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Optimal Solution of Structural Engineering Design Problems using Crow Search Algorithm

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Abstract
In real world, the structural engineering design problems are large scale non-linear constrained problems. In the present study, crow search algorithm (CSA) is applied to find the optimal solution of structural engineering design problems such as pressure vessel design problem, welded beam design problem and tension/ compression string design problem. The numerical results are compared with the existing results reported in the literature including metaheuristic algorithms and it is found that the results obtained by the crow search algorithm are better than other existing algorithms. Further, the effectiveness of the algorithm is verified to be better than the existing algorithms by statistical analysis using mean, median, best case, and worst case scenarios. The present study confirms that the crow search algorithm may be easily and effectively applied to various structural design problems.

Keywords- Metaheuristic algorithm, Crow search algorithm, Constrained optimization.

1. Introduction

In everyday life, we come across various problems for which we need to find the optimum solution. Design optimization is the process of finding the optimal values of various parameters defining the problem, which provides the best value of the objective function. Depending upon the problem, the objective function can be as simple as a single variable linear function to very complex multi variable non-linear function. Generally, the objective function is accompanied by various constraints related to the nature of the problems. The number of decision variables and constrains can vary from a very small number to a very large number.

Mathematically, the most general optimization problem can be defined as:

Maximize/Minimize $f(\vec{X})$ where $\vec{X} = (x_1, x_2, x_3, ..., x_n)$
subject to: $g_k(\vec{X}) \geq 0,\ k = 1,2,3,..., m$ and $h_j(\vec{X}) = 0,\ j = 1,2,3,..., p.$
where f, g and h are non-linear real valued functions.

Structural design problems include designing a product in most optimal way such that it satisfies given constraints. There can be a large number of feasible solutions for the given problem and an appropriate optimization technique is used to find the most optimal solution. Though some of these problems can be solved using traditional methods but a lot of problems are non-linear and include mixed design variables under complex constraints, due to which traditional methods fail (Michalewicz, 1994).

For such problems, Metaheuristic algorithms (Lee and Geem, 2005), which have been found effective for global optimization, are applied (Yang, 2011). These Metaheuristic algorithms generally have their inspiration from processes in nature (Holland, 1975; Glover, 1986). These algorithms have been able to overcome the drawbacks and restrictions faced by traditional numerical methods. Some of the examples of Metaheuristic algorithms developed and used in past have been, Genetic Algorithms (GA) (Michalewicz, 1994; Deb, 2000; Coello and Montes, 2002), Particle Search Optimization (PSO) (Cagnina et al., 2008; Dos Santos Coelho, 2010; Kennedy and Eberhart, 1995; Rao, 1996), Cuckoo Search (Yang and Deb, 2009), Harmony Search (Geem et al., 2001). A lot of people have developed and applied various Metaheuristic algorithms to such structural design problems. Many authors such as Deb (1991), Coello (2000), Dimopoulos (2007), Hwang and He (2006) used Genetic Algorithms to solve various problems. An advanced version of differential evolution algorithms have been applied by Montes et al. (2007). Particle Search Optimization (PSO) has been used by Shi and Eberhart (1998), He et al. (2004). Moreover, Harmony Search algorithms have been used by Lee and Geem (2005) for solving design optimization problems. Artificial bee colony algorithm has been used by Garg (2014) for solving structural engineering design optimization problem. Further, two challenging engineering design problems are solved using salp swarm algorithm by Mirjalili et al. (2017). In the present study, nature inspired algorithm known as the crow search algorithm (Zolghadr and Bozorg, 2018) has been applied for solving the structural engineering design problems. Crows are considered one of the intelligent organisms who can remember things. Crows are also smart; they can fool other crows who are following them by deviating from their paths and following a new path but at the same time remembering their previous path. These characteristics of crows have been utilized to search for the next optimal solution while remembering the previous best solution and in this way reaching the global optimal solution.

