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Improved Estimators for Estimating Average Yield Using Auxiliary Variable

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Abstract
In this paper, we consider the improved estimation of average production of peppermint at block level of Barabanki district of Uttar Pradesh State (India). We suggest certain estimators for population-mean. Here, population refers to production population as study variable and auxiliary-variable refers to Area of field. We study the sampling properties naming bias and MSE of estimators, which are presently proposed by us in the paper. We compare our proposed estimators with other ones existing in literature. For the support of the theoretical findings, we carry out a numerical study for the natural population on primary data collected from Banikodar Block of Barabanki District situated in Uttar Pradesh State.

Keywords- Population variable, Auxiliary variable, Ratio-estimator, MSE, PRE.

1. Introduction

Literature-review reveals that applying auxiliary-information enhances estimator's efficiency under consideration whenever we estimate any parameter. It has been now evident that auxiliary-variable technique improves the estimation process for target-population. Primary and the secondary variables have a high correlation to each other. They may have both negative and positive correlations. Ratio type estimators are preferred when primary and secondary variables are highly positively correlated while product types estimators when they have high negative correlation. As production (primary) and the area (secondary) are highly positively correlated so we consider the ratio types estimators only in the present study.

Watson (1937) used subsidiary variable and suggested the traditional regression-estimator of population mean of main variable. Usual Ratio estimator utilizing positively correlated auxiliary-information was given in Cochran (1940). The well-known product estimators was independently introduced by Robson (1957) and Murthy (1964) using negatively correlated auxiliary variable.

After Cochran (1940), various modified ratio-type-estimators are already played an important role in improving the estimation process for mean of population on the basis of auxiliary-variable having positively correlated. Some of the latest references include Yadav and Kadilar (2013), Subramani and Kumarapandiyan (2012a, 2012b), Yadav and Mishra (2015), Yadav et al. (2016), Subramani (2016), Abid et al. (2016), Cekim and Cingi (2017), Gupta and Yadav (2017, 2018), Subramani and Ajith (2017), Cekim and Kadilar (2018), Srija and Subramani (2018), Yadav et al. (2018).

The whole paper has been presented in various sections including introduction given above and the rest sections are review of existing estimators of population mean, proposed estimators, theoretical comparison of the efficiencies of various estimators with the proposed estimators, numerical study, acknowledgement of the funding agency CST UP for financial assistance for the work and finally paper ends with the references.

2. Review of Existing Estimators

The sample mean that does not use auxiliary information and various modified ratio-estimators of population mean applying auxiliary information are presented in the following Table 1. The variance of sample mean and the Mean Squared Error of various estimators are also given in the following Table 1.

Table1. Various estimators of population mean along with their mean squared errors

S.N.	Estimator	Variance/Mean Squared Error
1.	$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ Sample mean	$\lambda S_y^2 = \lambda \bar{Y}^2 C_y^2$
2.	$t_1 = \bar{y} + \beta(\bar{X} - \bar{x})$ Watson (1937)	$\lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$
3.	$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}}$ Cochran (1940)	$\lambda \bar{Y}^2 [C_y^2 + C_x^2 - 2C_{yx}]$
4.	$t_3 = \bar{y}\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$ Bahl and Tuteja (1991)	$\lambda \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4} - C_{yx}]$
5.	$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^2$ Kadilar and Cingi (2003)	$\lambda \bar{Y}^2 [C_y^2 + 4C_x^2 - 4C_{yx}]$
6.	$t_5 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}$ Srivastava (1967)	$\lambda \bar{Y}^2 [C_y^2 + \alpha^2 C_x^2 + 2\alpha C_{yx}]$ $\lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$ minimum for $\alpha_{opt} = -C_{yx}/C_x^2$
7.	$t_6 = \bar{y}\left[\frac{\bar{X}}{\bar{X} + \alpha(\bar{x} - \bar{X})}\right]$ Reddy (1974)	$\lambda \bar{Y}^2 [C_y^2 + \alpha^2 C_x^2 - 2\alpha C_{yx}]$ $\lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2)$ minimum for $\alpha_{opt} = C_{yx}/C_x^2$

