International Journal of Mathematical, Engineering and Management Sciences
Vol. 10, No. 2, 368-388, 2025
https://doi.org/10.33889/IJMEMS.2025.10.2.019

Ram Arti Publishers

Optimizing Ordering Policies for Imperfect Inventory Systems Integrating Trade Credit and Carbon Emission

Sahil Bhardwaj
Department of Mathematics,
Amity Institute of Applied Sciences, Amity University, Noida, 201303, Uttar Pradesh, India.
E-mail: sahilbhardwaj3@yahoo.com

Sudhir Kumar Chauhan
Department of Mathematics,
Amity Institute of Applied Sciences, Amity University, Noida, 201303, Uttar Pradesh, India.
E-mail: skchauhan@amity.edu

Mandeep Mittal
School of Computer Science Engineering and Technology,
Bennet University, Greater Noida, 201310, Uttar Pradesh, India.
Corresponding author: mittal_mandeep@yahoo.com

Leopoldo Eduardo Cárdenas-Barrón
Tecnológico de Monterrey, School of Engineering and Sciences,
E. Garza Sada 2501 Sur, 64849, Monterrey, Mexico.
E-mail: lecarden@tec.mx

Riju Chaudhary
Department of Mathematics,
Amity Institute of Applied Sciences, Amity University, Noida, 201303, Uttar Pradesh, India.
E-mail: riju.chaudhary@gmail.com

(Received on September 22, 2024; Revised on December 18, 2024 & January 19, 2025; Accepted on January 21, 2025)

Abstract
Reducing carbon emissions is about protecting the future, in which the greatest treasures are clean air, a stable temperature and a thriving planet, not only about saving the environment. Using Hill's (1997) model, this study investigates how environmental factors might be included in inventory control—more especially, how trade credit, fuel prices, and pollution taxes might be used. This paper explores the continuous difficulty of keeping a regular supply of high-quality products in the dynamic market of today, where the possibility of providing faulty goods never disappears. This study aims primarily to minimise the predicted total cost by means of a derivative-based technique and to identify the ideal order quantity. These findings highlight a strong correlation between lowering carbon emissions and attaining financial savings. This paper emphasises the critical part environmental awareness plays in efficient inventory control. A sensible framework is presented for concurrently lowering prices and carbon emissions by including trade credit into Hill's (1997) model. A numerical example together with a sensitivity analysis shows the model's resilience. Finally, this study emphasises the need to include environmental elements into inventory control plans and provides a workable road to reach both environmental responsibility and financial economy.

Keywords- EOQ, Defective items, Inspection, Trade credit, Carbon emission.

1. Introduction

An inventory is a comprehensive list of items that includes raw materials, semi-finished products in various stages of production and finished goods that are intended for sale. Public and private sector companies both extensively uses inventory control. In 1913 Harris created the Economic-Order-Quantity (EOQ) model, which grew to be the first widely used inventory model. As a result, there has been a lot of interest in this topic for research. Shipments to the store should be made during the manufacturer's cycle's production phase, as per the methodology proposed by Hill (1997). A Mathematical model for coordinating a two-level supply chain, emphasizing a joint replenishment lot size problem with a variable delay in payments was presented by Jaber and Osman (2006). Their approach considers the total holding cost as a composite of opportunity cost and stocking cost, offering a realistic trade-off. The model prevents unbounded order sizes and explores extending the delay period. Coordination with permissible delay proves advantageous in numerical examples, showcasing benefits for both retailers and suppliers. The introduction of a profit-sharing scenario highlights the potential for sustained long-term relationships and the possibility of offering quantity discounts under permissible delay. The paper suggests extending the model to incorporate multiple credit schemes and expanding from a 2-level to a 3-level supply chain.

The limitations of the Economic-Order-Quantity (EOQ) model have been extensively explored, with one notable concern being the assumption that every received item is without defects. Many scholars have investigated inventory models considering products with diverse qualities. The introduction of defective items in a batch, along with batch inspection for possible defects, was pioneered by Jaggi et al. (2011). Many firms globally are being influenced by global difficulties linked with creating eco-friendly supply chains and lowering emissions. Businesses are progressively implementing sustainable practices to improve their strategy and production schedules. Emphasising sustainability, the aim is now to reduce the negative environmental impact and minimise the expenses. This idea of including carbon emissions into manufacturing models so emerged. Wahab et al. (2011) investigated an Economic-Order- Quantity (EOQ) model considering products of different quality. They investigated the cost of carbon emissions in both domestic and global supply networks.

Recently, several methods of emission control have been applied to control emissions resulting from supply chain activities. In their 2013 research on cooperative supply chains including buyers and sellers, Jaber et al. found that production is the main cause of emissions. They advocated utilising legislation implementing a carbon fee as a way to limit these emissions. Additionally, trade-credit policies for fixed-life products are investigated by Sarkar et al. (2015) from the supplier and retailer viewpoints. While Gurtu et al. (2015) model covers transportation expenses and carbon emissions from the movement of goods, Jaber et al. (2013) included carbon emissions from the manufacturing process in their model. By integrating transportation expenses and carbon emission costs incurred during the production process into their suggested model, Aljazzar et al. (2017) extended upon Hill's method. Yadav et al. (2018) proposed Economic-Order-Quantity (EOQ) models for both cooperative and non-cooperative relationships between sellers and purchasers, accounting for faulty items. They concluded that the marketing budget and the buyer's selling price have an impact on the demand function. When compared to non-cooperative games, cooperative games resulted in higher demand and order quantities but lower selling prices and marketing expenses for the buyer. In their 2018 study, Tiwari et al. (2018) assessed a single vendor-buyer supply chain for low-quality, degrading commodities. The carbon emissions created by carrying, storing and retaining items were taken into consideration in their model. Fuel usage, pollutants produced and trip distance all have an effect on transportation-related emissions. Both the total inventory and the energy required by each item in the warehouse had an impact on carbon emissions.

A multi-item perishable inventory model that addresses deterministic needs, returns, and all-unit discounts from a single supplier is provided by Lesmono et al. (2020). They look at the optimal times to order returns and quantities to order in order to cut costs for merchants, expanding on prior work. Insights into cost dynamics are provided by their algorithmic approach, which highlights the efficiency of individual return times without discounts for reducing total costs and is supported by numerical examples and sensitivity analysis. Referring to the real variables of defective production and capacity requirements, Aastha et al. (2020) concern the suspicion of great item quality and boundless distribution centre limit of the conventional stock models. Their approach includes products of low quality and shortages and separates owned from rented warehouses with different storage capacity. Especially, following inspection, every faulty component is removed. Finding the ideal order numbers and backorder quantities can help to maximise profits by means of which the work offers solved numerical examples and a sensitivity study to offer still another perspective on the implications of the model.

