

## Signature of $A$ -Within- $B$ -From- $D/G$ Sliding Window System

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### Abstract

In this study, we have proposed a model of the sliding window coherent system in case of multiple failures. The considered model consists of  $G$  linearly required multi-state elements and  $G$  number of parallel elements in  $A$ -within- $B$ -from- $D/G$  for each multi-state. The system fails if at least  $A$  group elements out of  $B$  consecutive of  $D$  consecutive multi-state elements have performance lower than the weight  $w$ . We have evaluated the signature reliability, expected value and system sensitivity on the basis of the extended universal generating function of the considered system.

**Keywords-** Sliding window system, Universal generating function, Signature reliability, Sensitivity.

### 1. Introduction

In the real life situation binary state system (BSS) depends on mainly two states namely completely working or total failure. To compute the reliability of any binary system many algorithms have been used including universal generating function (UGF). Levitin (2005) analyzed the computation of reliability of different binary and consecutive  $A$ -out-of- $G$  systems by UGF. Levitin and Ben-Haim (2011) computed the reliability of the consecutive sliding window system (SWS) which have many possible states namely total failure and completely working using UGF algorithm. Sun et al. (2012) obtained the optimal solution for a transportation system by the analytical method. Ram (2013) discussed the survey of reliability evaluation of engineering system by various methods. Xiao et al. (2014) considered a  $B$ -gap-consecutive  $A$ -out-of- $D$ -from- $G:F$  system and computed the reliability for various elements causing failure. Negi and Singh (2015) studied the non-repairable complex system which has two binary subsystems namely weighted  $A$ -out-of- $G:G$  and weighted  $l$ -out-of- $b:G$  and evaluated the reliability, mean time to failure and sensitivity using UGF.

It is well known fact that many engineering systems are not binary but they are a multi-state and multi-state system (MSS) are in general more reliable than BSS. MSS is based on the performance rate of the system. Further, Multi-state SWS is widely used in oil pipeline system, telecommunication system, mobile communication system, radar detection, quality control system etc. Levitin (2002) evaluated the reliability of linear multi-state SWS with multi-state elements using UGF technique. Levitin (2003) also studied linear multi-state SWS, which have  $G$  linear multi-state elements and evaluated the reliability of common supply failures in the system. Habib et al. (2007) generalized linear consecutive  $A$ -out-of- $D$ -from- $G:G$  system having multi-state elements. They also have calculated the reliability of the system, which consists of  $G$

linearly required multi-state elements. Yueqin et al. (2010) computed the reliability of multi-state  $A$ -out-of- $G$ : $G$  system using UGF algorithm. Xiang and Levitin (2012a, 2012b) proposed a linear multi-state SWS model which consists of  $G$  linearly required multi-state elements and evaluated the reliability of the combination of  $B$ -consecutive and  $A$ -out-of- $G$  SWS. Levitin and Dai (2012) evaluated the reliability of  $A$ -out-of- $G$  SWS. They proposed new linear multi-state SWS in case of multiple failures in which system is failed if the performance rate is less than total demand. Xiang et al. (2013) evaluated the optimal solution of multi-state  $A$ -out-of- $G$  of consecutive SWS and computed the system reliability with the application of the genetic algorithm. Faghieh-Roohi et al. (2014) discussed the availability and capacity of a dynamic system for multi-state weighted  $A$ -out-of- $G$  system and given optimal solution by UGF and genetic algorithm. Li et al. (2014) analyzed the reliability of the multi-state system using UGF approach. Yu et al. (2014) evaluated the availability of a repairable multi-state system on the basis of the UGF and stochastic process. Xiao et al. (2015) studied the reliability of multi-state elements of the SWS having multiple failures, which have performance smaller than the allocation and optimized the system availability and cost analysis. Peng et al. (2017) considered a multi-state system having performance sharing groups of limited size and determined the reliability of the series multi-state system with the help of UGF technique.

Further, in the context of the signature reliability of the coherent system is widely used to calculate the expected lifetime of any kind of system with independent and identically distributed (i.i.d.) elements. Navarro et al. (2007) introduced the family of univariate distribution. They computed the minimal and the maximal signature of a coherent system along with distribution, bounds and moments of lifetime distribution. Samaniego (2007) discussed the signature of different systems and applied signature in many engineering fields. Bhattacharya and Samaniego (2008) appraised the optimal arrangement of the element in the coherent system and evaluated the optimal solution of parallel and series-parallel systems. Navarro and Hernandez (2008) studied the mean residual lifetime functions of the finite mixture, ordering properties and limiting behaviors. They evaluated the meantime and signature of the coherent system. Navarro and Rubio (2009) computed the signature reliability and expected a lifetime of the coherent system with  $n$  elements. Eryilmaz (2010) evaluated the reliability of consecutive  $k$ -system with some exchangeable element with the help of order stochastic of mixture representation. Navarro and Rychlik (2010) compared the expected lifetime of different systems and estimated the lifetime of i.i.d. elements in the lower and upper form. Mahmoudi and Asadi (2011) considered a coherent system and studied the dynamic signature with different properties of the signature. Marichal and Mathonet (2013) evaluated the weighted mean in case of an independent continuous lifetime and obtained the signature reliability of extension dependent lifetime of the coherent system. Eryilmaz (2012) determined the signature of a coherent system with the repairable element and calculated the expected lifetime for systems like linear consecutive  $A$ -within- $B$ -out-of- $G$ : $F$  and  $B$ -consecutive  $A$ -out-of- $G$ : $F$ . Da et al. (2012) computed the signature of the coherent system which decomposed into two or more subsystems and also using the redundancy of the backup system. Marichal and Mathonet (2013) evaluated the reliability function, signature, tail signature of the coherent system from the diagonal section by derivatives and with the help of structure function. Da Costa Bueno (2013) obtained the structure function of the multi-state monotone system by using a decomposition of multi-state systems. Franko and Tutuncu (2016) computed the reliability of repairable weighted  $A$ -out-of- $G$ : $G$  system in case of signature. Kumar and Singh (2017a, 2017b) studied the complex  $A$ -out-of- $G$  coherent system and sliding window coherent system (SWCS) with i.i.d. elements and calculated various reliability measures such as signature, mean time to failure (MTTF), Barlow-Proschan index using UGF technique.

