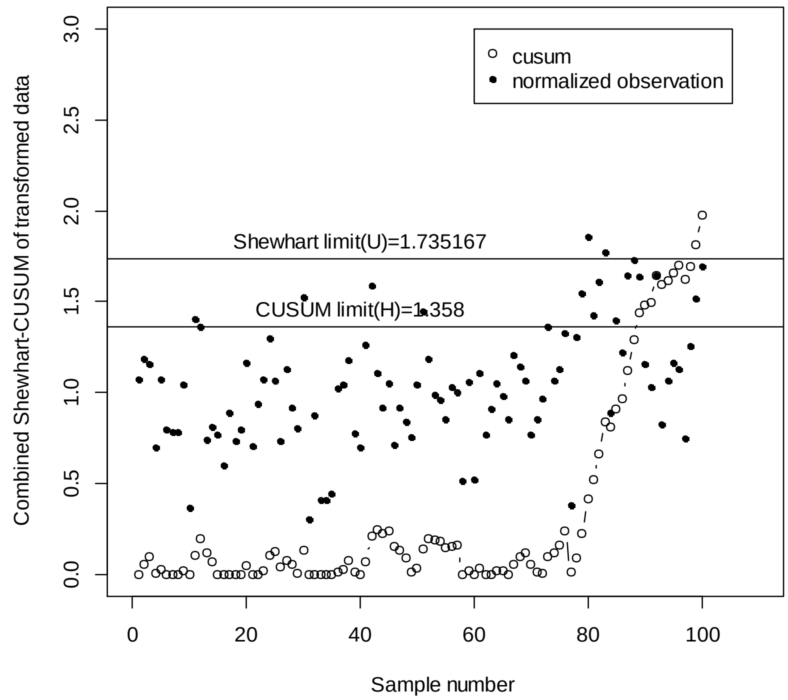

Figure 1. Combined Shewhart-CUSUM chart for transformed exponentially distributed data with ARL = 250

5. Calculation of ARL with Monte Carlo Simulation

The Statistical performance of a control chart shall be measured by the ARL. The run length is a random variable that denotes the number of points required before an out of control signal is shown by the chart. The ARL is the expected value of the run length. It measures on an average how much points the chart takes to detect the shift in the process. The ARL for without shift in the process is called in-control ARL. A well-designed control chart should offer large ARL value when there is no change in the process and small ARL value when there is shift/ change in the process.

To study the performance of the proposed Combined Shewhart-CUSUM Scheme for the exponential variable, we provide the approximate value of ARL for the pre-specified shift. The proposed Combined Shewhart-CUSUM Scheme is based on certain approximations, so we have used Monte Carlo simulation to compute the ARL associated with particular parameters. ARL calculated for the proposed chart is based on 1,00,0000 simulated observations. We estimated the values of ARLs to monitor the process from the in control mean $\mu_0 = 1$ to out of control mean $\mu_1 = 2$, based on in control ARL of 200, 350 and 500 and shown in Table 2.

A CUSUM chart with a small value of K is sensitive to small shifts. A CUSUM chart becomes less sensitive to small shifts but more sensitive to larger shifts as K increases. This can be explained by the fact that a CUSUM chart will stay inactive (i.e. $C_i + = 0$) as long as the shift is less than K. Thus, a CUSUM chart with a large K value is insensitive to small shifts. The Shewhart chart, which based its decision on the most recent sample mean, is the CUSUM chart with the largest possible value of K for a fixed in-control ARL and is thus the most sensitive for detecting the very large shift. In order to investigate the performance of the proposed Combined Shewhart-CUSUM Scheme to detect the shift of different magnitude, that means for varying out of control mean (μ_1) for fixed in control mean (μ_0) and in control ARL (ARL_0) the reference value k, decision interval h, out of control ARL has been calculated and shown in Table 3. For all numerical computations, the programs have been developed in R-software.

Table 2. Simulated ARLs for the combined Shewhart-CUSUM scheme for normalized exponentially distributed data where $\mu_0 = 1$ to $\mu_1 = 2$

True Mean(μ)	**ARL_0 = 200**	**ARL_0 = 350**	**ARL_0 = 500**
	k = 0.344079	**k = 0.344079**	**k = 0.344079**
	h = 1.274416	**h = 1.578622**	**h = 1.755029**
1.00	199.856100	349.454900	492.368300
1.10	113.199200	156.081700	202.069200
1.20	56.657220	85.437950	105.033200
1.30	39.184950	53.445640	63.673180
1.40	27.670170	37.306750	42.994110
1.50	21.505380	27.913940	31.678450
1.60	17.400380	22.131040	24.645600
1.70	14.845610	18.208570	20.124040
1.80	12.686010	15.424290	16.884930
1.90	11.178060	13.352600	14.515300
2.00	9.977252	11.789510	12.733790
2.10	9.021281	10.552930	11.371030
2.20	8.259616	9.549402	10.228780
2.30	7.571054	8.726689	9.315567
2.40	7.025975	8.027473	8.561996
2.50	6.551105	7.453631	7.911180
2.60	6.160556	6.964632	7.362393
2.70	5.810406	6.528293	6.903369
2.80	5.476181	6.150455	6.476466
2.90	5.210151	5.819380	6.116792
3.00	4.961351	5.523995	5.791904

Table 3. Combined Shewhart- CUSUM scheme parameters and out of control ARLs with the varying value of out of control mean when target mean $\mu_0 = 1$