By means of an economic production model for a two-echelon supply chain with supplier-offered trade credit, Gilotra et al. (2020) solves environmental problems in inventory management. Their methodology takes into account greenhouse gas emissions, energy utilisation, and human mistakes during inspections. Supported by a mathematical model and numerical examples, they investigate, by optimising environmental and economic performance, the effects of human errors, emission costs, transportation expenses, and payment delays on order replenishment sizes and retailer profit. Combining in-process inspection, learning effects and trade-credit financing with acceptable payment delays, Jayaswal et al. (2021) presents an inventory model for declining items with faulty quality. It departs from the ideal economic order quantity model and shows, by sensitivity analysis and numerical examples, the beneficial effects of trade-credit policies on the proposed model.

Yadav et al. (2021) investigate sustainable preservation strategies in the context of deteriorating products and environmentally sensitive demand. Their work emphasizes eco-friendly operations and manages deterioration losses through salvage trade. They assume upcoming demand to be price and environment-sensitive, aiming to maximize the seller's total profit by optimizing unit selling price and minimizing investments in environmental product performance and preservation technology. The paper includes a numerical example and graphs to validate the model.

Designed especially to handle detected defective items in product batches, Pimsap and Srisodaphol (2022) presented a bespoke Economic Order Quantity (EOQ) system for 2022. Their approach follows a single sampling plan, whereby a random sample picked from the lot is matched to specific criteria. Should the sample's defective products satisfy or meet the necessary criteria, the order is authorised. Every item in the rejected lots is carefully inspected and any problems are quickly fixed with fixes. Examining the impact of crumble and trade credit systems helps Tiwari et al. (2022) explore the nuances of stock control for blemished goods, therefore questioning traditional suspicions of universal item rules. Ada (2022) under agri-food supply chain management stresses the need for supplier commitment and looks at sustainable supplier selection. Considering financial, natural, and social supportability elements, their review offers a mixed strategy coordinating Fuzzy Insightful Organisation Cycle and fuzzy VIKOR. This approach is validated by the research using a thorough case study of a Turkish agri-food company. It gives supply chain managers who wish to strengthen their ties with suppliers and make decisions better for the company and the surroundings direction.

In their most recent investigation, Ahmad et al. (2023) concentrate on encouraging a special EOQ stock model custom fitted for stores handling benign to the ecosystem things. Their approach considers credit issues, carbon emissions, environmental sustainability measures, learning effects, inflationary impacts, and other elements to determine the whole cost of inventory. Chaudhary et al. (2023) widen the conversation by stressing the inclusion of carbon emissions from warehouse electricity use into their EOQ model. Furthermore, covered in their studies are triangular fuzzy demand issues and faulty goods, thereby illustrating a thorough inventory control system that considers environmental issues and uncertainty. To help reduce overall expenses, including costs associated with emissions, Raiya and Mittal (2024) suggest an Economic Order Quantity (EOQ) model that includes factors such as carbon emission taxes, defect rates, and imperfect products. The model also addresses issues like scrap, backordering, and selling prices, and is supported by a numerical example and sensitivity analysis to show its effectiveness.

This paper combines both economical and environmental factor into inventory control by keeping the reality of production in mind as 100% good production is not feasible so keeping the imperfect items in lot size is moreover practical where as some recent researchers also missed either trade credit or imperfect items. This paper combines all the factors which includes: Trade credit, Imperfect items & Environmental factors and delves into a thorough analysis of total costs, encompassing a spectrum of elements such as setup, holding, transportation, production, interest accrual, carbon emissions during manufacturing, vendor-to-buyer transit and the repercussions of defective quality items. Additionally, it examines the impact of a trade credit financing policy on the overall cost framework within the proposed model. This all-encompassing methodology offers a comprehensive insight into the myriad factors influencing total costs within the realm of inventory management. Moreover, it sheds light on the intricacies of financial decision-making, shedding further insight into effective cost management strategies and their implications on organizational efficiency and sustainability.

2. Research Gap

The study reported in the previous review consists of EOQ models considering trade credit, carbon emissions, and defective items. **Table 1** lists the numerous authors' contributions, aggregating their findings and points of view in different fields.

Table 1. Table of contribution.

Authors	EOQ-model	Imperfect-items	Trade credit	Carbon emission
Sarkar et al. (2015)		✓	✓	
Bazan et al. (2015)	✓			✓
Aljazzar et al. (2017)	✓		✓	✓
Yadav et al. (2018)	✓			✓
Jayaswal et al. (2019a)	✓	✓		
Jayaswal et al. (2019b)		✓	✓	
Jayaswal et al. (2021)	✓	✓	✓	
Sarkar et al. (2022)	✓	✓		
Yadav et al. (2022)	✓	✓		
Pimsap and Srisodaphol (2022)	✓	✓		
Tiwari et al. (2022)		✓	✓	
Singh and Chaudhary (2023)	✓			✓
Chaudhary et al. (2023)	✓	✓		✓
Ahmad et al. (2023)	✓			✓
Bhardwaj et al. (2023)	✓	✓		✓
Raiya and Mittal (2024)	✓	✓		✓
This Paper	✓	✓	✓	✓

3. Notations and Assumptions

The model development incorporates the following notations and assumptions. While additional assumptions are extracted from the Aljazzar et al. (2017) model, the fundamental assumptions are taken from Hill (1997).

3.1 Notations

Notations, along with their corresponding descriptions elucidating their meanings, are presented in **Table 2** below:

Table 2. Notations.

Q	the order lot, functioning as the decision variable, measured in units.
P	the manufacturer^' s production rate, subject to the condition D<P.
D	the retailer's demand rate, expressed in units per year.
C_r	the item unit cost for the retailer, measured in dollars per unit.
A_r	the ordering cost incurred by the retailer, quantified in dollars per shipment.
s_r	the storage holding cost for the retailer, quantified in dollars per unit per year.
h_r	the holding cost for the retailer, expressed in dollars per unit per year.
C_m	the item unit cost for the manufacturer, denominated in dollars.
A_m	the ordering cost for the manufacturer, expressed in dollars per shipment.
s_m	the storage holding cost for the retailer, measured in dollars per unit per year.
h_m	the holding cost for the manufacturer, measured in dollars per unit per year.
λ	the vendor cycle's shipment lot size multiplier.
δ	the percentage of defective items.
T_{ci}	the emission tax rate, measured in dollars per ton.
T_e	for the transport emission tax, measured in dollars.
T_{cap}	denotes the truck capacity, measured as an integer.
T_m	the manufacturer's cycle time, calculated as $\lambda Q/D$ and measured in years.
T_r	retailer's cycle = Q/D.
η	the truck count for each shipment., calculated as Q/T_{cap}
E_m	the manufacturing emission, measured in grams per unit.
d	the distance travelled on one side, measured in kilometres.
α_1	Screening rate.
TAC_m	the manufacturer's total annual cost, measured in dollars.
TAC_R	the retailer's total annual cost, measured in dollars.
$\Psi(Q)$	the supply chain system's integrated total annual cost, expressed in dollars.

