

One Point Conventional Model to Optimize Trapezoidal Fuzzy Transportation Problem

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Abstract

This article puts forward a new one point approach to optimize trapezoidal fuzzy transportation problem. It proposes the method having point wise breakup of the trapezoidal number in such a way, that fuzzy transportation problem is converted into four crisp transportation problems. The method is equipped with minimum of supply and demand approach. In the end, the solutions are combined to construct the optimal solution. Modified distribution is applied on each crisp problem to develop optimal solution. The scheme presented is compared with competitive methods available in literature and it is found to be in good coordination with these. The scheme is equally good to be applied on unbalanced problems. Two numerical problems are considered to test the performance of the proposed approach.

Keywords- One point approach, Trapezoidal fuzzy number, Minimum demand supply, Modified distribution, Fuzzy transportation problem.

1. Introduction

An appropriate shipment of commodities from factories to distributors is a crucial step in business management. To deal with this cost-effectively transportation models are developed. A properly modelled transportation problem confirms the well-organized movement of goods. In the transportation modelling, the key task is to decide the way of allocation goods from several origins to several terminuses. The elementary transportation problems were initially established by Hitchcock (1941). In traditional transportation problems, decision makers have a clear idea about the amount of demand, supply and the transportation cost. In the true scenario, decision maker may not have a precise idea about the demand, supply or the cost. These data may be vague because of many affecting factors. In such cases, classical transportation methods are useless. A generalization of classical methods is needed to deal with such type of vagueness. Zadeh (1965) gave a concept to deal with this vagueness by introducing notion of fuzzy sets. This theory is based on grades, where everything is a matter of degree (Bisht et al., 2018; Jain et al., 2018; Srivastava et al., 2018; Bisht and Srivastava, 2019; Srivastava and Bisht, 2019). Since its inception, the theory has given rise to various techniques and concepts to deal with many complex problems (Jain et al., 2017; Goyal and Bisht, 2019).

Bellman and Zadeh (1970) used fuzzy theory in solution of transportation problems. In real life, different type of uncertainty may arise e.g. decisive person may not have correct idea of the transportation cost, or decision maker has a vague idea about the market demands, or there can be ambiguity in the supply of the goods (Kaur and Kumar, 2011; Kumar and Kaur, 2011). To deal

with above cases a more generalized transportation problem solving technique is needed. It can be done by treating the problem as fuzzy transportation problem (FTP) since the decision maker is not clear about the exact inputs (Chhibber et al., 2019).

In FTP, the parameters are represented in fuzzy numbers. It may lead a set of possible optimal solution or fuzzy optimal solution, when decision variables are considered as a fuzzy number. Researchers support the fact for preferring to take fuzzy numbers when decision variables are vague (Buckley and Feuring, 2000; Tanaka et al., 2000; Buckley et al., 2001; Nasrabadi and Nasrabadi, 2004; Dehghan et al. 2006; Allahviranloo et al., 2008; Ghazanfari et al. 2009; Lotfi et al., 2009; Baykasoglu and Gocken, 2012). The FTP is described by special nature of fuzzy numbers (Akyar et al., 2012; Mathur et al., 2016; Bisht and Srivastava, 2017, 2018; Mathur et al., 2018).

Zimmermann (1978) proposed the fuzzy method to solve vector maximization problem, obtained solutions by above presented method found to be more competent. A fuzzy constrained transportation problem is solved to get relationship between settled and fuzzy solution (ÓhÉigearthaigh, 1982). Chanas et al. (1984) presented a fuzzy method using parameters for fuzzy transportation with fuzzy demand and supply. This method gave alternate solution to the problem. Chanas and Kuchta (1996) furnished a technique to get best results of the FTP. The method offered a very useful set of elementary solutions for decision makers. Chanas and Kuchta (1998) solved transportation problem by striking integrality condition on the solution of a problem with fuzzy demand and supply. Saad and Abass (2002) have done a parametric study to the FTPs. Liu and Kao (2004) made advancement to the method on the basis of extension principle. Dinagar and Palanivel (2009) projected fuzzy modified distribution scheme to get the finest fuzzy solutions. Pandian and Natarajan (2010) projected fuzzy zero point technique to find optimal solution to the trapezoidal FTPs. Further, Natarajan (2010) gave a separation scheme for finding a finest solution of interval valued FTPs. Kumar and Kaur (2012) presented a method to deal with unbalanced fuzzy transportation.