It is clear from the above discussions that many researchers computed the reliability, MTTF, cost of binary and multi-state systems with various techniques, but the signature of SWCS is yet to be studied. Keeping this fact in view, in the present work we propose to study the  $A$ -within- $B$ -from- $D/G$  SWCS with  $G$  parallel i.i.d. elements consisting of the multi-state element (MSE) of the system. In this study, we have used UGF and Owen's method to estimate the different characteristics such as signature, tail signature, sensitivity, Barlow-Proschan index and expected lifetime having structure or reliability function.

*Nomenclature*

$G$	MSE in the system
$D$	consecutive elements in a group
$w$	weight for a group of $D$ consecutive multi element (ME)
$b_j$	random performance of ME $j$
$E_a$	multi-state element $a$
$B$	consecutive groups in the considered system
$A$	failure groups within $B$ consecutive groups
$s_A$	signature of the $A$ -within- $B$ -from- $D/G$ SWCS with $A$ elements
$T$	lifetime of system
$R/H$	reliability/reliability function of the $A$ -within- $B$ -from- $D/G$ SWCS
$U_a(z)$	UGF of $E_a$
$\tilde{U}_a(z)$	UGF of modified failure counter of $N_a$
$c_{a,i}$	the vector of size $B$ consecutive groups of $E_a$ in state $i$
$g_{a,i}$	state performance of elements of $E_a$ in state $i$
$F / \bar{S} / S$	failure probability/tail signature/sensitivity of the $A$ -within- $B$ -from- $D/G$ SWCS
$E(T)$	expected lifetime of the system elements
$C / \bar{p}_i$	minimal signature/probability function of the $A$ -within- $B$ -from- $D/G$ SWCS
$R_{a,i}$	the probability of the $A$ -within- $B$ -from- $D/G$ SWCS with $E_a$ in state $i$

**2. Assessment of Signature Reliability of  $A$ -Within- $B$ -From- $D/G$ SWCS (Xiao et al., 2015).**

Consider an  $A$ -within- $B$ -from- $D/G$  SWCS which contains  $G$  ordered MSE in which every element consists of  $G$  number of parallel elements. The failure element of the  $A$ -within- $B$ -from- $D/G$  system is presented when at least  $A$ -out of  $B$ -consecutive groups of  $D$  consecutive elements are greater than supply  $w$ . If the performance of  $D$  consecutive elements is less than weight  $w$  and a consecutive element consists of MSE, then the system fails if  $\sum_{j=a}^{a+r-1} b_j < w$ , where  $b_j$  is the random performance of MSE  $E_j$ . Consider a set of  $B$  consecutive elements from MSE ( $B + D - 1$ ) consisting of MSE  $E_a, E_{a+1}, \dots, E_{B+a+r-1}$ . The system fails if events are lower than  $A$  groups

within  $B$  consecutive groups which can be expressed as  $\sum_{e=a}^{a+B-1} I\left(\sum_{j=a}^{e+r-1} b_j < w\right) < a$ , where  $I(z)$  is an indicator function defined as

$$I(z) = \begin{cases} I(z) = 1, & \text{if } z \text{ is true} \\ I(z) = 0, & \text{otherwise} \end{cases}$$

Further, the system reliability of the sets of  $B$  consecutive groups of  $D$  consecutive MSE  $E_1, E_2, \dots, E_{G-D+B+2}$  can be written as

$$R = P\left\{\prod_{a=1}^{G-D-B+2} \left[ I\left(\sum_{e=a}^{a+B-1} \left[ I\left(\sum_{j=e}^{e+D-1} b_j < w\right) \right] < A\right) \right] = 1\right\} \quad (1)$$

From equation (1), we can evaluate the signature of the system having i.i.d. elements as  $s_A = p_D(T_s = T_{AG})$ , where  $T$  is system lifetime and  $s_A$  is the probability of the system failure. Boland (2001) obtained structure function  $R$  of the system having i.i.d. elements as

$$s_A = \frac{1}{\binom{G}{G-A+1}} \sum_{a \subseteq [G]} \phi(R) - \frac{1}{\binom{G}{G-A}} \sum \phi(R) \quad (2)$$

### 3. Evaluating the Failure Reliability of A-Within-B-From-D/G Failure Groups (Xiao et al., 2015)

The failure element of the A-within-B-from-D/G system is discussed as if at least A-groups of B-consecutive groups of D consecutive elements are not less than supply  $w$ .