Table 1 continued …

8.	$t_7 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta} \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ Kadilar (2016)	$\lambda \bar{Y}^2\left[C_y^2 + \left(\delta^2+\delta+\frac{1}{4}\right)C_x^2 + (2\delta+1)C_{yx}\right]$ $\lambda \bar{Y}^2 C_y^2(1-\rho_{yx}^2)$ minimum for $\delta_{opt} = \left(\frac{1}{2} - \rho_{yx}C_y / C_x\right)$
9.	$t_8 = \bar{y}\left\{2-\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\xi(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right)\right\}$ Solanki et al. (2012)	$\lambda \bar{Y}^2 C_y^2(1-\rho_{yx}^2)$, minimum for $(2\alpha+\xi)=2\kappa$
10.	$t_9 = \bar{y}\left(\frac{\bar{X}+Q_1}{\bar{x}+Q_1}\right)$ Al-Omari et al. (2009)	$\lambda \bar{Y}^2[C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_{yx}]$, $\theta_9 = \frac{\bar{X}}{\bar{X}+Q_1}$
11.	$t_{10} = \bar{y}\left(\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right)$ Al-Omari et al. (2009)	$\lambda \bar{Y}^2[C_y^2 + \theta_{10}^2 C_x^2 - 2\theta_{10} C_{yx}]$, $\theta_{10} = \frac{\bar{X}}{\bar{X}+Q_3}$
12.	$t_{11} = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$ Sisodia and Dwivedi (1981)	$\lambda \bar{Y}^2[C_y^2 + \theta_{11}^2 C_x^2 - 2\theta_{11} C_{yx}]$, $\theta_{11} = \frac{\bar{X}}{\bar{X}+C_x}$
13.	$t_{12} = \bar{y}\left[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right]$ Singh et.al (2004)	$\lambda \bar{Y}^2[C_y^2 + \theta_{12}^2 C_x^2 - 2\theta_{12} C_{yx}]$, $\theta_{12} = \frac{\bar{X}}{\bar{X}+\beta_2}$
14.	$t_{13} = \bar{y}\left[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right]$ Yan and Tian (2010)	$\lambda \bar{Y}^2[C_y^2 + \theta_{13}^2 C_x^2 - 2\theta_{13} C_{yx}]$, $\theta_{13} = \frac{\bar{X}}{\bar{X}+\beta_1}$
15.	$t_{14} = \bar{y}\left[\frac{\bar{X}+\rho}{\bar{x}+\rho}\right]$ Singh and Tailor (2003)	$\lambda \bar{Y}^2[C_y^2 + \theta_{14}^2 C_x^2 - 2\theta_{14} C_{yx}]$, $\theta_{14} = \frac{\bar{X}}{\bar{X}+\rho}$
16.	$t_{15} = \bar{y}\left[\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right]$ Upadhyaya and Singh (1999)	$\lambda \bar{Y}^2[C_y^2 + \theta_{15}^2 C_x^2 - 2\theta_{15} C_{yx}]$, $\theta_{15} = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}$
17.	$t_{16} = \bar{y}\left[\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}\right]$ Upadhyaya and Singh (1999)	$\lambda \bar{Y}^2[C_y^2 + \theta_{16}^2 C_x^2 - 2\theta_{16} C_{yx}]$, $\theta_{16} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+C_x}$
18.	$t_{17} = \bar{y}\left[\frac{\bar{X}\beta_1+\beta_2}{\bar{x}\beta_1+\beta_2}\right]$ Yan and Tian (2010)	$\lambda \bar{Y}^2[C_y^2 + \theta_{17}^2 C_x^2 - 2\theta_{17} C_{yx}]$, $\theta_{17} = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$
19.	$t_{18} = \bar{y}\left[\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right]$ Yan and Tian (2010)	$\lambda \bar{Y}^2[C_y^2 + \theta_{18}^2 C_x^2 - 2\theta_{18} C_{yx}]$, $\theta_{18} = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_1}$