In the solution of FTP, defuzzification or fuzzy number ranking is a key task. A proper ranking of fuzzy numbers facilitates appropriate results, at the same time improper ranking can mislead the solutions. Jain (1977) projected a technique based on the concept of maximization set for ranking of the fuzzy numbers. The algebraic operations of crisp numbers are further improved to fuzzy numbers by Dubois and Prade (1978). Yager (1980) projected a technique for the choice of fuzzy subsets on the unit interval based on adjoining to a linear association on interval [0,1]. Many authors have explored numerous ranking procedures to yield diverse ranked values for the FTPs (Akyar et al., 2012; Rezvani, 2013; Ganesh and Jayakumar, 2014; Bisht and Srivastava, 2017, 2018; Srivastava and Bisht, 2018, 2019; Mathur and Srivastava, 2019; Nagar et al., 2019).

The central developments of this study are summarized as:

- (i) Trapezoidal FTP with fuzzy parameters is considered.
- (ii) A point to point method is discussed briefly, which is easy to apply.
- (iii) Minimum demand-supply technique is used to get initial basic feasible solution (IBFS).
- (iv) Modified distribution (MODI) technique is applied to test the optimality.
- (v) A fuzzy optimal solution is produced to give flexibility to the decision makers.

2. Trapezoidal Fuzzy Number and Corresponding Transportation Problem

A fuzzy number is characterized by notion of membership function. A number $X^* = (x_1, x_2, x_3, x_4)$ is termed as trapezoidal if its membership function $\mu_{x^*}(\alpha)$ satisfies the condition

$$\mu_{x^*}(\alpha) = \begin{cases} \frac{\alpha - x_1}{x_2 - x_1}, & x_1 \leq \alpha \leq x_2 \\ 1, & x_2 \leq \alpha \leq x_3 \\ \frac{\alpha - x_4}{x_3 - x_4}, & x_3 \leq \alpha \leq x_4 \\ 0, & \text{otherwise.} \end{cases}$$

A transportation problem defined in the form of fuzzy trapezoidal number is called trapezoidal FTP. In such FTPs demand, supply and transportation cost will be in terms of fuzzy trapezoidal number. If (c_1, c_2, c_3, c_4) is the cost of a unit transportation from resource to target then it means the actual transportation cost may vary from c_1 to c_4 and maximum possibility of cost will be in between c_2 to c_3 . A similar way of assumption is for supply and demand.

The FTPs may be balanced or unbalanced. The unbalanced problems are made balanced through introduction of dummy rows and dummy columns.

3. One-Point Conventional Approach Through Minimum of Supply and Demand Algorithm

The one-point approach is based on the concept of the possible optimized outcome of a FTP if all the four entries of trapezoidal cost with the corresponding demand and supply are covered one by one at a time. It provides all possible optimal transportation costs in appearance of a fuzzy trapezoidal number. This approach is combined with minimum of demand and supply algorithm to solve each crisp transportation problem as described below:

Consider a crisp balanced transportation problem having table of order (p, r) with supply $a_i, i = 1, 2, \dots, p$ and demand $b_j, j = 1, 2, \dots, r$ along with corresponding cost c_{ij} . The minimum of demand and supply algorithm is applied through following steps:

Step I: Choose the $\min(a_i, b_j)$ and then find the lowest cost corresponding to this $\min(a_i, b_j)$ value and make allocation. It may have three cases

- (i) If $\min(a_i, b_j) = a_i$, allocate a_i in lowest cost cell of the i^{th} row and remove this i^{th} row. It will give a table of order $(p-1) \times r$. Reduce b_j by $b_j - a_i$ in new transportation problem.

- (ii) If $\min(a_i, b_j) = b_j$, allocate b_j in lowest cost block of the j^{th} column and remove this j^{th} column. It will give a table of order $p \times (r-1)$. Reduce a_i by $a_i - b_j$ in new transportation problem.
- (iii) if $\min(a_i, b_j) = a_i = b_j$, then select the box having lowest cost corresponding to i^{th} row and j^{th} column. Tie in transportation cost will be considered arbitrarily.

Step II: Apply Step I repeatedly till a 1×1 table is obtained. It will make allocation in all possible cells.