UGF of the failure element of the system with probability  $R_{a,i}$  and state performance  $g_{a,i}$  is given by

$$U_a(z) = \sum_{i=1}^{A_a} R_{a,i} \cdot z^{g_{a,i}} \quad (3)$$

Now, with the help of equation (3), the UGF of the groups  $N_a$ ,  $a \geq B$  can be expressed as

$$\tilde{U}_a(z) = \sum_{i=1}^{A_a} R_{a,i} \cdot z^{c_{a,i} \cdot g_{a,i}} \quad (4)$$

where,  $c_{a,i}$  is a vector which belongs to  $\{0,1\}$  and shows the working state of groups  $N_{a-B+j}$ .

Further, the UGF of the group  $N_{a+1}$ ,  $a \geq B$  using operator  $\underset{\leftarrow}{\otimes}$  can be evaluated as

$$\begin{aligned} \tilde{U}_{a+1}(z) &= \underset{\leftarrow}{\otimes}(\tilde{U}_a(z), u_{a+D}(z)) \\ &= \sum_{i=1}^{B_a} \sum_{l=1}^{L_{a+D}} (R_{a,i} \cdot p_{a+D,l}) z^{\rho(c_{a,i}, g_{a+D,l}) \cdot \phi(g_{a,i}, g_{a+D-1,l})} \end{aligned} \quad (5)$$

where,

$$\phi(g_{a,i}, g_{a+D,l}) = \{g_{a,i}(2), g_{s,i}(3), \dots, g_{a,i}(r), g_{a+D,l}\} \quad (6)$$

and

$$\rho(c_{a,i}, g_{a+D-1,l}) = \{c_{a,i}(2), c_{a,i}(3), \dots, c_{a,i}(B), I(\delta(\phi(c_{a,i}, c_{a+D-1,l})) < w)\} \quad (7)$$

Now, the failure probability  $F_i$  of  $A$ -within- $B$ -from- $D/G$  SWCS if at least  $A$  groups out of  $B$  consecutive groups of  $D$  fail can be calculated as

$$F = F_1 + F_2(1 - F_1) + \dots + F_{G-D-B+2} \prod_{i=1}^{G-D-B+1} (1 - F_i) \quad (8)$$

Using equation (8) one can compute the failure probability  $F_1$  of modified UGF of the group  $N_B$  as

$$F_1 = \theta(\tilde{U}_B(z)) = \sum_{i=1}^{M_B} R_{B,i} \cdot I(\delta(c_{B,i}) \geq A) \quad (9)$$

Further, we can assessment the failure probability of  $F_2(1 - F_1)$  and eliminate failure terms from  $\tilde{U}_B(z)$  and then compute  $U_B(z)$  by using the operator  $\varphi$  as

$$\begin{aligned} U_B(z) &= \varphi(\tilde{U}_B(z)) \\ &= \sum_{i=1}^{M_B} R_{B,i} \cdot I(\delta(c_{B,i}) < A) z^{c_{B,i} \cdot g_{B,i}} \end{aligned} \quad (10)$$

Evaluating  $\tilde{U}_{B+1}(z)$  with the help of an operator  $\underset{\leftarrow}{\otimes}$ , we have

$$\tilde{U}_{B+1}(z) = \underset{\leftarrow}{\otimes}(U_B(z), u_{B+D}(z)) \quad (11)$$

Similarly, one can evaluate the value of  $F_a \prod_{i=1}^{a-1} (1 - F_i)$  for  $a = 2, 3, \dots, G - D - B + 2$  from the equations (10-11).

#### 4. Algorithm for Evaluating Reliability of A-Within-B-From-D/G Siding Window System (Xiao et al., 2015)

Step 1. Construct UGF  $\tilde{U}_{B+1}(z)$  for  $ME_j$  and obtain  $U_1(z)$  with the help of an operator  $\Omega$ .

Step 2. Modify  $U_1(z)$  to  $\tilde{U}_1(z)$  and calculate the number of failure groups.

Step 3. For  $j = D + 1, \dots, D + B - 1$

Evaluate  $U_{j+1-D}(z) = \underset{\Leftarrow}{\otimes}(U_{j-D}(z), u_j(z))$  and collect the like terms.

Step 4. For  $j = D + B, \dots, G$

Eliminate failure terms from  $\tilde{U}_B(z)$  and compute  $\tilde{U}_{j-D}(z)$ .

Step 5. Find  $\tilde{U}_{j+1-D}(z) = \underset{\Leftarrow}{\otimes}(U_{j-D}(z), u_j(z))$  and collect the like terms.

Step 6. Calculate  $F = F + R(\tilde{U}_{j+1-D}(z))$ .

Step 7. Find system reliability  $R = 1 - F$ .