The present paper is organized as follows: in Section 2, the crow search algorithm is presented. In Section 3, the mathematical model of three structural engineering design problems and their numerical results are presented. The paper closes with conclusion in Section 4.

2. Algorithm

The crow search algorithm (CSA) is a metaheuristic optimization algorithm. This algorithm was initially introduced by Askarzadeh (2016). It is based on intelligent behavior of crow flocks. CSA attempts to imitate the social intelligence of crow flock and their food gathering process. A crow individual has a tendency to tap in to the food resources of other species, including the other crow members of the flock. In fact each crow attempts to hide its excess food in a hideout spot and retrieve the stored food in the time of need. However the other members of the flock, which have their own food reservation spots as well, try to tail one another to find these hidings spots and plunder the stored food. In the standard CSA, the flock of crows spread and search throughout the decision space for the perfect hideout spots (global optima). Subsequently, each crow individual shall make a motion based on the two basic principles of the CSA: (i) protecting its own hideout spot and (ii) detecting the other members' hideout spot (Zolghadr and Bozorg, 2018). The stepwise procedure is given below:

(i) Define decision variables, objective function and constraints.
(ii) Define adjustable parameters. The adjustable parameters include flock size (N), maximum number of iterations (maxit), flight length (f_1) and awareness probability (AP).

International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 4, 968–981, 2019
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(iii) Initialize the position of crows. Using random function, N crows are randomly positioned in a d dimensional search space. Each crow denotes a feasible solution of the problem and d is the number of decision variables.
(iv) Initialize and record the memory of each crow. Since at the initial iteration, the crows have no experience, it is assumed that they have hidden their food at their initial position.
(v) Check the feasibility of the initial locations generated. Check if they satisfy constraints. Reinitialize the locations till it reaches a set of locations that satisfy the constraints giving a feasible solution.
(vi) Randomly select one crow (called *j*), and other crows would follow this crow to update their locations using equation
$X_{(i,iter+1)} = X_{(i,iter)} + r_{(i)} \times f_{1(i,iter)} \times (m_{(j,iter)} - x_{(i,iter)}) \quad if \quad r(j) \geq AP(i,iter)$.
Else generate a random (position) feasible solution in search space if $r(j) < AP(i,iter)$, where $r(j)$ is a random number with uniform distribution between 0 and 1 and $AP(i,iter)$ denotes the awareness probability of crow *j* at iteration iter.
(vii) Check the feasibility of the new positions of each crow in decision space. If not feasible, generate a feasible random solution.
(viii) If the new position is feasible, the crow updates its position. Otherwise, the crow stays in the current position and does not move to the new generated position.
(ix) Evaluate the new position of the crows. The fitness function value for the new position of each crow is computed. If the fitness (objective function) value of the new position is better than the fitness function value of the memorized position, the crow updates its memory by new position.
(x) Reiterate till maximum number of input iterations. When termination criterion is satisfied, the best position of the memory in terms of the objective function value is reported as the solution of the optimization problem.

3. Structural Optimization Problems

The crow search algorithm has been applied on various structural engineering design problems, including their different versions.

3.1 Design of Pressure Vessel

It consists of a compressed air storage tank having a volume of 750 ft^3 and pressure of 2000 psi. Hemispherical heads are placed at both ends. The cylindrical shape is formed by two halves of longitudinal welds. We have to minimize the total cost, which includes the cost of the material, forming process and welding process (Kannan and Kramer, 1994). The variables in the problem are thickness of pressure vessel x_1, thickness of head x_2, inner radius x_3 of the vessel and the length x_4 of the vessel excluding the head.