3.2 Assumptions

a. This model strictly prohibits shortages, ensuring that the quantity of goods produced and supplied consistently matches consumer demand. This balance is preserved by careful inventory control, efficient supply chain management and well-planned manufacturing.
b. The demand rate persists unchanged over the specified time horizon, indicating that consumer purchasing remains unaffected by factors like price, income, or market conditions.
c. The demand rate is less than the production rate, resulting in a surplus of goods. Companies produce more goods than buyers are prepared to buy at the going rate in the market, which could result in an oversupply.
d. The supply chain exclusively deals with a single type of item, simplifying logistics and reducing the intricacies of inventory management and transportation. This minimizes the probability of errors and miscommunications.
e. The expenses related to holding units are recorded in non-production stages and include things like transportation, storage and inventory control. This covers the costs related to storing goods or resources during these particular stages.
f. All trucks possess identical capacities and follow the same predefined path throughout the supply chain. Standardizing truck paths and capacities simplifies logistics, enhances predictability and reduces variability. However, this approach may lack adaptability in responding to unforeseen changes or disruptions.
g. Products with inferior quality are offered at a discount in a different market. It is strictly forbidden for retailers to sell defective things to customers; thus, they must first inspect and classify the defective

items before trying to sell them in another market.

h. The financial cost for retailers surpasses that of manufacturers at the retailer's end. Manufacturers receive their complete cost after production, whereas retailers have additional investments in holding costs, setup costs, item costs and potential expenses linked to defective items if they are not sold at a favourable price.
i. Delivered lots may contain defective items due to the inherent challenge of achieving 100% flawless production. The ramifications of defective items within production lots necessitate vigilant monitoring.
j. The price of keeping units is known as a financial cost and it stays the same all the way down the supply chain.
k. The manufacturer's emission cost is not always fixed and may change along the supply chain. This variability is contingent on factors such as production efficiency, transportation methods and energy sources.

4. Proposed Model

The methodology of Hill's model (1997) is followed by the proposed model and **Figure 1** shows how inventory is represented in it. Where, D denotes the rate of demand for the store, the manufacturer creates Q units based on the retailer's demand. Considering the Jaber et al. (2013) model in this context, concentrating on manufacturer emissions as $E_m = aP^2 - bP + c$. Gurtu et al. (2015) described the cost of transportation-related carbon emissions using the formula $(2d_e T_{ci})$.

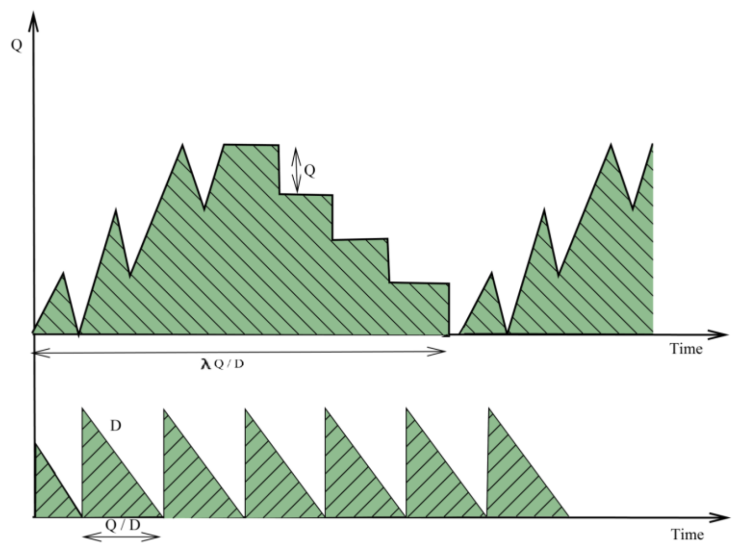

Figure 1. The process of inventory system.

4.1 Proposed Model with Permissible Delay in Payments

Within this section, Delays in payment has been integrated into the model adopted from Bhardwaj et al. (2023). Three separate scenarios concerning late payments are outlined as follows: The payment can be done in one of three ways: (1) on the last day of the grace period; (2) after the grace period but before the next lot is received; or (3) after subsequent lot is purchased.

4.1.1 Case 1 $(0 \leq t = \tau \leq T_r)$

Here, the scenario illustrates a situation in which the amount of time that can pass before interest is charged on a payment (called "t") is equal to the amount of time that a retailer needs to settle its account with a supplier (called "τ"). This duration, however, is less than the retailer's total cycle time ($\frac{Q}{D}$) but still larger than zero. The annual cost for the retailer encompasses

Setup cost (SC_r) is the cost that includes all expenses related to placing the actual order for the inventory, including those for packaging, delivery, shipping and handling here it is given as:

Setup cost, $\left(SC_r^1 = \frac{(A_r)D}{Q}\right)$ (1)

Item cost (IC_r) encompasses the payment made for the item and it incorporates any supplementary direct costs associated with transporting the item to the facility for a purchased item and here it is given as:

Item cost, $(IC_r^1 = C_r D)$ (2)

Holding cost (HC_r) refers to the expenses linked with maintaining inventory or commodities in a warehouse. These holding costs encompass various elements such as facility rent, security fees, depreciation costs and insurance and here it is given as:

Holding cost, $\left(HC_r^1 = (h_r\left(1 - \frac{2Dt}{Q} + \frac{(Dt)^2}{Q}\right) + S_r\right)\left(\frac{(1-\boldsymbol{\delta})\mathbf{Q}}{2} + \frac{\boldsymbol{\delta}\mathbf{Q}^2}{\alpha_1}\right)\right)$ (3)

Transportation cost (TC_r) represents the overall expenses related to the transportation of workers, finished goods and raw materials. This includes all costs associated with ensuring that all components are correctly positioned at the designated locations and times, guaranteeing the timely delivery of goods or services to consumers, which incurs associated costs and here it as calculated as:

Transportation cost, $\left(TC_r^1 = \left(\frac{\eta D A_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2}\right)\right)$ (4)

Emission tax (ET_r) is the tax amount integrated into the retailer's cost, where Te represents the transportation emission tax used due to the retailer's cycle. The transportation emission data is sourced from Gurtu et al. (2015) and here it is calculated as:

Cost associated with emission for retailer's cycle, $\left(ET_r^1 = \left(\frac{\eta D T_e}{Q}\right)\right)$ (5)

Interest earning (IR_r) for retailers involves strategically managing payment terms with suppliers, potentially earning additional income and optimizing cash flow and here it is calculated as:

Interest earning during this case can be expressed as, $\left(IR_r^1 = \left(C_r D\left(e^{k_r \tau} - 1\right)\right)\right)$ (6)

For a detailed expansion on the interest earning factor, please refer to Jaber et al. (2006).