#### 4.1 Algorithm for Evaluating Signature of A-Within-B-From-D/G SWCS with Its Reliability Function

Step 1: Calculate the signature of the reliability function by (Boland, 2001)

$$s_l = \frac{1}{\binom{B}{B-l+1}} \sum_{\substack{H \subseteq [B] \\ |H|=B-l+1}} \varphi(H) - \frac{1}{\binom{B}{B-l}} \sum_{\substack{H \subseteq [B] \\ |H|=B-l}} \varphi(H) \quad (12)$$

and compute polynomial function of system  $H(p) = \sum_{j=1}^B C_j \binom{B}{j} p^j q^{G-j}$  where,

$$C_j = \sum_{i=B-j+1}^B s_i, \quad j = 1, 2, \dots, B.$$

Step 2: Evaluate the tail signature of the system, i.e.,  $(B+1)$ -tuple  $\bar{S} = (\bar{S}_0, \dots, \bar{S}_B)$  using

$$\bar{S}_l = \sum_{i=l+1}^B s_i = \frac{1}{\binom{B}{B-l}} \sum_{\substack{H \subseteq [B] \\ |H|=B-l}} \varphi(H) \quad (13)$$

Step 3: Calculate the reliability function with the help of Taylor expansion from polynomial function about  $x=1$  by

$$p(x) = x^B H\left(\frac{1}{x}\right) \quad (14)$$

Step 4: Find the tail signature of the system reliability function from equation (12) as (Marichal and Mathonet, 2013).

$$\bar{S}_l = \frac{(B-l)!}{B!} D^l p(1), \quad l = 0, \dots, B \quad (15)$$

Step 5: Determine the signature of the system using equation (14)

$$s = \bar{S}_{l-1} - \bar{S}_l, l = 1, \dots, B. \quad (16)$$

#### 4.2 Algorithm to Determine Expected Lifetime of A-Within-B-From-D/G System with Minimum Signature

Step 1: Evaluate the expected lifetime of i.i.d. element system, which is exponentially distributed with mean  $\mu$ .

Step 2: Calculate the minimum signature of the A-within-B-from-D/G system with the expected lifetime of the reliability function by using

$$H_T(t) = \sum_{i=1}^G C_i H_{li}(t) = \sum_{i=1}^G d_i H_{ii}(t) \quad (17)$$

where,  $H_{li}(t) = P_D(z_{li} > t)$  and  $H_{ii}(t) = P_D(z_{ii} > t)$  for  $i = 1, 2, \dots, n$ .

Step 3: Compute the expected lifetime  $E(T)$  of the systems, which have i.i.d. elements by (Navarro and Rubio, 2009).

$$E(T) = \mu \sum_{i=1}^G \frac{C_i}{i} \quad (18)$$

where,  $C_i (i = 1, 2, \dots, G)$  is a vector coefficient of minimal signature.

#### 4.3 Algorithm to Calculate Barlow-Proschan index of SWCS

Estimate the Barlow and Proschan (1975) index of the i.i.d. elements are given by its reliability function in equation (2) as (Shapley, 1953; Owen, 1975, 1988).

$$I_{BP}^{(i)} = \int_0^1 (\partial_i R)(x) dx = \int_0^1 (\partial_i H)(x) dx, i = 1, 2, \dots, G \quad (19)$$

where,  $R$  and  $H$  are structure and reliability functions of SWCS respectively.

#### 4.4 Algorithm for Determining Expected Value of Element X and Expected Cost Rate of System When Working Elements are Failed (Eryilmaz, 2012)

Step 1: Evaluate the amount of failed elements at the time of system failure from signature

$$E(X) = \sum_{i=1}^G i.s_i, i = 1, 2, \dots, G \quad (20)$$

Step 2: Calculate the  $E(X)$  and  $E(X)/E(T)$  of A-within-B-from-D/GSWCS with minimum signature.

#### 4.5 Sensitivity of A-Within-B-From-D/GSWCS

The sensitivity of reliability function is defined as the rate of change in output due to an input of the system. If  $R$  and  $\lambda$  are the reliability and parameter of the system respectively, then sensitivity  $S$  with the parameter is expressed as

$$S = \frac{\partial R}{\partial \lambda}$$

(21)

### 5. Illustration

Consider a 2-within-3-from-3/5 SWCS with supply  $w=4$ . Each of the MSE consists of 3 number of parallel elements having a failure and working performance 0 and 1,1,2,2,1.

The probability function  $\bar{p}_i (i = 1,2,3,4,5)$  for each inner parallel element is given by

$$\bar{p}_i = 1 - \prod_{B=1}^G (1 - R_{iB}).$$

$$\bar{p}_1 = 1 - \{(1 - R_{11})(1 - R_{12})(1 - R_{13})\} \quad (22)$$

$$\bar{p}_2 = 1 - \{(1 - R_{21})(1 - R_{22})(1 - R_{23})\} \quad (23)$$

$$\bar{p}_3 = 1 - \{(1 - R_{31})(1 - R_{32})(1 - R_{33})\} \quad (24)$$

$$\bar{p}_4 = 1 - \{(1 - R_{41})(1 - R_{42})(1 - R_{43})\} \quad (25)$$

$$\bar{p}_5 = 1 - \{(1 - R_{51})(1 - R_{52})(1 - R_{53})\} \quad (26)$$

Now, UGF of each MSE is obtained as

$$U_i(z) = \bar{p}_i z^i + (1 - \bar{p}_i) z^0 \quad (27)$$

where,  $\bar{p}_i$  is the probability function and  $z^i$  is the working rate and  $z^0$  failure rate.