The fitness function is given by:

Minimize $f(X) = 0.6224x_1x_2x_3 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$

subject to:

$g_1(X) = -x_1 + 0.0193x_3 \leq 0,\ \ g_2(X) = -x_2 + 0.00954x_3 \leq 0,$

$g_3(X) = -\pi x_3^2 x_4 - (4/3)\pi x_3^3 + 1296000 \leq 0,\ g_4(X) = x_4 - 240 \leq 0.$

International Journal of Mathematical, Engineering and Management Sciences
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Version 1: Design of pressure vessel problem has been solved by various authors in the following bounded region:

Search Space: $1\times 0.0625 \le x_1, x_2 \le 99\times 0.0625;\ 10 \le x_3, x_4 \le 200.$

The optimal solution in this bounded region using the crow search algorithm is:
$X = (0.7784145664935,\ 0.3847801710354,\ 40.3319303998310, 199.8303931318342)$
and optimum (minimum) fitness function value: $f(X) = 5596.18641830762.$

Constraint values:

$G = (g_1, g_2, g_3, g_4) = (-0.53319600, -2.33772754, -20516404.40392017, -118.87315251).$

The result obtained using the proposed algorithms are better than any of the existing results. A comparison of the same is given in the Table 1.

Version 2:
The upper bound of the fourth variable (length of the vessel) is updated to 240 i.e. $10 \le x_4 \le 240$ [Dimopoulos (2007)]. Rest all the things remain the same.

$X = (0.7280931726201,\ 0.3599225433937,\ 37.7241665248285,\ 239.5857630173754).$
Optimum (minimum) fitness function value: $f(X) = 5501.57955042116$.

Constraints value:
$G = (g_1, g_2, g_3, g_4) = (-1.86901631,\ -2.63361733,\ -25611830.76570778,\ -173.16317732).$

The result for version 2 are also illustrated in Table 1 with comparison to other existing algorithms. Further, statistical analysis has been done and a comparison with other existing algorithms has been shown in the Table 2.

The comparisons of the simulated results are shown in the Table 1 and the corresponding statistical results are summarized in Table 2. The results obtained by crow search algorithm are better than any other solutions reported in the literature.

International Journal of Mathematical, Engineering and Management Sciences
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Table 1. Comparison of the best solution for pressure vessel design problem obtained by different methods

Version	Method	Design Variables				Cost
		x_1	x_2	x_3	x_4	$f(x)$
I	Sandgren (1988)	1.125000	0.625000	47.700000	117.701000	8129.1036
	Kannan and Kramer (1994)	1.125000	0.625000	58.291000	43.690000	7198.0428
	Deb (1997)	0.937500	0.50000	48.329000	112.67900	6410.3811
	Coello (2000)	0.812500	0.437500	40.323900	200.00000	6288.7445
	Coello and Montes (2002)	0.812500	0.437500	42.097398	176..654050	6059.946
	He and Wang (2007)	0.812500	0.437500	42.091266	176.746500	6061.0777
	Montes and Coello (2008)	0.812500	0.437500	42.098087	176.640518	6059.7456
	Kaveh and Talatahari (2009)	0.812500	0.437500	42.103566	176.573220	6059.0925
	Kaveh and Talathari (2010)	0.812500	0.437500	42.098353	176.637751	6059.7258
	Zhang and Wang (1993)	1.125000	0.625000	58.290000	43.6930000	7197.7000
	Cagnina et al. (2008)	0.812500	0.437500	42.098445	176.6365950	6059.714335
	Dos Santos Coelho (2010)	0.812500	0.437500	42.098400	176.6372000	6059.7208
	He et al. (2004)	0.812500	0.437500	42.098445	176.6365950	6059.7143
	Lee and Geem (2005)	1.125000	0.625000	58.278900	43.75490000	7198.433
	Montes et al. (2007)	0.812500	0.437500	42.098446	176.6360470	6059.701660
	He et al. (2003)	0.812500	0.437500	42.098450	176.6366000	6059.131296
	Gandomi et al. (2003)	0.812500	0.437500	42.0984456	176.6365958	6059.7143348
	Akay and Karaboga (2012)	0.812500	0.437500	42.098446	176.636596	6059.714339
	Garg (2014)	0.7781977	0.3846656	40.3210545	199.9802367	5885.4032828
	Present Study	**0.778414566493 5**	**0.38478017 10354**	**40.33193039 98310**	**199.830393131 8342**	**5596.186418**
II	Dimopoulos (2007)	0.75	0.375	38.86010	221.36549	5850.38306
	Mahdavi et al. (2007)	0.75	0.375	38.86010	221.36553	5849.76169
	Hedar and Fukushima (2006)	0.7683257	0.3797837	39.8096222	207.2255595	5868.764836
	Gandomi et al. (2011a)	0.75	0.375	38.86010	221.36547	5850.38306
	Garg (2014)	0.7275958	0.3596552	37.6991359	239.999805	5804.448670
	Present Study	**0.728093172620 1**	**0.35992254 33937**	**37.72416652 48285**	**239.585763017 3754**	**5501.57955**