For this case the Retailer's annual cost (TAC_r^1) can be defined as follows:

$$TAC_r^1 = SC_r^1 + IC_r^1 + HC_r^1 + TC_r^1 + ET_r^1 - IR_r^1 \quad (7)$$

$$TAC_r^1 = \frac{(A_r)D}{Q} + (C_r D) + (h_r\left(1 - \frac{2Dt}{Q} + \frac{(Dt)^2}{Q}\right) + S_r\right)\left(\frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{\alpha_1}\right) + \frac{\eta D A_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2} + \frac{\eta D T_e}{Q} - C_r D\left(e^{k_r \tau} - 1\right) \quad (8)$$

And the manufacturer annual cost includes,

Manufacturer setup cost (SC_m) is the cost associated with manufacturer for setup:

Setup cost, $\left(SC_m^1 = \frac{(A_m)D}{\lambda Q}\right)$ (9)

Manufacturer item cost (IC_m) represents the item cost specifically associated with the manufacturer:
Item cost, $(IC_m^1 = C_m D)$ (10)

Manufacturer holding cost (HC_m) is the holding cost specifically associated with the manufacturer:
Holding cost, $\left(HC_m^1 = \frac{(s_m+h_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_m Dt\right)$ (11)

Manufacturer Emission cost can be referred as Emission tax during manufacturing process (ET_m). E_m is the emission constraint for the manufacturer, as utilized by Jaber et al. (2013), with its value expressed as $E_m = aP^2 - bP + c$ & here emission cost is calculated as:
Cost associated for emission during manufacturer's cycle, $(ET_m^1 = E_m DT_{ci})$ (12)

Manufacturer interest earning (IR_m) involve strategically managing payment terms to potentially earn additional income on delayed payments from buyers, optimizing cash flow and enhancing financial performance and here it is given as:
Interest earning during this case can be expressed as, $\left(IR_m^1 = (C_r - C_m)D\left(e^{k_m t} - 1\right)\right)$ (13)

For this case the Manufacturer's annual cost (TAC_m^1) can be defined as follows:

$$TAC_m^1 = SC_m^1 + IC_m^1 + HC_m^1 + ET_m^1 + IR_m^1 \tag{14}$$

$$TAC_m^1 = \frac{(A_m)D}{\lambda Q} + (C_m D) + \frac{(s_m+h_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_m Dt + (C_r - C_m)D\left(e^{k_m t} - 1\right) + E_m DT_{ci} \tag{15}$$

The integrated total annual cost for the supply chain system in case 1 is calculated as the sum of the retailer's annual cost and the manufacturer's cost. This summation is derived from Equations (8) and (15), results in,

$$\Psi(Q) = \frac{(A_r)D}{Q} + \frac{(A_m)D}{\lambda Q} + D(C_r + C_m) + \left(h_r\left(1 - \frac{2Dt}{Q} + \frac{(Dt)^2}{Q}\right) + S_r\right)\left(\frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{\alpha_1}\right) + \frac{(s_m+h_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) - C_r D\left(e^{k_r \tau} - 1\right) + h_m Dt + (C_r - C_m)D\left(e^{k_m t} - 1\right) + \frac{\eta DA_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2} + \frac{\eta DT_e}{Q} + E_m DT_{ci} \tag{16}$$

The following is the expression for the derivative of the total cost function for Case 1 with respect to Q:

$$\Psi'(Q) = \frac{-(A_m + \lambda A_r)D}{\lambda Q^2} + (h_r + S_r)\left(\frac{(1-\delta)}{2} + \frac{2\delta Q}{\alpha_1}\right) + \frac{h_r \delta Dt}{\alpha_1}(Dt - 2) + \frac{(s_m+h_m)}{2}\left\{\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right\} - \frac{\eta DA_0}{Q^2} - \frac{\eta DT_e}{Q^2} \tag{17}$$

The convexity of the joint total for the supply for the case 1 is depicted in **Figure 2** below in solution procedure, using Mathematica software. It can be seen that $\Psi(Q)$ is convex function and the complete solution procedure is explained below in detail.

4.1.2 Case 2 $(0 \le t < \tau \le T_r)$

In this case, the retailer settles its payment with the supplier after the approved timeframe but before the next shipment arrives. Additionally, the annual costs for both the retailer and the manufacturer are computed in a same way as mentioned in the case 1.

The retailer annual cost, includes

Setup cost, $\left(SC_r^2 = \frac{(A_r)D}{Q}\right)$ (18)

Item cost, $(IC_r^2 = C_r D)$ (19)

Holding cost, $\left(HC_r^2 = \left(h_r\left(1 - \frac{2D\tau}{Q} + \frac{(D\tau)^2}{Q}\right) + S_r\right)\left[\frac{(1-\boldsymbol{\delta})\mathbf{Q}}{2} + \frac{\boldsymbol{\delta}\mathbf{Q}^2}{\alpha_1}\right]\right)$ (20)

Transportation cost, $\left(TC_r^2 = \left(\frac{\eta D A_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2}\right)\right)$ (21)

Cost associated with emission for retailer's cycle, $\left(ET_r^2 = \left(\frac{\eta D T_e}{Q}\right)\right)$ (22)

Interest earning during this case can be expressed as, $\left(IR_r^2 = \left(C_r D\left(e^{k_r \tau} - 1\right)\right)\right)$ (23)

In this case, the Retailer's annual cost (TAC_r^2) can be defined as follows:

$$TAC_r^2 = SC_r^2 + IC_r^2 + HC_r^2 + TC_r^2 + ET_r^2 - IR_r^2 \tag{24}$$

$$TAC_r^2 = \frac{(A_r)D}{Q} + (C_r D) + \left(h_r\left(1 - \frac{2D\tau}{Q} + \frac{(D\tau)^2}{Q}\right) + S_r\right)\left[\frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{\alpha_1}\right] + \frac{\eta D A_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2} + \frac{\eta D T_e}{Q} - C_r D\left(e^{k_r \tau} - 1\right) \tag{25}$$

And the manufacturer annual cost includes,

Setup cost, $\left(SC_m^2 = \frac{(A_m)D}{\lambda Q}\right)$ (26)

Item cost, $(IC_m^2 = C_m D)$ (27)

Holding cost, $\left(HC_m^2 = \frac{(s_m + h_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_m D\tau\right)$ (28)

Cost associated for emission during manufacturer's cycle, $(ET_m^2 = E_m D T_{ci})$ (29)

Interest earning during this case can be expressed as, $\left(IR_m^2 = (C_r - C_m)D\left(e^{k_m t} - 1\right)\right)$ (30)

In this case the manufacturer's annual cost (TAC_m^2) can be defined as follows:

$$TAC_m^2 = SC_m^2 + IC_m^2 + HC_m^2 + ET_m^2 + IR_m^2 \tag{31}$$

$$TAC_m^2 = \frac{(A_m)D}{\lambda Q} + (C_m D) + \frac{(s_m + h_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_m D\tau + (C_r - C_m)D\left(e^{k_m t} - 1\right) + E_m D T_{ci} \tag{32}$$