Further, UGF of SWCS for each MSE  $\bar{p}_i (i = 1,2,3,4,5)$  of sliding window can be computed as

$$U_1(z) = \bar{p}_1 z^1 + (1 - \bar{p}_1) z^0 \quad (28)$$

$$U_2(z) = \bar{p}_2 z^1 + (1 - \bar{p}_2) z^0 \quad (29)$$

$$U_3(z) = \bar{p}_3 z^2 + (1 - \bar{p}_3) z^0 \quad (30)$$

$$U_4(z) = \bar{p}_4 z^2 + (1 - \bar{p}_4) z^0 \quad (31)$$

$$U_5(z) = \bar{p}_5 z^1 + (1 - \bar{p}_5) z^0 \quad (32)$$

Now, using step1 of algorithm 1, we have UGF as

$$U_{-2}(z) = z^{\{0,0,0\}}$$

For  $i = 1$

$$U_{-1}(z) = \bar{p}_1 z^{\{0,0,1\}} + (1 - \bar{p}_1) z^{\{0,0,0\}}.$$

For  $i = 2$

$$U_0(z) = \bar{p}_1 \bar{p}_2 z^{\{0,1,1\}} + \bar{p}_1 (1 - \bar{p}_2) z^{\{0,1,0\}} + \bar{p}_2 (1 - \bar{p}_1) z^{\{0,0,1\}} + (1 - \bar{p}_1)(1 - \bar{p}_2) z^{\{0,0,0\}}.$$

For  $i = 3$

$$\begin{aligned}
 U_1(z) = & \bar{p}_1 \bar{p}_2 \bar{p}_3 z^{\{1,1,2\}} + \bar{p}_1 (1 - \bar{p}_2) \bar{p}_3 z^{\{1,0,2\}} + \bar{p}_2 \bar{p}_3 (1 - \bar{p}_1) z^{\{0,1,2\}} \\
 & \bar{p}_1 (1 - \bar{p}_2) (1 - \bar{p}_3) z^{\{1,0,0\}} + \bar{p}_2 (1 - \bar{p}_1) (1 - \bar{p}_3) z^{\{0,1,0\}} + (1 - \bar{p}_1) (1 - \bar{p}_2) (1 - \bar{p}_3) z^{\{0,0,0\}} \\
 & + (1 - \bar{p}_1) (1 - \bar{p}_2) \bar{p}_3 z^{\{0,0,2\}} + \bar{p}_1 \bar{p}_2 (1 - \bar{p}_3) z^{\{1,1,0\}}.
 \end{aligned}$$

Now with the help of equation (4), one can have the modified UGF  $\tilde{U}_1(z)$  of  $U_1(z)$  as

$$\begin{aligned}
 \tilde{U}_1(z) = & \bar{p}_1 \bar{p}_2 \bar{p}_3 z^{\{0,0,0\},\{1,1,2\}} + \bar{p}_1 (1 - \bar{p}_2) \bar{p}_3 z^{\{0,0,1\},\{1,0,2\}} \\
 & + \bar{p}_2 \bar{p}_3 (1 - \bar{p}_1) z^{\{0,0,1\},\{0,1,2\}} + (1 - \bar{p}_1) (1 - \bar{p}_2) \bar{p}_3 z^{\{0,0,1\},\{0,0,2\}} \\
 & + \bar{p}_1 (1 - \bar{p}_2) (1 - \bar{p}_3) z^{\{0,0,1\},\{1,0,0\}} + \bar{p}_1 \bar{p}_2 (1 - \bar{p}_3) z^{\{0,0,1\},\{1,1,0\}} + \bar{p}_2 (1 - \bar{p}_1) (1 - \bar{p}_3) z^{\{0,0,1\},\{0,1,0\}} \\
 & + (1 - \bar{p}_1) (1 - \bar{p}_2) (1 - \bar{p}_3) z^{\{0,0,1\},\{0,0,0\}}
 \end{aligned}$$

For  $i = 4$

$$\begin{aligned}
 \tilde{U}_2(z) = & \bar{p}_1 \bar{p}_2 \bar{p}_3 \bar{p}_4 z^{\{0,0,0\},\{1,2,2\}} + (1 - \bar{p}_2) \bar{p}_3 \bar{p}_4 z^{\{0,1,0\},\{0,2,2\}} + \bar{p}_2 \bar{p}_3 \bar{p}_4 (1 - \bar{p}_1) z^{\{0,1,0\},\{1,2,2\}} \\
 & + \bar{p}_2 (1 - \bar{p}_3) \bar{p}_4 z^{\{0,1,1\},\{1,0,2\}} + (1 - \bar{p}_2) (1 - \bar{p}_3) \bar{p}_4 z^{\{0,1,1\},\{0,0,2\}} + \bar{p}_1 \bar{p}_2 \bar{p}_3 (1 - \bar{p}_4) z^{\{0,0,1\},\{1,2,0\}} \\
 & + (1 - \bar{p}_2) \bar{p}_3 (1 - \bar{p}_4) z^{\{0,1,1\},\{0,2,0\}} + (1 - \bar{p}_1) \bar{p}_2 \bar{p}_3 (1 - \bar{p}_4) z^{\{0,1,1\},\{1,2,0\}} \\
 & + \bar{p}_2 (1 - \bar{p}_3) (1 - \bar{p}_4) z^{\{0,0,1\},\{1,0,0\}} + (1 - \bar{p}_2) (1 - \bar{p}_3) (1 - \bar{p}_4) z^{\{0,1,1\},\{0,0,0\}}
 \end{aligned}$$