Table 2. Statistical results of different methods for pressure vessel problem

Version	Method	Best	Mean	Worst	Std. Dev.	Median
I	Sandgren (1998)	8129.1036	NA	NA	NA	NA
	Kannan and Kramer (1994)	7198.0428	NA	NA	NA	NA
	Deb (1997)	6410.3811	NA	NA	NA	NA
	Coello (2000)	6288.7445	6293.8432	6308.1497	7.4133	NA
	Coello and Montes (2002)	6059.9463	6177.2533	6469.3220	130.9297	NA
	He and Wang (2007)	6061.0777	6147.1332	6363.8041	86.4545	NA
	Montes and Coello (2008)	6059.7456	6850.0049	7332.8798	426.0000	NA
	Kaveh and Talatahari (2009)	6059.7258	6081.7812	6150.1289	67.2418	NA
	Kaveh and Talatahari (2010)	6059.0925	6075.2567	6135.3336	41.6825	NA
	Gandomi et al. (2003)	6059.714	6447.7360	6495.3470	502.693	NA
	Cagnina et. al. (2008)	6059.714335	6092.0498	NA	12.1725	NA
	Dos Santos Coelho (2010)	6059.7208	6440.3786	7544.4925	448.4711	6257.5943
	He et al. (2004)	6059.7143	6289.92881	NA	305.78	NA
	Akay and Karaboga (2012)	6059.714339	6245.308144	NA	205	NA
	Garg (2014)	5885.40328280	5887.5570240	5895.126804	2.745290	5886.149289
	Present Study	**5596.18642**	**5599.28868**	**5648.15212**	**11.5203421**	**5596.44649**
II	Dimopoulos (2007)	5850.38306	NA	NA	NA	NA
	Mahdavi et al. (2007)	5849.7617	NA	NA	NA	NA
	Hedar and Fukushima (2006)	5868.764836	6164.585867	6804.328100	257.473670	NA
	Gandomi et al. (2011a)	5850.38306	5937.33790	6258.96825	164.54747	NA
	Garg (2014)	5804.4486708	5805.4739140	5811.977127	1.41146216	5805.0737979
	Present Study	**5501.57955**	**5504.44726**	**5524.91931**	**5.79839866**	**5502.342012**

International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 4, 968–981, 2019
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3.2 Design of Welded Beam

The objective of this design problem is to minimize the cost of fabrication of welded beam given the constraints on shear stress, bending stress in the beam, buckling load on the bar, end deflection of the beam and side constraints. The variables in the problem are thickness of the weld $h = x_1$, length of joint $l = x_2$, width of the beam $t = x_3$, thickness of beam $b = x_4$. The problem has been taken from Rao (1996).