The integrated total annual cost for the supply chain system in case 2 is calculated as the sum of the retailer's annual cost and the manufacturer's cost. This summation is derived from Equations (25) and (32), results is,

$$\Psi(Q)=\frac{(A_r)D}{Q}+\frac{(A_m)D}{\lambda Q}+D(C_r+C_m)+\left(h_r\left(1-\frac{2D\tau}{Q}+\frac{(D\tau)^2}{Q}\right)+S_r\right)\left(\frac{(1-\delta)Q}{2}+\frac{\delta Q^2}{\alpha_1}\right)+\frac{(s_m+h_m)Q}{2}\left(\frac{2D}{P}+\frac{\lambda(P-D)}{P}-1\right)+h_m D\tau-C_rD\left(e^{k_r\tau}-1\right)+(C_r-C_m)D\left(e^{k_m t}-1\right)+\frac{\eta DA_0}{Q}+\frac{\eta A_1(2\lambda n-\lambda-1)}{2}+\frac{\eta DT_e}{Q}+E_mDT_{ci} \tag{33}$$

The following is the expression for the derivative of the total cost function for case 2 with respect to Q:

$$\Psi'(Q)=\frac{-(A_m+\lambda A_r)D}{\lambda Q^2}+(h_r+S_r)\left(\frac{(1-\delta)}{2}+\frac{2\delta Q}{\alpha_1}\right)+\frac{h_r\delta D\tau}{\alpha_1}(D\tau-2)+\frac{(s_m+h_m)}{2}\left(\frac{2D}{P}+\frac{\lambda(P-D)}{P}-1\right)-\frac{\eta DA_0}{Q^2}-\frac{\eta DT_e}{Q^2} \tag{34}$$

The convexity of the joint total for the supply for the case 2 is depicted in **Figure 3** below in solution procedure section, using Mathematica software. It can be seen that $\Psi(Q)$ is convex function and the complete solution procedure is explained below in detail.

4.1.3 Case 3 $(O \le t \le T_r < \tau)$

In this case, the retailer pays its dues after the allowed timeframe and subsequent to receiving the next shipment. The annual costs for both the retailer and the manufacturer are calculated in a similar manner as done for the case 1 and case 2.

The retailer annual cost includes,

Setup cost, $\left(SC_r^3=\frac{(A_r)D}{Q}\right)$ (35)

Item cost, $(IC_r^3=C_rD)$ (36)

Holding cost, $\left(HC_r^3=S_r\left[\frac{(1-\delta)Q}{2}+\frac{\delta Q^2}{\alpha 1}\right]\right)$ (37)

Transportation cost, $\left(TC_r^3=\left(\frac{\eta DA_0}{Q}+\frac{\eta A_1(2\lambda n-\lambda-1)}{2}\right)\right)$ (38)

Cost associated with emission for retailer's cycle, $\left(ET_r^3=\left(\frac{\eta DT_e}{Q}\right)\right)$ (39)

Interest earning during this case can be expressed as, $\left(IR_r^3=C_rD\left(e^{k_r(\tau-t)}-1\right)\right)$ (40)

In this case the retailer's annual cost (TAC_r^3) can be defined as follows:

$$TAC_r^3=SC_r^3+IC_r^3+HC_r^3+TC_r^3+ET_r^3-IR_r^3 \tag{41}$$

$$TAC_r^3=\frac{(A_r)D}{Q}+(C_rD)+S_r\left(\frac{(1-\delta)Q}{2}+\frac{\delta Q^2}{\alpha_1}\right)+\frac{\eta DA_0}{Q}+\frac{\eta A_1(2\lambda n-\lambda-1)}{2}+\frac{\eta DT_e}{Q}-C_rD\left(e^{k_r\tau}-1\right) \tag{42}$$

And the manufacturer annual cost includes,

Setup cost, $\left(SC_m^3=\frac{(A_m)D}{\lambda Q}\right)$ (43)

Item cost, $(IC_m^3=C_mD)$ (44)

Holding cost, $\left(HC_m^3=\frac{(s_m+h_m)Q}{2}\left(\frac{2D}{P}+\frac{\lambda(P-D)}{P}-1\right)+h_mD\tau\right)$ (45)

Cost associated for emission during manufacturer's cycle, $(ET_m^3=E_mDT_{ci})$ (46)

Interest earning during this case can be expressed as, $\left(IR_m^3 = (C_r - C_m)D\left(e^{k_m t} - 1\right)\right)$ (47)

In this case the Manufacturer's annual cost (TAC_m^3) can be defined as follows:

$$TAC_m^3 = SC_m^3 + IC_m^3 + HC_m^3 + ET_m^3 + IR_m^3 \tag{48}$$

$$TAC_m^3 = \frac{(A_m)D}{\lambda Q} + (C_m D) + \frac{(s_m + h_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_m D\tau + (C_r - C_m)D\left(e^{k_m t} - 1\right) + E_m DT_{ci} \tag{49}$$

The integrated total annual cost for the supply chain system in case 3 is calculated as the sum of the retailer's annual cost and the manufacturer's cost. This summation is derived from Equations (42) and (49), results in,

$$\Psi(Q) = \frac{(A_r)D}{Q} + \frac{(A_m)D}{\lambda Q} + D(C_r + C_m) + S_r\left(\frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{\alpha_1}\right) + \frac{(h_m + s_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) - C_r D\left(e^{k_r(\tau - t)} - 1\right) + h_m D\tau + (C_r - C_m)D\left(e^{k_m t} - 1\right) + \frac{\eta D A_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2} + \frac{\eta D T_e}{Q} + E_m DT_{ci} \tag{50}$$

The following is the expression for the derivative of the total cost function for case 3 with respect to Q:

$$\Psi'(Q) = \frac{-(A_m + \lambda A_r)D}{\lambda Q^2} + (S_r)\left(\frac{(1-\delta)}{2} + \frac{2\delta Q}{\alpha_1}\right) + \frac{(h_m + s_m)}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) - \frac{\eta D A_0}{Q^2} - \frac{\eta D T_e}{Q^2} \tag{51}$$

The convexity of the joint total for the supply for the case 3 is illustrated in **Figure 4** below in solution procedure section, using Mathematica software. It can be seen that $\Psi(Q)$ is convex function and the complete solution procedure is explained below in detail.