For  $i = 5$

$$\begin{aligned}
 \tilde{U}_3(z) = & \bar{p}_1 \bar{p}_2 \bar{p}_3 \bar{p}_4 \bar{p}_5 z^{\{0,0,0\},\{2,2,1\}} + \bar{p}_1 \bar{p}_2 \bar{p}_3 \bar{p}_4 (1 - \bar{p}_5) z^{\{0,0,0\},\{2,2,0\}} + (1 - \bar{p}_1 \bar{p}_2) \bar{p}_3 \bar{p}_4 \bar{p}_5 z^{\{1,0,0\},\{2,2,1\}} \\
 & + (1 - \bar{p}_1 \bar{p}_2) \bar{p}_3 \bar{p}_4 (1 - \bar{p}_5) z^{\{1,0,0\},\{2,2,0\}} + \bar{p}_1 \bar{p}_2 \bar{p}_3 (1 - \bar{p}_4) \bar{p}_5 z^{\{0,1,1\},\{2,0,1\}} \\
 & + \bar{p}_1 \bar{p}_2 \bar{p}_3 (1 - \bar{p}_4) (1 - \bar{p}_5) z^{\{0,1,1\},\{2,0,0\}} \\
 & + (1 - \bar{p}_1 \bar{p}_2) \bar{p}_3 (1 - \bar{p}_4) \bar{p}_5 z^{\{1,1,1\},\{2,0,1\}} + (1 - \bar{p}_1 \bar{p}_2) \bar{p}_3 (1 - \bar{p}_4) (1 - \bar{p}_5) z^{\{1,1,1\},\{2,0,0\}} \\
 & + (1 - \bar{p}_3) \bar{p}_4 (1 - \bar{p}_5) z^{\{1,1,1\},\{0,2,0\}} + (1 - \bar{p}_3) \bar{p}_4 \bar{p}_5 z^{\{1,1,1\},\{0,2,1\}} \\
 & + (1 - \bar{p}_3) (1 - \bar{p}_4) \bar{p}_5 z^{\{1,1,1\},\{0,0,1\}} + (1 - \bar{p}_3) (1 - \bar{p}_4) (1 - \bar{p}_5) z^{\{1,1,1\},\{0,0,0\}}
 \end{aligned}$$

Hence, the failure probability of the SWCS is obtained as

$$\begin{aligned}
 F = \delta(\tilde{U}_3(z)) = & \bar{p}_1 \bar{p}_2 \bar{p}_3 (1 - \bar{p}_4) \bar{p}_5 + \bar{p}_1 \bar{p}_2 \bar{p}_3 (1 - \bar{p}_4) (1 - \bar{p}_5) + (1 - \bar{p}_1 \bar{p}_2) \bar{p}_3 (1 - \bar{p}_4) \bar{p}_5 \\
 & + (1 - \bar{p}_1 \bar{p}_2) \bar{p}_3 (1 - \bar{p}_4) (1 - \bar{p}_5) + (1 - \bar{p}_3) \bar{p}_4 (1 - \bar{p}_5) + (1 - \bar{p}_3) \bar{p}_4 \bar{p}_5 \\
 & + (1 - \bar{p}_3) (1 - \bar{p}_4) \bar{p}_5 + (1 - \bar{p}_3) (1 - \bar{p}_4) (1 - \bar{p}_5) \\
 = & 1 - \bar{p}_3 \bar{p}_4
 \end{aligned}$$

Now, the reliability of the SWCS is

$$R = 1 - F = \bar{p}_3 \bar{p}_4 \tag{33}$$

$$\begin{aligned}
 = & (R_{31} + R_{32} + R_{33} - R_{31}R_{32} - R_{31}R_{33} - R_{32}R_{33} + R_{31}R_{32}R_{33}) \\
 & (R_{41} + R_{42} + R_{43} - R_{41}R_{42} - R_{41}R_{43} - R_{42}R_{43} + R_{41}R_{42}R_{43})
 \end{aligned} \tag{34}$$