Version 1: The fitness function is given by:

Minimize $f(X) = 1.1047x_1^2x_2 + 0.04811x_3x_4(14 + x_2)$

subject to:

$$g_1(X) = T(X) - T_{\max} \le 0,\ g_2(X) = \sigma(X) - \sigma_{\max} \le 0,\ g_3(X) = x_1 - x_4 \le 0,$$
$$g_4(X) = 0.125 - x_1 \le 0, g_5(X) = \delta(X) - 0.25 \le 0, g_6(X) = P - P_c(X) \le 0,$$
$$0.1 \le x_1 \le 2;\ 0.1 \le x_2 \le 10;\ 0.1 \le x_3 \le 10;\ 0.1 \le x_4 \le 2.$$

where shear stress within the weld is given by T, allowable shear stress which is equal to 13600 psi is given by $T_{\max}$, the normal stress within the beam which is equal to 30000 psi is given by $\sigma_{\max}$, whereas the buckling load of the bar is given by P_c, the load equal to 6000 lb is given by P and the end deflection is noted by δ.

$$T(X) = T_1^2 + 2T_1T_2\left(\frac{x^2}{2R}\right) + r_2^2 \text{ where } T_1 = \frac{P}{\sqrt{2}x_1x_2},\ T_2 = \frac{MR}{J},$$

where $M = P\left(L + \frac{x_2}{2}\right)$, $J(X) = 2\left[\frac{x_1x_2}{\sqrt{2}}\left\{\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right\}\right]$, $R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$,

$$\sigma(X) = \frac{6PL}{x_4x_3^2},\ \delta(X) = \frac{4PL^3}{Ex_3^3x_4},\ P_c(X) = \frac{4.013\sqrt{\frac{EGx_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right).$$

$$G = 12\times10^6\ psi,\ \ E = 30\times10^6\ psi,\ \ P = 6000\,lb,\ \ L = 14\,inch.$$

The solution obtained for this problem using crow search algorithm:
$X = (0.244343277069188, 6.213952517706234, 8.296218570994581, 0.244345831688385)$.
Optimum (minimum) fitness function value: $f(X) = 2.38122873948154$.

Constraints value:

$$G = (g_1, g_2, g_3, g_4, g_5, g_6) = (-8402.2936827306, -23694.6260885614, -0.2081091708, -0.8905141940, -0.2466020386, -733939.8720985127).$$

International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 4, 968–981, 2019
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A comparison with other algorithms has been shown in the Table 3 and it found that the results obtained using proposed algorithms are better than existing results.

Version 2:
In this version a new constraint, along with the constraints used in version 1, has been added which includes deflection, polar moment of inertia and buckling load.

$$g_7(X) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0, \quad \delta(X) = \frac{6PL^3}{Ex_3^2x_4},$$

$$P_c(X) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right), \; J(X) = 2\left[\sqrt{2}x_1x_2\left\{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right\}\right].$$

The solution obtained for this problem using proposed algorithm:
$X = (0.205671231425633, 3.254535728596008, 9.036762134420712, 0.205729413100657)$.
Optimum (minimum) fitness function value: $f(X) = 1.69527215466093$.

Constraints values:
$G = (g_1, g_2, g_3, g_4, g_5, g_6, g_7) = (-2650.9695371295, -13840.0862642908, -0.4891852108,$
$-0.7480686955, -0.0838556961, -0.2342050396, -182819.6954371415)$.

A comparison with other algorithms has been shown in the Table 3 and it is found that the results obtained using proposed algorithms are better than other algorithms. Further, statistical analysis has been done for version 1 and version 2 and a comparison with other algorithms has been shown in Table 4.

International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 4, 968–981, 2019
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Table 3. Comparison of the best solution for welded beam design problem obtained by different methods (NA means not available)