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Table 1 continued …

20.	$t_{19}=\bar{y}\left[\frac{\bar{X}+M_d}{\bar{x}+M_d}\right]$ Subramani and Kumarpandiyan (2012a)	$\lambda\bar{Y}^2[C_y^2+\theta_{19}^2C_x^2-2\theta_{19}C_{yx}]$, $\theta_{19}=\frac{\bar{X}}{\bar{X}+M_d}$
21.	$t_{20}=\bar{y}\left(\frac{\bar{X}C_x+M_d}{\bar{x}C_x+M_d}\right)$ Subramani and Kumarpandiyan (2012a)	$\lambda\bar{Y}^2[C_y^2+\theta_{20}^2C_x^2-2\theta_{20}C_{yx}]$, $\theta_{20}=\frac{\bar{X}C_x}{\bar{X}C_x+M_d}$
22.	$t_{21}=\bar{y}\left(\frac{\bar{X}\beta_1+QD}{\bar{x}\beta_1+QD}\right)$ Jeelani et al. (2013)	$\lambda\bar{Y}^2[C_y^2+\theta_{21}^2C_x^2-2\theta_{21}C_{yx}]$, $\theta_{21}=\frac{\bar{X}\beta_1}{\bar{X}\beta_1+QD}$
23.	$t_{22}=\bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right)$ Jerajuddin and Kishun (2016)	$\lambda\bar{Y}^2[C_y^2+\theta_{22}^2C_x^2-2\theta_{22}C_{yx}]$, $\theta_{22}=\frac{\bar{X}}{\bar{X}+n}$
24.	$t_{23}=\bar{y}\left(\frac{\bar{X}+Q_r}{\bar{x}+Q_r}\right)$ Subramani and Kumarpandiyan (2012b)	$\lambda\bar{Y}^2[C_y^2+\theta_{23}^2C_x^2-2\theta_{23}C_{yx}]$, $\theta_{23}=\frac{\bar{X}}{\bar{X}+Q_r}$
25.	$t_{24}=\bar{y}\left(\frac{\bar{X}+Q_d}{\bar{x}+Q_d}\right)$ Subramani and Kumarpandiyan (2012b)	$\lambda[C_y^2+\theta_{24}^2C_x^2-2\theta_{24}C_{yx}]$, $\theta_{24}=\frac{\bar{X}}{\bar{X}+Q_d}$
26.	$t_{25}=\bar{y}\left(\frac{\bar{X}+Q_a}{\bar{x}+Q_a}\right)$ Subramani and Kumarpandiyan (2012b)	$\lambda[C_y^2+\theta_{25}^2C_x^2-2\theta_{25}C_{yx}]$, $\theta_{25}=\frac{\bar{X}}{\bar{X}+Q_a}$
27.	$t_{26}=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{b_1}$ Nangsue (2009)	$\lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2)$
28.	$t_{27}=\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)^{b_1}$ $t_{28}=\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)^{b_1}$ Soponviwatkul and Lawson (2017)	$\lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2)$
29.	$t_{29}=\left[\omega_1\bar{y}+(1-\omega_1)\left(\bar{y}\frac{\bar{X}}{\bar{x}}\right)\right]$ $t_{30}=\left[\omega_1\bar{y}+(1-\omega_1)\left(\bar{y}\exp\frac{\bar{X}-\bar{x}}{\bar{x}-\bar{X}}\right)\right]$ Ijaz and Ali (2018)	$\lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2)$ for $\omega_{1(opt)}=(1-\rho_{yx}C_y/C_x)$ and $\lambda\bar{Y}^2C_y^2(1-\rho_{yx}^2)$ for $\omega_{1(opt)}=(-2\rho_{yx}C_y/C_x)$

3. Proposed Class of Estimators

Motivated by various authors in the literature, we propose the following class of estimators of population-mean using auxiliary information as,

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$$t_p = \bar{y}\left(\frac{a.b\,\bar{X} + c.d}{a.b\,\bar{x} + c.d}\right) \tag{1}$$

where, a, b, c and d are either constants or the parameters of auxiliary variable under consideration.

Following Table 2 represents various members of the proposed class of estimators for different values of a, b, c and d putting as either constant or M_x, β_1, β_2, C_x, ρ, n.