The structure function of the 2-within-3-from-3/5SWCS when all elements are identical ( $R_i \equiv R$ ) is given by

$$R = 9R^2 - 18R^3 + 15R^4 - 6R^5 + R^6$$

### 5.1 Signature of 2-Within-3-From-3/5 SWCS

Using equation (14), we get a polynomial function  $H(x)$  of 2-within-3-from-3/5 SWCS as

$$H(x) = 9x^2 - 18x^3 + 15x^4 - 6x^5 + x^6.$$

The tail signature of the 2-within-3-from-3/5 SWCS can be obtained by equation (15) as

$$\bar{s} = \left( 1, 1, 1, \frac{9}{10}, \frac{3}{5}, 0 \right).$$

One can get the signature of the system from equation (16) as

$$s = \left( 0, 0, \frac{1}{10}, \frac{3}{10}, \frac{3}{5}, 0 \right).$$

### 5.2 Expected Lifetime of 2-Within-3-From-3/5 SWCS

Using equation (17), we get expected lifetime from minimal signature as

$$H(x) = 9x^2 - 18x^3 + 15x^4 - 6x^5 + x^6 \tag{35}$$

Finally, one can calculate the minimal signature by equation (35) as

$$\text{Minimal signature} = (0, 9, -18, 15, -6, 1).$$

Hence, expected lifetime is

$$E(T) = 1.2167.$$

### 5.3 Expected Cost Rate of 2-Within-3-From-3/5 SWCS

By using equation (20), we get the expected value of  $X$  is  $E(X)$   $E(X)$  as  $E(X)=4.5$ .

$$\text{Cost rate} = \frac{E(X)}{E(T)} = 0.270.$$

### 5.4 Barlow- Proschan Index for 2-Within-3-From-3/5 SWCS

Barlow-Proschan index of the considered system is obtained using equation (19) as

$$I_{BP}^{(1)} = \int_0^1 (d_1 H) dR = \int_0^1 (3R - 9R^2 + 10R^3 - 5R^4 + R^5) dR = \frac{1}{6}.$$

Similarly, obtain all Barlow-Proschan index of the 2-within-3-from-3/5 SWCS as

$$I_{BP} = \left( \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right).$$

### 5.5 Sensitivity of 2-Within-3-From-3/5 SWCS

To obtain the sensitivity of 2-within-3-from-3/5 SWCS let us take value  $R_{31} = 0.5, R_{32} = 0.6, R_{33} = 0.65, R_{41} = 0.7, R_{42} = 0.8, R_{43} = 0.55$ . Now differentiating the equation (34) with respect to different parameters, we have sensitivities as

$$S_1 = \frac{\partial R}{\partial R_{31}} = 0.1362, S_2 = \frac{\partial R}{\partial R_{32}} = 0.1703, S_3 = \frac{\partial R}{\partial R_{33}} = 0.1652$$

$$S_4 = \frac{\partial R}{\partial R_{41}} = 0.1064, S_5 = \frac{\partial R}{\partial R_{42}} = 0.1401, S_6 = \frac{\partial R}{\partial R_{43}} = 0.0774.$$

## 6. Result and Discussion

In this study, we deal with *A*-within-*B*-from-*D/G* SWCS and computed different reliability measures, viz. signature, expected lifetime, Barlow-Proschan Index and sensitivity of the proposed system.

## 7. Conclusion

In the present paper, we have studied the *A*-within-*B*-from-*D/G* SWCS incorporating multiple failures. An algorithm for evaluating signature estimation on the basis of Owen's method and UGF technique has been used for the considered system. The algorithm is based on structure function, which has been used to evaluate the signature of the proposed system. The results show that the system signature is increasing w.r.t. the price value of the expected cost. Sensitivity with respect to parameters  $R_{32}$  and  $R_{42}$  is found to be highest and lowest respectively.

### Conflict of Interest

The authors confirm that this article contents have no conflict of interest.

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## References

- Barlow, R. E., & Proschan, F. (1975). Importance of system components and fault tree events. *Stochastic Processes and their Applications*, 3(2), 153-173.
- Bhattacharya, D., & Samaniego, F. J. (2008). On the optimal allocation of elements within coherent systems. *Statistics & Probability Letters*, 78(7), 938-943.