Version	Method	Design Variables				$f(x)$
		x_1	x_2	x_3	x_4	
I	Ragsdell and Phillips (1976)	0.245500	6.196000	8.273000	0.245500	2.385937
	Rao (1996)	0.245500	6.196000	8.273000	0.245500	2.3860
	Deb (1991)	0.248900	6.173000	8.178900	0.253300	2.433116
	Deb (2000)	NA	NA	NA	NA	2.38119
	Ray and Liew (2003)	0.244438276	6.237967234	8.288576143	0.2445661820	2.3854347
	Lee and Geem (2005)	0.2442	0	0	0.2443	2.38
	Hwang and He (2006)	0.223100	6.2231	8.2915	0.2245	2.25
	Mehta and Dasgupta (2012)	0.24436895	1.5815	12.84680	0.24436895	2.3811341
	Garg (2014)	0.24436198	6.21767407	8.29163558	0.24436883	2.38099617
	Present Study	**0.244343277069188**	**6.213952517706234**	**8.296218570994581**	**0.244345831688385**	**2.381228739**
II	Coello (2000)	0.208800	3.420500	8.997500	0.210000	1.748309
	Coello and Montes (2002)	0.205986	3.471328	9.020224	0.206480	1.728226
	Hu et al. (2003)	0.20573	3.47049	9.03662	0.20573	1.72485084
	Hedar and Fukushima (2006)	0.205644261	3.472578742	9.03662391	0.2057296	1.7250022
	He and Wang (2007)	0.202369	3.544214	9.048210	0.205723	1.728024
	Dimopoulos (2007)	0.2015	3.5620	9.041398	0.205706	1.731186
	Mahdavi et al. (2007)	0.20573	3.47049	9.03662	0.20573	1.7248
	Montes et al. (2007)	0.205730	3.470489	9.036624	0.205730	1.724852
	Montes and Coello (2008)	0.199742	3.612060	9.037500	0.206082	1.73730
	Cagnina et al. (2008)	0.205729	3.470488	9.036624	0.205729	1.724852
	Fesanghary et al. (2008)	0.20572	3.47060	9.03682	0.20572	1.7248
	Kaveh and Talatahari (2009)	0.205729	3.469875	9.036805	0.205765	1.724849
	Kaveh and Talatahari (2010)	0.205700	3.471131	9.036683	0.205731	1.724918
	Gandomi et al. (2011a)	0.2015	3.562	9.0414	0.2057	1.73121
	Mehta and Dasgupta (2012)	0.20572885	3.47050567	9.03662392	0.20572964	1.724855
	Akay and Karaboga (2012)	0.205730	3.470489	9.036624	0.205730	1.724852
	Garg (2014)	0.20572450	3.25325369	9.03664438	0.20572999	1.69526388
	Present Study	**0.205671231425633**	**3.254535728596008**	**9.036762134420712**	**0.205729413100657**	**1.695272155**

Table 4. Statistical results of different methods for welded beam design problem

Version	Method	Best	Mean	Worst	Std. Dev.	Median
I	Regsdell and Phillips (1976)	2.385937	N A	N A	N A	N A
	Rao (1996)	2.3860	N A	N A	N A	N A
	Deb (1991)	2.433116	N A	N A	N A	N A
	Deb (2000)	2.38119	N A	N A	NA	N A
	Ray and Liew (2003)	2.3854347	3.2551371	6.3996785	0.9590780	3.0025883
	Lee and Geem (2005)	2.38	N A	N A	N A	N A
	Hwang and He (2006)	2.25	2.26	2.28	N A	N A
	Mehta and Dasgupta (2012)	2.381134	2.3811786	2.3812614	NA	2.3811641
	Garg (2014)	2.38099617	2.38108932	2.38146999	1.01227E-4	2.38107233
	Present Study	**2.38122874**	**2.38141637**	**2.38232915**	**0.00036676**	**2.381299106**
II	Coello (2000)	1.748309	1.771973	1.785835	0.011220	N A
	Coello and Montes (2002)	1.728226	1.792654	1.993408	0.07471	N A
	Dimopoulos (2007)	1.731186	N A	N A	N A	N A
	He and Wang (2007)	1.728024	1.748831	1.782143	0.012926	N A
	Hedar and Fukushima (2006)	1.7250022	1.7564428	1.8843960	0.0424175	N A
	Montes et. al. (2007)	1.724852	1.725	N A	1 E-15	N A
	Montes and Coello (2008)	1.737300	1.813290	1.994651	0.070500	N A
	Cagnina et al. (2008)	1.724852	2.0574	N A	0.2154	N A
	Kaveh and Talatahari (2010)	1.724918	1.729752	1.775961	0.009200	N A
	Kaveh and Talatahari (2009)	1.724849	1.727564	1.759522	0.008254	N A
	Gandomi et al. (2011a)	1.7312065	1.8786560	2.3455793	0.2677989	N A
	Mehta and Dasgupta (2012)	1.724855	1.724865	1.72489	N A	1.724861
	Akay and Karaboga (2012)	1.724852	1.741913	N A	0.031	N A
	Garg (2014)	1.69526388	1.69530842	1.69537060	2.836238E-5	1.69530879
	Present Study	**1.69527215**	**1.6953409**	**1.69551879**	**6.9996E-05**	**1.695323792**