4.2 Integrated Total Annual Cost for Supply Chain System (Bhardwaj et al., 2023)

In this section, the model formed by Bhardwaj et al. (2023) is discussed which clearly shows that if the trade credit policy has been revoked from the proposed model and then the proposed model has been reduced to model formed by Bhardwaj et al. (2023) their total joint cost for supply chain which is the sum of manufacturer's cost and retailer's annual cost is:

$$\Psi(Q) = \frac{(A_m + \lambda A_r)D}{\lambda Q} + D(C_r + C_m) + (h_r + S_r)\left(\frac{(1-\delta)Q}{2} + \frac{\delta Q^2}{\alpha_1}\right) + \frac{(h_m + s_m)Q}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + \frac{\eta D A_0}{Q} + \frac{\eta A_1(2\lambda n - \lambda - 1)}{2} + \frac{\eta D T_e}{Q} + E_m DT_{ci} \tag{52}$$

Bhardwaj et al. (2023) model has been considered as base model for this paper.

The following is the expression for the derivative of the total cost function for this Case with respect to Q:

$$\Psi'(Q) = \frac{-(A_m + \lambda A_r)D}{\lambda Q^2} + (h_r + S_r)\left(\frac{(1-\delta)}{2} + \frac{2\delta Q}{\alpha_1}\right) + \frac{(h_m + s_m)}{2}\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) - \frac{\eta D A_0}{Q^2} - \frac{\eta D T_e}{Q^2} \tag{53}$$

4.3 Solution Procedure

In this section, the detailed solution approach is discussed for all three cases.

Step 1: step 1 is to compute the cost function's derivative. $\frac{d\Psi(Q)}{dQ} = 0$.

Step 2: step 2 is figuring out the cost function's second derivative. $\frac{d^2\Psi(Q)}{dQ^2}$.

Step 3: After the function's second derivative is calculated, it is found to be greater than zero. $\frac{d^2\Psi(Q)}{dQ^2} > 0$, indicating point of minima.

For case 1: $(0 \leq t = \tau \leq T_r)$

$$\Psi''(Q) = \frac{2(A_m+\lambda A_r)D}{\lambda Q^2} + (h_r + S_r)\left(\frac{\delta}{\alpha 2}\right) + 2\eta D\left(\frac{A_0}{Q^3} + \frac{T_e}{Q^3}\right) > 0 \tag{54}$$

For case 2: $(0 \leq t < \tau \leq T_r)$

$$\Psi''(Q) = \frac{2(A_m+\lambda A_r)D}{\lambda Q^3} + (h_r + S_r)\left(\frac{\delta}{\alpha 2}\right) + 2\eta D\left(\frac{A_0}{Q^3} + \frac{T_e}{Q^3}\right) > 0 \tag{55}$$

For case 3: $(0 \leq t \leq T_r < \tau)$

$$\Psi''(Q) = \frac{2(A_m+\lambda A_r)D}{\lambda Q^3} + (S_r)\left(\frac{\delta Q}{\alpha 2}\right) + 2\eta D\left(\frac{A_0}{Q^3} + \frac{T_e}{Q^3}\right) > 0 \tag{56}$$

Step 4: For getting the minimum value of (*Q*) continue solving the equation by assigning a zero value to the first derivative. $\frac{d\Psi(Q)}{dQ} = 0$.

Step 5: Utilize mathematical software to determine the value of Q = Q* (considered) and perform the calculation $\frac{d^2\Psi(Q)}{dQ^2}$ and if it results $\frac{d^2\Psi(Q)}{dQ^2} > 0$, then Q = Q* is called optimal value of *Q* and denoted by Q*.

The convexity of the expected total cost functions is also established graphically; graphs for all three cases are shown in see **Figures 2** to **4**.

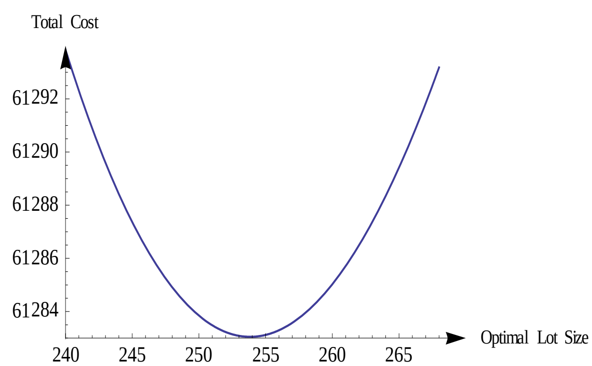

Figure 2. Convexity of aggregate total cost for supply chain in case 1.

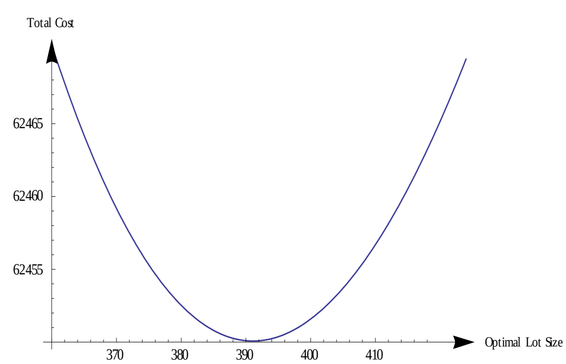

Figure 3. Convexity of aggregate total cost for supply chain in case 2.

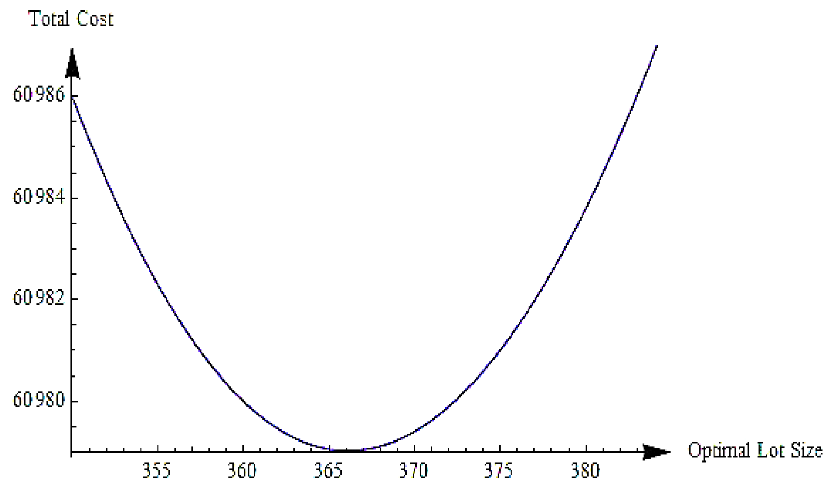

Figure 4. Convexity of aggregate total cost for supply chain in case 3.