- Boland, P. J. (2001). Signatures of indirect majority systems. *Journal of Applied Probability*, 38(2), 597-603.
- Da Costa Bueno, V. (2013). A multistate monotone system signature. *Statistics & Probability Letters*, 83(11), 2583-2591.
- Da, G., Zheng, B., & Hu, T. (2012). On computing signatures of coherent systems. *Journal of Multivariate Analysis*, 103(1), 142-150.
- Eryilmaz, S. (2010). Mixture representations for the reliability of consecutive-k systems. *Mathematical and Computer Modelling*, 51(5), 405-412.
- Eryilmaz, S. (2012). The number of failed elements in a coherent system with exchangeable elements. *IEEE Transactions on Reliability*, 61(1), 203-207.
- Faghih-Roohi, S., Xie, M., Ng, K. M., & Yam, R. C. (2014). Dynamic availability assessment and optimal element design of multi-state weighted  $k$ -out-of- $n$  systems. *Reliability Engineering & System Safety*, 123(2014), 57-62.
- Franko, C., & Tütüncü, G. Y. (2016). Signature based reliability analysis of repairable weighted  $k$ -out-of- $n$ : G systems. *IEEE Transactions on Reliability*, 65(2), 843-850.
- Habib, A., Al-Seedy, R. O., & Radwan, T. (2007). Reliability evaluation of multi-state consecutive  $k$ -out-of- $r$ -from- $n$ : G system. *Applied Mathematical Modelling*, 31(11), 2412-2423.
- Kumar, A. & Singh, S. B. (2017a). Computations of signature reliability of coherent system, *International Journal of Quality & Reliability Management*, 34(6), 785-797.
- Kumar, A., & Singh, S. B. (2017b). Signature reliability of sliding window coherent system. In *Mathematics Applied to Engineering*, Elsevier, London, pp.83-95.
- Levitin, G. (2002). Optimal allocation of elements in a linear multi-state sliding window system. *Reliability Engineering & System Safety*, 76(3), 245-254.
- Levitin, G. (2003). Common supply failures in linear multi-state sliding window systems. *Reliability Engineering & System Safety*, 82(1), 55-62.
- Levitin, G. (2005). *The universal generating function in reliability analysis and optimization* (p. 442). London: Springer. DOI 10.1007/1-84628-245-4.
- Levitin, G., & Ben-Haim, H. (2011). Consecutive sliding window systems. *Reliability Engineering & System Safety*, 96(10), 1367-1374.
- Levitin, G., & Dai, Y. (2012).  $k$ -out-of- $n$  sliding window systems. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 42(3), 707-714.
- Li, Y. F., Ding, Y., & Zio, E. (2014). Random fuzzy extension of the universal generating function approach for the reliability assessment of multi-state systems under aleatory and epistemic uncertainties. *IEEE Transactions on Reliability*, 63(1), 13-25.
- Mahmoudi, M., & Asadi, M. (2011). The dynamic signature of coherent systems. *IEEE Transactions on Reliability*, 60(4), 817-822.
- Marichal, J. L., & Mathonet, P. (2013). Computing system signatures through reliability functions. *Statistics & Probability Letters*, 83(3), 710-717.
- Navarro, J., & Hernandez, P. J. (2008). Mean residual life functions of finite mixtures, order statistics and coherent systems. *Metrika*, 67(3), 277-298.
- Navarro, J., & Rubio, R. (2009). Computations of signatures of coherent systems with five elements. *Communications in Statistics-Simulation and Computation*, 39(1), 68-84.

- Navarro, J., Ruiz, J. M., & Sandoval, C. J. (2007). Properties of coherent systems with dependent elements. *Communications in Statistics-Theory and Methods*, 36(1), 175-191.
- Negi, S., & Singh, S. B. (2015). Reliability analysis of non-repairable complex system with weighted subsystems connected in series. *Applied Mathematics and Computation*, 262, 79-89.
- Owen, G. (1975). Multilinear extensions and the Banzhaf value. *Naval Research Logistics Quarterly*, 22(4), 741-750.
- Owen, G. (1988). Multilinear extensions of games. *The Shapley Value. Essays in Honor of Lloyd S. Shapley*, 139-151.
- Peng, R., Xiao, H., & Liu, H. (2017). Reliability of multi-state systems with a performance sharing group of limited size. *Reliability Engineering & System Safety*, 166, 164-170.
- Ram, M. (2013). On system reliability approaches: a brief survey. *International Journal of System Assurance Engineering and Management*, 4(2), 101-117.
- Samaniego, F. J. (2007). System signatures and their applications in engineering reliability (Vol. 110). *Springer Science & Business Media*. Doi.10.1007/978-0-387-71797-5.
- Shapley, L. S. (1953). A value for  $n$ -person games. In: *Contributions to the Theory of Games*, Vol. 2. In: *Annals of Mathematics Studies*, vol. 28. Princeton University Press, Princeton, NJ, 307–317.
- Sun, Z. L., Xie, M., Ng, K. M., & Habibullah, M. S. (2012). A study of lifetime optimization of transportation system. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 42(4), 1013-1019.
- Xiang, Y., & Levitin, G. (2012a). Combined  $m$ -consecutive and  $k$ -out-of- $n$  sliding window systems. *European Journal of Operational Research*, 219(1), 105-113.
- Xiang, Y., & Levitin, G. (2012b). Linear-gap sliding window systems. *IEEE Transactions on Reliability*, 61(2), 560-568.
- Xiang, Y., Levitin, G., & Dai, Y. (2013). Optimal allocation of multistate elements in consecutive sliding window systems. *IEEE Transactions on Reliability*, 62(1), 267-275.
- Xiao, H., Peng, R., & Levitin, G. (2015). Optimal replacement and allocation of multi-state elements in  $k$ -within- $m$ -from- $r/n$  sliding window systems. *Applied Stochastic Models in Business and Industry*, 32(2), 184-198.
- Xiao, H., Peng, R., Wang, W., & Zhao, F. (2014). Linear  $m$ -gap-consecutive  $k$ -out-of- $r$ -from- $n$  system with common supply failures. *International Conference on Reliability, Maintainability and Safety (ICRMS)*, 2014 (pp. 348-353).
- Yu, H., Yang, J., & Mo, H. (2014). Reliability analysis of repairable multi-state system with common bus performance sharing. *Reliability Engineering & System Safety*, 132, 90-96.
- Yueqin, W., Yun, F., & Qihua, W. (2010). A universal generating function approach for reliability analysis of multi-state systems. *2010 Second WRI Global Congress on Intelligent Systems*; Wuhan, China. Los Alamitos; IEEE Computer Society; 2010. Vol. 3, pp. 207-210.
- Navarro, J., & Rychlik, T. (2010). Comparisons and bounds for expected lifetimes of reliability systems. *European Journal of Operational Research*, 207(1), 309-317.