Step III: The IBFS and corresponding cost of transportation acquired through these steps will be

$$x_{ij} \text{ and } \sum_{i=1}^{i=p} \sum_{j=1}^{j=r} c_{ij} x_{ij} \text{ respectively.}$$

4. Methodology

The overall methodology projected in this study comprises following steps to solution of trapezoidal FTPs:

Step 1. A trapezoidal FTP with fuzzy demand, fuzzy supply and fuzzy cost is considered. The parameters are in expressions of trapezoidal fuzzy numbers.

Step 2. Check the FTP for balance.

Step 3. Apply one-point conventional method (discussed in section 3) to get four individual classical transportation problems.

Step 4. Solve crisp transportation problems obtained in step 2 using minimum demand and supply method as described in section 3, to obtain the IBFS.

Step 5. Apply MODI technique to test the optimality of the crisp solutions in all four problems obtained in step 2.

Step 6. Combine the allocations obtained through point wise calculation of each crisp transportation problem, which gives a trapezoidal fuzzy number as an allocation.

5. Numerical Illustration

This section covers two numerical problems to signify the methodology of the proposed algorithm. The first numerical problem represented in Table 1 is directly adopted from Kaur and Kumar (2011) and the results acquired by projected technique are judged against the techniques proposed by Kaur and Kumar (2011) and Mathur et al. (2016). The second numerical problem represented in Table 7 is directly adopted from Ebrahimnejad (2016) and the results acquired by projected technique are judged against the techniques proposed by him. Proposed technique is found in good harmony with these previous techniques.

5.1 Example 1

Table 1. FTP in trapezoidal form

	R ₁	R ₂	R ₃	Supply
A	(1, 4, 9, 19)	(1, 2, 5, 9)	(2, 5, 8, 18)	(1, 5, 7, 9)
B	(8, 9, 12, 26)	(3, 5, 8, 12)	(7, 9, 13, 28)	(4, 7, 8, 10)
C	(11, 12, 20, 27)	(0, 5, 10, 15)	(4, 5, 8, 11)	(4, 5, 8, 11)
Demand	(3, 5, 8, 12)	(4, 8, 9, 10)	(2, 4, 6, 8)	

Solution:

First Point Calculation

The crisp transportation problem obtained from first point value is displayed in Table 2.

Table 2. Transportation cost for first point transportation problem

	R ₁	R ₂	R ₃	Supply
A	1	1	2	1
B	8	3	7	4
C	11	0	4	4
Demand	3	4	2	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm the IBFS is

$$x_{11} = 1, x_{21} = 2, x_{22} = 2, x_{32} = 2, x_{33} = 2.$$

The transportation cost against this IBFS will be 31 units.

Applying MODI technique on this IBFS it is confirmed as optimal solution in first iteration.

Second Point Calculation

The crisp transportation problem obtained from second point value is put on view in Table 3.

Table 3. Transportation cost for second point transportation problem

	R ₁	R ₂	R ₃	Supply
A	4	2	5	5
B	9	5	9	7
C	12	5	5	5
Demand	5	8	4	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{12} = 1, x_{13} = 4, x_{21} = 5, x_{22} = 2, x_{32} = 5.$$

Applying MODI technique on this IBFS the favorable solution is achieved in third iteration. The optimal transportation cost will be 80 units.

Third Point Calculation

The crisp transportation problem obtained from third point value is displayed in Table 4.

Table 4. Transportation cost for third point transportation problem

	R ₁	R ₂	R ₃	Supply
A	9	5	8	7
B	12	8	13	8
C	20	10	8	8
Demand	8	9	6	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{12} = 7, x_{21} = 8, x_{22} = 0, x_{32} = 2, x_{33} = 6.$$

Applying MODI technique on this IBFS it is confirmed as optimal solution in first iteration. The corresponding transportation cost will be 199 units.

Fourth Point Calculation

The crisp transportation problem obtained from fourth point value is presented in Table 5.

Table 5. Transportation cost for fourth point transportation problem

	R ₁	R ₂	R ₃	Supply
A	19	9	18	9
B	26	12	28	10
C	27	15	11	11
Demand	12	10	8	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{11} = 2, x_{12} = 7, x_{21} = 10, x_{32} = 3, x_{33} = 8.$$

Applying MODI technique on this IBFS it is confirmed as optimal solution in third iteration. The corresponding transportation cost will be 460 units.