4.3.1 Algorithm used for Solution

Step 1: Enter the values of C_m, P, A_m, D, h_m, k_m, s_m, A_r, h_r, s_r, C_r, k_r, T, D, λ, δ, T_e, T_{ci}, T_{cap}, a, b, c, E_m, η & α_1 into Equation (16).
Step 2: Set Q = Q*
Step 3: Finding the value of Q* from Equation (17).
Step 4: Using the value of Q* and then evaluate $\Psi(Q)$.
Step 5: If $0 \leq t = \tau \leq T_r$ (Case 1) then find the expected total cost, $\Psi(Q)$ from Equation (16).
Step 6: Repeating the step-1 for Case 2 and again set Q = Q*
Step 7: Finding the value of Q* from Equation (34).
Step 8: Using the value of Q* and then evaluate $\Psi(Q)$.
Step 9: If $0 \leq t < \tau \leq T_r$ (case 2) then find the expected total cost, $\Psi(Q)$ from Equation (33).
Step 10: Repeating the step-1 for Case 3 and again set Q = Q*
Step 11: Finding the value of Q* from Equation (51).
Step 12: Using the value of Q* and then evaluate $\Psi(Q)$.
Step 13: If $0 \leq t \leq T_r < \tau$ (case 3) then find the expected total cost, $\Psi(Q)$ from Equation (50).
Step 14: Compare the expected total costs for cases 1, 2, 3, and check for minimum expected total cost.

Select the case with the lowest anticipated supply chain total cost and the correspondingly best Q* values.

5. Numerical Problem

This study's numerical analysis mostly uses data from an earlier publication (Aljazzar et al., 2017). Additionally, Gurtu et al. (2015) provided statistics on fuel use and transport emissions, while Jaber et al. (2013) provided data on manufacturing emissions. **Table 3** provides a full breakdown of the input parameter values used in the investigation.

Mathematica 11.0 software was used in the calculation process to maximize the quantity of orders as well as the overall incorporated cost in the supply chain. The basic model's optimal order quantity is Q* = 223 units, and the optimal integrated cost relative to this optimal order quantity is $\Psi^*(Q)$=60517 \$. The below **Table 4** include information on the trade credit policy for case 1, case 2, and case 3.

Table 3. Parameters for input.

Variable	Value	Units	Variable	Value	Units
C_m	$15	per unit	δ	1-6	%
P	3200	units per year	T_e	2d*0.00077344*30	$
A_m	$200	per order	T_{ci}	30	$/Ton
D	1000	units per year	T_{cap}	80	units/truck
h_m	$ 3	per unit per year	a	0.0000003	unit less
k_m	0.1	%	b	0.0012	unit less
s_m	$ 9	per unit per year	c	1.4	unit less
A_r	$ 30	per order	E_m	aP^2-bP +c	g/unit
h_r	$ 4	per unit per year	Q^*	223	units
s_r	$ 12	per unit per year	$\Psi(Q^*)$	60517	$
C_r	$ 20	per unit	η	3	unit less
k_r	0.2	%	α_1	175200	unit less
T	5	years			
d	500	km			
λ	2	unit less			

Table 4. Proposed model comparison with conventional model.

Model	Defective item percentage	Lot size (Q)	Total cost ($)
Base Paper	0.00	222	60487
	0.01	222	60535
	0.02	223	60517
Case 1	Without Trade Credit at (0.02)	223	60517
	With Trade Credit at (0.02)	251	61361
Case 2	Without Trade Credit at (0.02)	223	60517
	With Trade Credit at (0.02)	361	61069
Case 3	Without Trade Credit at (0.02)	223	60517
	With Trade Credit at (0.02)	385	62548

6. Sensitivity Analysis

A crucial part of research is analyzing model parameters and sensitivity analysis that can help to evaluate and so that the judgments could be clear. **Tables 5-7** and **Figures 5-10** support this analysis where the impact on total cost is clearly visible as **Tables 5-7** which represents the effect of defective item percentage on lot size and aggregated total cost for all the three cases mentioned above and to support the findings their graphical representations has shown below from the **Figures 5-10**. It has been observed that with the impact of defective items lot size increases and the total cost decreases from which it is easy to opt for the best case and also it is required to balance the carbon emission factor for which ideal case is found to be case 2.

Table 5. Effect of defective item percentage on lot size and aggregate total cost in case 1.

Defective item percentage	Corresponding lot size (Q)	Total cost ($)
0.010	250	61381
0.020	251	61361
0.030	252	61342
0.040	252	61322
0.050	253	61303
0.060	254	61283

Table 6. Effect of defective item percentage on lot size and aggregate total cost in case 2.

Defective item percentage	Corresponding lot size (Q)	Total cost ($)
0.010	360	61091
0.020	361	61069
0.030	362	61047
0.040	363	61024
0.050	365	61002
0.060	366	60979

Table 7. Effect of defective item percentage on lot size and aggregate total cost in case 3.

Defective item percentage	Corresponding lot size (Q)	Total cost ($)
0.010	384	62566
0.020	385	62548
0.030	387	62520
0.040	388	62497
0.050	390	62473
0.060	391	62450

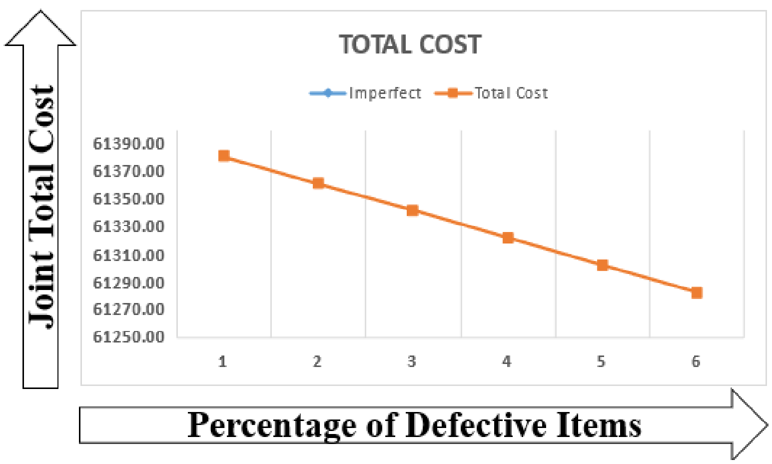

Figure 5. Effect of defective items percentage on total cost for case 1.

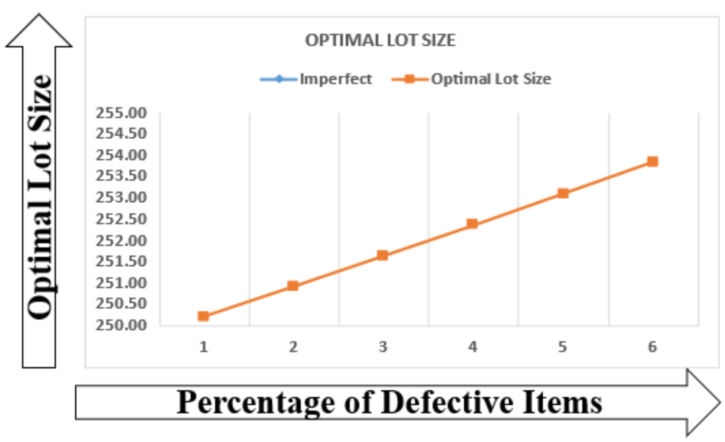

Figure 6. Effect of defective items percentage on lot size for case 1.