	ARL_0=200			**ARL_0=350**			**ARL_0 = 500**		
μ_1	**k**	**h**	**ARL_1**	**k**	**h**	**ARL_1**	**k**	**h**	**ARL_1**
1.1	0.0435	3.1367	95.5602	0.0435	4.1996	139.2952	0.0435	5.0183	171.9395
1.2	0.0842	2.6513	57.2856	0.0842	3.4181	77.4894	0.0842	3.9766	91.4746
1.3	0.1225	2.3141	39.3413	0.1225	2.9144	50.5669	0.1225	3.3389	58.3444
1.4	0.1588	2.0655	29.1966	0.1588	2.5608	36.4620	0.1588	2.9050	41.2514
1.5	0.1932	1.8739	22.9030	0.1932	2.2975	27.8895	0.1932	2.5886	31.2727
1.6	0.2260	1.7213	18.5867	0.2260	2.0928	22.2823	0.2260	2.3463	24.6580
1.7	0.2574	1.5964	15.5416	0.2574	1.9286	18.3691	0.2574	2.1539	20.2032
1.8	0.2874	1.4920	13.3379	0.2874	1.7934	15.5850	0.2874	1.9970	16.9678
1.9	0.3163	1.4033	11.6415	0.3163	1.6799	13.4499	0.3163	1.8661	14.6133
2.0	0.3441	1.3267	10.3127	0.3441	1.5829	11.8008	0.3441	1.7550	12.7516
2.1	0.3709	1.2599	9.2248	0.3709	1.4991	10.5000	0.3709	1.6594	11.2728
2.2	0.3968	1.2010	8.3584	0.3968	1.4257	9.4611	0.3968	1.5760	10.1208
2.3	0.4219	1.1485	7.6561	0.4219	1.3607	8.5935	0.4219	1.5025	9.1795
2.4	0.4461	1.1015	7.0507	0.4461	1.3028	7.8843	0.4461	1.4372	8.3893
2.5	0.4697	1.0590	6.5453	0.4697	1.2508	7.2691	0.4697	1.3787	7.7265

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6. Conclusion

In this paper, we provide the properties of the Combined Shewhart-CUSUM control scheme for exponentially distributed data. The simulated value of ARL shows that after transformation of the data into normal approximation and adding Shewhart control limits to existing CUSUM control Chart improves ARL in compare to existing charts. The proposed scheme is more effective than the Shewhart and CUSUM chart individually in detecting smaller, moderate and large changes in process quality characteristics. The out-of-control ARL values are provided at various shifts and various in-control ARL for the proposed Combined Shewhart-CUSUM scheme. The frequency of false alarm is higher for the smaller value of in-control ARL and more observations are required to detect the shift in the process if higher in-control ARL is selected therefore Quality control practitioner has to select the parameters of the scheme after considering the false alarm cost as well as the cost of defective production carefully. If a company is already using CUSUM control charts to monitor normally distributed data then it is not hard to implement the proposed scheme. Scheme parameters (value of k and h) and out of control ARL to detect δ standard deviation shifts are provided in Table 4, where $\delta = 0.01$, 0.02, 0.03............, and in-control ARL values are 200, 350 and 500, which may be helpful to the quality control practitioners in designing the Combined Shewhart-CUSUM when data is highly skewed according to the required efficiency of the process. If we use traditional charts for heavy tails distributed observations then false alarms are more than expected.

Table 4. Combined Shewhart-CUSUM scheme parameters and out of control ARL to detect δ standard deviation shift.

	ARL_0=200			ARL_0=350			ARL_0 = 500		
δ	K	h	ARL_1	k	h	ARL_1	k	h	ARL_1
0.1	0.0435	3.2571	99.4293	0.0435	4.1996	137.7145	0.0435	4.9617	170.4245
0.2	0.0842	2.7408	59.5500	0.0842	3.4181	77.7037	0.0842	3.9388	90.4675
0.3	0.1225	2.3853	40.7438	0.1225	2.9144	50.9580	0.1225	3.3104	57.7267
0.4	0.1588	2.1249	30.1127	0.1588	2.5608	36.4323	0.1588	2.8821	40.9529
0.5	0.1932	1.9251	23.4759	0.1932	2.2975	27.8805	0.1932	2.5692	30.9173
0.6	0.2260	1.7664	19.0841	0.2260	2.0928	22.2638	0.2260	2.3295	24.4584
0.7	0.2574	1.6369	15.8800	0.2574	1.9286	18.4261	0.2574	2.1390	20.0585
0.8	0.2874	1.5288	13.6215	0.2874	1.7934	15.6189	0.2874	1.9835	16.9201
0.9	0.3163	1.4371	11.8836	0.3163	1.6799	13.4546	0.3163	1.8538	14.4911
1.0	0.3441	1.3581	10.4948	0.3441	1.5829	11.8051	0.3441	1.7437	12.6652
1.1	0.3709	1.2893	9.3984	0.3709	1.4991	10.5125	0.3709	1.6488	11.2464
1.2	0.3968	1.2286	8.5175	0.3968	1.4257	9.4447	0.3968	1.5661	10.0765
1.3	0.4219	1.1746	7.7876	0.4219	1.3607	8.6060	0.4219	1.4932	9.1420
1.4	0.4461	1.1263	7.1533	0.4461	1.3028	7.8783	0.4461	1.4284	8.3452
1.5	0.4697	1.0827	6.6388	0.4697	1.2508	7.2684	0.4697	1.3703	7.6919
1.6	0.4926	1.0431	6.1823	0.4926	1.2038	6.7626	0.4926	1.3179	7.1225
1.7	0.5149	1.0069	5.8123	0.5149	1.1610	6.3090	0.5149	1.2704	6.6521
1.8	0.5366	0.9737	5.4766	0.5366	1.1219	5.9308	0.5366	1.2270	6.2187
1.9	0.5577	0.9432	5.1995	0.5577	1.0860	5.5823	0.5577	1.1872	5.8549
2.0	0.5783	0.9149	4.9401	0.5783	1.0528	5.2991	0.5783	1.1505	5.5387

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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