Final Allocation

Finally analyzing the combination of the allocations obtained through point wise calculation of each crisp transportation problem it is concluded that the optimum transportation cost for the FTP will be obtained if the receivers R₁, R₂ and R₃ receive manufactured goods from dealers A and B, from dealers B and C and from dealer C respectively (Table 6) and the corresponding fuzzy transportation cost will be (31, 80, 199, 460) units (Figure 1).

Table 6. Decision table

	R ₁	R ₂	R ₃
A	✓	×	×
B	✓	✓	×
C	×	✓	✓

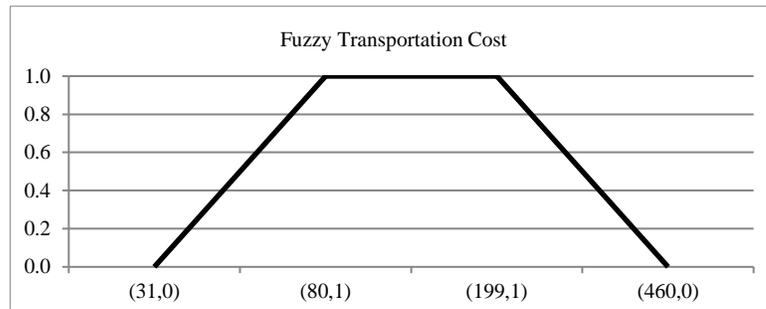


Figure 1. Optimal Fuzzy Transportation Cost

5.2 Example 2

Table 7. FTP in trapezoidal form

	R ₁	R ₂	R ₃	Supply
A	(10, 20, 30, 40)	(50, 60, 70, 90)	(80, 90, 110, 120)	(70, 90, 90, 100)
B	(60, 70, 80, 90)	(70, 80, 100, 120)	(20, 30, 50, 60)	(40, 60, 70, 80)
Demand	(30, 40, 50, 70))	(20, 30, 40, 50)	(40, 50, 50, 80)	

Solution:

This is an unbalanced FTP, so making it balanced it becomes as described in Table 8.

Table 8. Balanced form of example 2

	R ₁	R ₂	R ₃	Dummy	Supply
A	(10,20,30,40)	(50,60,70,90)	(80,90,110,120)	(0,0,0,0)	(70,90,90,100)
B	(60,70,80,90)	(70,80,100,120)	(20, 30,50,60)	(0,0,0,0)	(40,60,70,80)
Dummy	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,10,50)
Demand	(30,40,50,70)	(20,30,40,50)	(40,50,50,80)	(20,30,30,30)	

First Point Calculation

The crisp transportation problem obtained from first point value is displayed in Table 9.

Table 9. Transportation cost for first point transportation problem

	R ₁	R ₂	R ₃	Dummy	Supply
A	10	50	80	0	70
B	60	70	20	0	40
Dummy	0	0	0	0	0
Demand	30	20	40	20	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{11} = 30, x_{12} = 20, x_{14} = 20, x_{23} = 40, x_{31} = 0, x_{32} = 0.$$

Applying MODI technique on this IBFS it is confirmed as optimal solution in second iteration. The corresponding transportation cost will be 2100 units.

Second Point Calculation

The crisp transportation problem obtained from second point value is put on view in Table 10.

Table 10. Transportation cost for second point transportation problem

	R ₁	R ₂	R ₃	Dummy	Supply
A	20	60	90	0	90
B	70	80	30	0	60
Dummy	0	0	0	0	0
Demand	40	30	50	30	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{11} = 40, x_{12} = 30, x_{13} = 20, x_{23} = 30, x_{24} = 30, x_{33} = 0.$$

Applying MODI technique on this IBFS it the favorable solution is achieved in second iteration. The optimal transportation cost will be 4100 units.

Third Point Calculation

The crisp transportation problem obtained from third point value is displayed in Table 11.

Table 11. Transportation cost for third point transportation problem

	R ₁	R ₂	R ₃	Dummy	Supply
A	30	70	110	0	90
B	80	100	50	0	70
Dummy	0	0	0	0	10
Demand	50	40	50	30	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{11} = 30, x_{12} = 30, x_{14} = 30, x_{21} = 20, x_{23} = 50, x_{32} = 10.$$

Applying MODI technique on this IBFS it is confirmed as optimal solution in second iteration. The corresponding transportation cost will be 6100 units.

Fourth Point Calculation

The crisp transportation problem obtained from fourth point value is put on view in Table 12.