Table 2. Various members of the proposed family

$t_{p1} = \bar{y}\left(\frac{\beta_2 M_x \bar{X} + \rho}{\beta_2 M_x \bar{x} + \rho}\right)$,	$t_{p2} = \bar{y}\left(\frac{\beta_2 M_x \bar{X} + \rho C_x}{\beta_2 M_x \bar{x} + \rho C_x}\right)$,	$t_{p3} = \bar{y}\left(\frac{\beta_1 M_x \bar{X} + \rho}{\beta_1 M_x \bar{x} + \rho}\right)$,	$t_{p4} = \bar{y}\left(\frac{\beta_1 M_x \bar{X} + \rho C_x}{\beta_1 M_x \bar{x} + \rho C_x}\right)$,	
$t_{p5} = \bar{y}\left(\frac{n\bar{X} + \rho}{n\bar{x} + \rho}\right)$,	$t_{p6} = \bar{y}\left(\frac{n\bar{X} + C_x}{n\bar{x} + C_x}\right)$,	$t_{p7} = \bar{y}\left(\frac{n\bar{X} + \rho C_x}{n\bar{x} + \rho C_x}\right)$,	$t_{p8} = \bar{y}\left(\frac{n\rho\bar{X} + C_x}{n\rho\bar{x} + C_x}\right)$,	$t_{p9} = \bar{y}\left(\frac{nC_x\bar{X} + \rho}{nC_x\bar{x} + \rho}\right)$,

Note: Many more members of the proposed family can be obtained by putting different values of the constants a, b, c and d of the suggested class.

3.1 Bias and MSE of the Suggested Class

The expressions for the bias and MSE of the suggested class are obtained using the following approximation, given as,

$\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$ such that $E(e_0) = E(e_1) = 0$, and $E(e_0^2) = \lambda C_y^2$, $E(e_1^2) = \lambda C_x^2$, $E(e_0 e_1) = \lambda C_{yx}$, where $\lambda = \frac{1-f}{n}$ and $f = \frac{n}{N}$.

Expressing t_p in terms of $e's\,(i = 0,1)$, we have $t_p = \bar{Y}(1 + e_0)(1 - \theta e_1)^{-1}$, where, $\theta = \frac{a.b\bar{X}}{a.b\bar{X} + c.d}$.

Now expanding the right hand side of the above equation, simplifying and retaining the terms up to the approximation of order one, we have

$$t_p = \bar{Y}(1 + e_0 - \theta e_1 - \theta e_0 e_1 + \theta^2 e_1^2).$$

Subtracting $\bar{Y}$ on both sides of the above equation, we get

$$t_p - \bar{Y} = \bar{Y}(e_0 - \theta e_1 - \theta e_0 e_1 + \theta^2 e_1^2) \tag{2}$$

Taking expectations on both sides of (2) and putting values of various expectations, we get the bias of the proposed class of estimators as,

$$B(t_p) = \lambda \bar{Y} (\theta^2 C_x^2 - \theta C_{yx}) \quad (3)$$

Squaring on both sides of (2), retaining the terms up to the approximation of order one and putting values of various expectations, we get the mean squared error of the proposed class of estimators as,

$$E(t_p - \bar{Y})^2 \approx \bar{Y}^2 E(e_0^2 + \theta^2 e_1^2 - 2\theta e_0 e_1)$$

$$MSE(t_p) = \lambda \bar{Y}^2 (C_y^2 + \theta^2 C_x^2 - 2\theta C_{yx}) \quad (4)$$

The optimum value of θ obtained by,

$\frac{\partial}{\partial \theta} MSE(t_p) = 0$, which gives, $\theta = \frac{C_{yx}}{C_x^2}$ and the minimum MSE for this optimum value of θ is given by,

$$MSE(t_p) = \lambda \bar{Y}^2 \left[C_y^2 - \frac{C_{yx}^2}{C_x^2} \right] \quad (5)$$

4. Efficiency Comparison

Under this section, the proposed class of estimators is compared with the existing ratio-type-estimators of population-mean of study-variable and the conditions under which it performs better than existing estimators are given in the following Table 3. The condition for the proposed class to be more efficient than the existing estimators is given by,

$$MSE(t_i) - MSE(t_p) > 0, \; i = 0,1,\ldots,30$$

Table 3. Conditions under which proposed estimators are better than existing ones.