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3.3 Design of Tension / Compression String

Tension/Compression string problem requires to optimize (minimize) the weight of the string and it has been given by Belegundu (1982) and Arora (1989). The string is constrained to minimum deflection, constrained diameter, shear stress etc. The variables that we have to optimize are coil diameter (x_1), the wire diameter (x_2) and the number of coil (x_3).

The fitness function is given by: Minimize $f(X) = (x_3 + 2)x_2 x_1^2$

$$\text{subject to}: g_1(X) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0,\ g_2(X) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^{4)}} + \frac{1}{5108 x_1^2} - 1 \le 0,$$

$$g_3(X) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0,\ g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \le 0.$$

$$0.05 \le x_1 \le 2;\ 0.25 \le x_2 \le 1.3;\ 2 \le x_3 \le 15.$$

The solution obtained for this problem using crow search algorithm:

$$X = (0.051686690341221, 0.356658764326658, 11.292540928654185).$$

Optimum (minimum) fitness function value: $f(X) = 0.012665248980102$.

Constraints value:

$$G = (g_1, g_2, g_3, g_4) = (-0.610136631957694, -0.093497586159253, -1.953085401792193, -0.623891361731687).$$

The results obtained using the proposed algorithms are found to be better than the other algorithms (See Table 5). Further, statistical analysis has been done and a comparison with other algorithms has been shown in the Table 6.

International Journal of Mathematical, Engineering and Management Sciences
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Table 5. Comparison of the best solution for tension/compression string design problem by different methods

Method	Design Variables			
	x_1	x_2	x_3	$f(x)$
Belegundu (1982)	0.05	0.315900	14.25000	0.0128334
Arora (1989)	0.053396	0.399180	9.185400	0.0127303
Coello (2000)	0.051480	0.351661	11.632201	0.01270478
Ray and Saini (2001)	0.050417	0.321532	13.979915	0.013060
Coello and Montes (2002)	0.051989	0.363965	10.890522	0.0126810
Ray and Liew (2003)	0.0521602170	0.368158695	10.6484422590	0.1266924934
Hu et al. (2003)	0.051466369	0.351383949	11.60865920	0.0126661409*
He et al. (2004)	0.05169040	0.35674999	11.28712599	0.0126652812*
Hedar and Fukushima (2006)	0.05174250340926	0.35800478345599	11.21390736278739	0.012665285
Raj et al. (2005)	0.05386200	0.41128365	8.68437980	0.01274840
Tsai (2005)	0.05168906	0.3567178	11.28896	0.01266523
Mahdavi et al. (2007)	0.05115438	0.34987116	12.0764321	0.0126706
Montes et al. (2007)	0.051688	0.356692	11.290483	0.012665
He and Wang (2007)	0.051728	0.357644	11.244543	0.0126747
Cagnina et al. (2008)	0.051583	0.354190	11.438675	0.012655
Zhang et al. (2008)	0.0516890614	0.3567177469	11.2889653382	0.012665233
Montes and Coello (2008)	0.051643	0.355360	11.397926	0.012698
Omran and Salman (2009)	0.0516837458	0.3565898352	11.2964717107	0.0126652375
Keveh and Talatahari (2010)	0.051865	0.361500	11.0000	0.0126432*
Dos Santos Coelho (2010)	0.051515	0.352529	11.538862	0.012665
Akay and Karaboga (2012)	0.051749	0.358179	11.203763	0.012665
Garg (2014)	0.051689156131	0.356720026419	11.288831695483	0.0126652327883
Present Study	**0.051686690341221**	**0.356658764326658**	**11.292540928654185**	**0.012665249**