Table 12. Transportation cost for fourth point transportation problem

	R ₁	R ₂	R ₃	Dummy	Supply
A	40	90	120	0	100
B	90	120	60	0	80
Dummy	0	0	0	0	50
Demand	70	50	80	30	

It's a balanced problem of transportation and applying the minimum of demand-supply algorithm IBFS is

$$x_{11} = 50, x_{12} = 50, x_{23} = 50, x_{24} = 30, x_{31} = 20, x_{33} = 30.$$

Applying MODI technique on this IBFS it is confirmed as optimal solution in third iteration. The corresponding transportation cost will be 7600 units.

Final Allocation

Considering the combination of the allocations it is concluded that the optimum transportation cost for the FTP will be obtained if the receivers R₁ and R₂ receive product from supplier A and R₃ receives product from supplier B only (Table 13) and the corresponding fuzzy transportation cost will be (2100, 4100, 6100, 7600) units (Figure 2).

Table 13. Decision table

	R ₁	R ₂	R ₃	Dummy Destination
A	✓	✓	×	✓
B	×	×	✓	×
Dummy Source	×	×	×	×

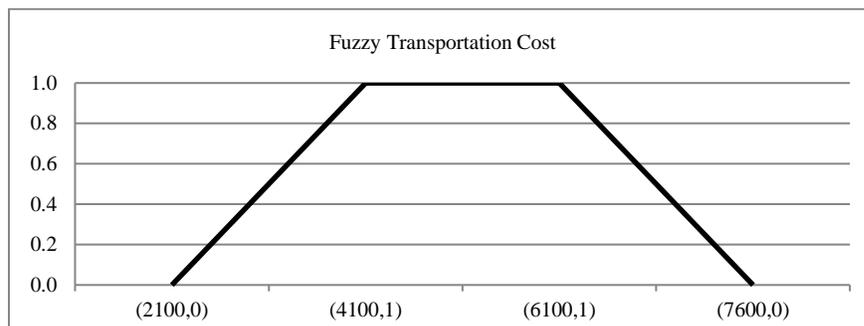


Figure 2. Optimal Fuzzy transportation cost

6. Comparative Study

Both the real life examples 1 and 2 under consideration are compared with other existing methods for the problem and comparison of allocation is depicted in Table 14 and 15. The proposed algorithm either provides better alternating solution or verifies the result.

Table 14. Comparative study of Example 1

	R ₁			R ₂			R ₃		
	Mathur et al. (2016)	Bisht and Srivastava (2018)	Proposed Technique	Mathur et al. (2016)	Bisht and Srivastava (2018)	Proposed Technique	Mathur et al. (2016)	Bisht and Srivastava (2018)	Proposed Technique
A	✓	✓	✓	✓	×	×	×	×	×
B	✓	✓	✓	×	✓	✓	×	×	×
C	×	×	×	✓	✓	✓	✓	✓	✓

Table 15. Comparative study of Example 2

	R ₁		R ₂		R ₃	
	Ebrahimnejad (2016)	Proposed Technique	Ebrahimnejad (2016)	Proposed Technique	Ebrahimnejad (2016)	Proposed Technique
A	✓	✓	✓	×	×	×
B	✓	✓	×	✓	×	×
C	×	×	✓	✓	✓	✓

7. Conclusion

This research article deals with an effortless and robust method to solve FTPs with trapezoidal fuzzy parameters. A point to point method is applied to get classical transportation problems out of FTP. Further minimum demand and supply method followed by MODI technique is used to get IBFS and optimal solution of crisp problems respectively. Hence obtained solutions of all four problems are clubbed to give a trapezoidal fuzzy solution. This fuzzy solution gives a flexibility to decision maker on particular decision. Two distinct types of the FTPs are discussed: one with balanced fuzzy transportation data and the other with un-balanced fuzzy transportation data. It is

found that this method is equally applicable on both types of problems. Results obtained by presented procedure are matched up to the existing methods. The proposed scheme is got to be more operative to solve such FTPs having data in trapezoidal fuzzy form. The final transportation cost is articulated by fuzzy number in place of crisp value, more flexible decisions can be carried out. This technique proposes noteworthy engineering and industrial relevance in decision making.

Conflict of Interest

Both authors have contributed equally in this work. The authors declare that there is no conflict of interest for this publication.

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