Efficiency condition	**Estimators**
$C_{yx}^2 > 0$	Mean per unit estimator
$[1-\rho_{yx}^2] - \left[C_y^2 - \frac{C_{yx}^2}{C_x^2} \right] > 0$	Watson (1937), Srivastava (1967), Reddy (1974), Nangsue (2009), Kadilar (2016), Soponviwatkul and Lawson (2017), Ijaz and Ali (2018) estimators
$[C_x^2 - 2C_{yx}] + \frac{C_{yx}^2}{C_x^2} > 0$	Cochran (1940) estimator
$[\frac{C_x^2}{4} - C_{yx}] + \frac{C_{yx}^2}{C_x^2} > 0$	Bahl and Tuteja (1991) estimator
$[4C_x^2 - 4C_{yx}] + \frac{C_{yx}^2}{C_x^2} > 0$	Kadilar and Cingi (2003) estimator
$[\theta_i^2 C_x^2 - 2\theta_i C_{yx}] + \frac{C_{yx}^2}{C_x^2} > 0$	Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et al. (2004), Al-Omari et al. (2009), Yan and Tian (2010), Subramani and Kumarpandiyan (2012a, 2012b), Jeelani et al. (2013), Jerajuddin and Kishun (2016) estimators

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5. Numerical Study

For the verification of the conditions under which the proposed estimators are better than the existing estimators of population mean, we have used the primary data of production of peppermint oil, obtained from the crop from Banikodar Block at Barabanki District of Uttar Pradesh State in India. The parameters of the population under consideration are given in Table 4. The dependent variable and the auxiliary variables are as follows:

Y : The production (Yield) of peppermint oil in kilogram
X : The area of the field in Bigha (2529.3 Square Meter)

Table 4. Parameters of the population

$N=150$, $n=40$, $\bar{X}=4.204667$, $\bar{Y}=33.462$, $S_x=3.080385$, $S_y=25.50316$, $C_x=0.732611$, $C_y=0.762153$, $M_x=3$, $M_y=25$, $\rho_{yx}=0.911241$, $\beta_1(x)=2.801407$, $\beta_2(x)=16.44023$, $\lambda=0.018333$, $Q_1(x)=2$, $Q_3(x)=5$, $QD=1.5$, $Q_a(x)=3.5$, $Q_r(x)=3$, $TM=3.25$

The following Table 5 represents the variance or mean squared error of the existing and the proposed estimators along with the efficiency of the proposed class of estimators over other estimators. For calculating percentage relative efficiency of the proposed class over competing estimators, we have considered the least mean squared error of the proposed class.

Table 5. MSE of various estimators and PRE of proposed class over competing estimators

Estimator	MSE	PRE	Estimator	MSE	PRE	Estimator	MSE	PRE
t_0	11.92	596.04	t_{14}	2.042	102.06	t_{28}	2.023	101.11
t_1	2.023	101.11	t_{15}	5.124	256.15	t_{29}	2.023	101.11
t_2	2.053	102.60	t_{16}	4.006	200.25	t_{30}	2.023	101.11
t_2	4.234	211.64	t_{17}	3.486	174.24	t_{p1}	2.016	100.80
t_4	14.22	710.63	t_{18}	4.169	208.40	t_{p2}	2.014	100.67
t_5	2.023	101.11	t_{19}	2.064	103.16	t_{p3}	2.011	100.54
t_6	2.023	101.11	t_{20}	10.04	501.71	t_{p4}	2.009	100.42
t_7	2.023	101.11	t_{21}	2.828	141.36	t_{p5}	2.007	100.31
t_8	2.023	101.11	t_{22}	4.681	233.99	t_{p6}	2.015	100.72
t_9	2.125	106.23	t_{23}	3.486	174.24	t_{p7}	2.011	100.53
t_{10}	8.127	406.22	t_{24}	2.513	125.62	t_{p8}	2.002	100.08
t_{11}	3.356	167.75	t_{25}	3.806	190.23	t_{p9}	2.004	100.19
t_{12}	2.198	109.87	t_{26}	2.023	101.11	$t_{p(\min)}$	2.001	100.00
t_{13}	8.902	444.98	t_{27}	2.023	101.11			

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Following Figure 1 and 2 show the MSE of various estimators along with the members of proposed class and the Percentage Relative Efficiency (PRE) of the proposed class of estimators over the competing estimators including its mentioned members as well. For PRE, we have used the least mean squared of the proposed class of estimators of population mean.