*Infeasible solution as they violate one of the constraints set.

Table 6. Statistical results of different methods for tension / compression string problem (NA means not available)

Method	Best	Mean	Worst	Std. Dev.	Median
Belegundu (1982)	0.0128334	NA	NA	NA	NA
Arora (1989)	0.0127303	NA	NA	NA	NA
Coello (2000)	0.01270478	0.01276920	0.01282208	3.9390×10^{-5}	0.01275576
Ray and Saini (2001)	0.0130600	0.015526	0.018992	NA	NA
Coello and Montes (2002)	0.0126810	0.012742	0.012973	5.9000×10^{-5}	NA
Ray and Liew (2003)	0.01266924934	0.012922669	0.016717272	5.92×10^{-4}	0.012922669
Hu et al. (2003)	0.0126661409	0.012718975	NA	6.446×10^{-5}	NA
He et al. (2004)	0.0126652812	0.01270233	NA	4.12439×10^{-5}	NA
He and Wang. (2007)	0.0126747	0.012730	0.012924	5.1985×10^{-5}	NA
Zhang et al. (2008)	0.012665233	0.012669366	0.012738262	1.25×10^{-5}	NA
Hedar and Fukushima (2006)	0.012665285	0.012665299	0.012665338	2.2×10^{-8}	NA
Montes et al. (2007)	0.012665	0.012666	NA	2.0×10^{-6}	NA
Montes and Coello (2008)	0.012698	0.013461	0.164850	9.6600×10^{-4}	NA
Cagnina et al. (2008)	0.012665	0.0131	NA	4.1×10^{-4}	NA
Kaveh and Talatahari (2010)	0.0126432	0.012720	0.012884	3.4888×10^{-5}	NA
Omran and Salman (2009)	0.0126652375	0.0126652642	NA	NA	NA
Dos Santos Coelho (2010)	0.012665	0.013524	0.017759	0.001268	0.012957
Akay and Karaboga (2012)	0.012665	0.012709	NA	0.012813	NA
Garg (2014)	0.0126652327883	0.0126689724845	0.012710407729	9.429426E-6	0.12665314728
Present Study	**0.01266525**	**0.01266604**	**0.01267091**	**1.5293E-06**	**0.012665449**

4. Conclusions

A crow search algorithm is applied to various structural engineering design problems such as Pressure vessel design optimization, Welded beam design optimization, Tension/Compression string design optimization. The numerical results are compared with the various existing optimization algorithms including metaheuristic algorithms and it is found that the results obtained by the crow search algorithm are better than others. Further, the effectiveness of the algorithm is verified to be better than the existing algorithms by statistical analysis using mean, median, best case, worst case scenarios and it is observed that crow search algorithm is better than other existing algorithms. It is also remarked that the standard deviation has been found to be pretty less than that by others method. The present study confirms that the crow search algorithm may be easily and effectively applied to various structural engineering design problems.

On observing the performance of crow search, this paper opens doors for new research directions in crow search algorithm. The power of the algorithm suggests that the crow search should be implemented in integer optimization problems and multi-objective optimization problems. Further, it can also be parallelized for solving large scale problems.

Conflict of Interest

The author confirms that there is no conflict of interest to declare for this publication.

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