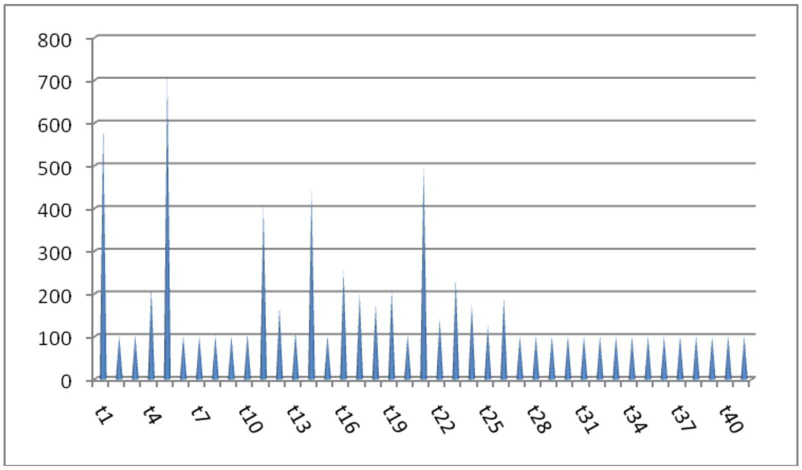

Figure 1. MSE of various estimators

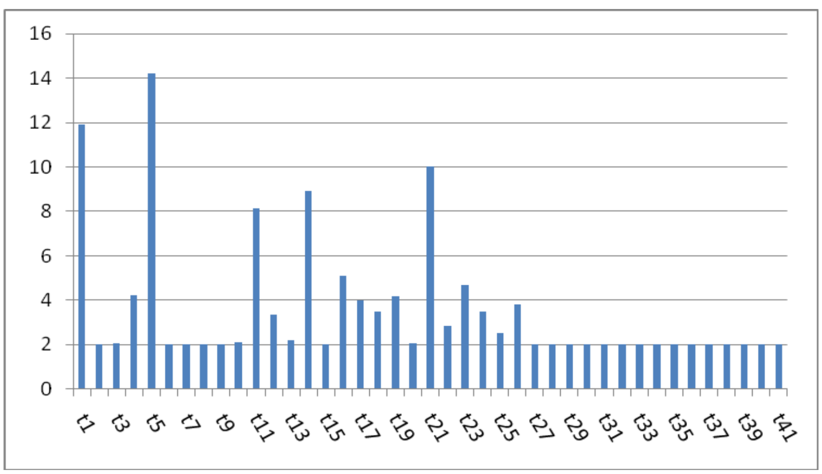

Figure 2. PRE of proposed over other estimators

6. Results and Discussions

From Table 5, it is evident that the members of the suggested class are having lesser MSE in comparison to all mentioned estimators of population-mean for the given population. The results are also presented in two figures namely Figure 1 and Figure 2, which respectively represent the MSE of various estimators along with the members of the proposed class and the PRE of the proposed class over other competing estimators of population-mean under consideration. The variance of the sample mean is 11.92 whereas the MSE of Kadilar and Cingi (2003) estimator is 14.22. The MSE of all competing estimators are lying between 8.13 to 2.02, whereas proposed class has their MSE in between 2.02 to 2.002. The least value of the MSE of the suggested class is 2.0006, which is least among the class of all mentioned competing estimators.

7. Conclusion

In the present study, we have been able to suggest a class of estimators for estimating the average yield of the peppermint crop using information on the area of the field as the auxiliary-variable. We study the bias and MSE of the proposed class up to single order of approximation. The suggested class has been compared with the competing estimators of population-mean and the conditions for the suggested class to be better than the competing ones have been referred. A numerical study is also carried out using primary data to verify the theoretical results. It is evident from Table 5 that the proposed class has the least MSE among the competing estimators of population-mean, thereby fulfilling the purpose of the study. Thus, the suggested class may be applied for enhanced estimation of population-mean utilizing auxiliary-information under simple random sampling scheme.

Conflict of Interests

It is declared by the authors that there is no conflict of interests regarding the publication of this paper.

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