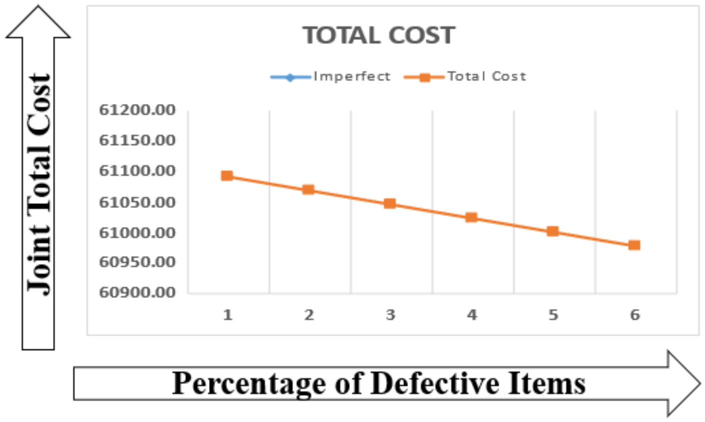

Figure 7. Effect of defective items percentage on total cost for case 2.

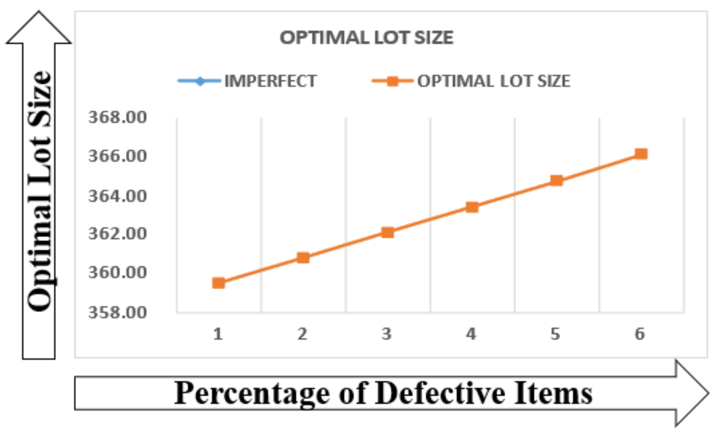

Figure 8. Effect of defective items percentage on lot size for case 2.

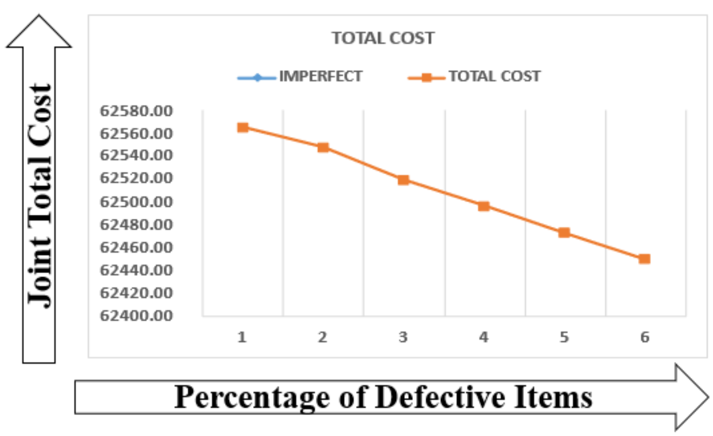

Figure 9. Effect of defective items percentage on total cost for case 3.

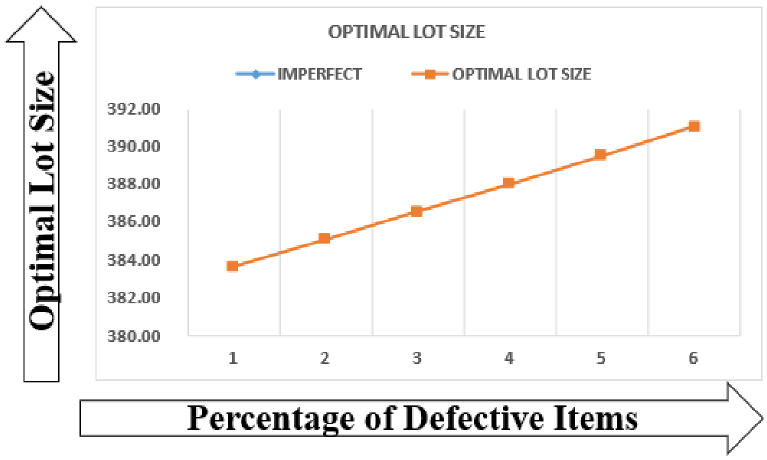

Figure 10. Effect of defective items percentage on lot size for case 3.

Out of all three cases evaluated, case 2 turns out to be the best option mostly because of its amazing cost efficiency and rather low carbon emissions, as Aljazzar et al. (2017) note. The results demonstrating that case 2 employs effective lot sizes and resource allocation techniques justify this choice since they help to lower running expenses. Aljazzar et al. also underline the environmental sustainability of the instance by stressing its much smaller carbon footprint than in the other cases. Case 2 is chosen since it provides both environmental responsibility and financial affordability. Case 2 is thus an ideal option for more investigation and discussion in pertinent decision-making procedures. **Table 7** shows, at a 0.02 level of defective goods, a comparison of expenses among the three cases. Examined for their respective initial investments, running expenses, and overall costs are cases 1, 2, and 3. This study offers insightful information on the financial results of every scenario, therefore supporting the assessment of cost-effectiveness under the given defect rate condition.

Table 7. Cost comparison.

Model	Defective item percentage	Lot size (Q)	Total cost ($)
Case 1	Without trade credit at (0.02)	223	60517
	With trade credit at (0.02)	251	61361
Case 2	Without trade credit at (0.02)	223	60517
	With trade credit at (0.02)	361	61069
Case 3	Without trade credit at (0.02)	223	60517
	With trade credit at (0.02)	385	62548

6.1 Managerial Insights

This study has practical consequences, emphasising that strategically applying authorised delay-in-payments can reduce the total cost of coordinated supply chains effectively. Even when factoring in costs such as emissions, transportation, manufacturing and fuel the resulting savings might be utilised to enhance environmental performance or support social and community projects. Reinvesting these savings into operational improvements, technology, processes, or equipment efficiency enhances sustainability, leading to greener supply chain practices. Additionally, directing savings towards research and development of eco-friendly materials and packaging further diminishes the carbon footprint while concurrently lowering overall system costs. Selecting the optimal scenario for offering a permissible delay in payments is crucial to enhance both the economic factor and the environmental performance of a supply chain. Among the evaluated scenarios, case 2 stands out as the optimal choice, demonstrating superior environmental and economic benefits according to sensitivity analysis results. Conversely, case 3 is deemed impractical, and case 1 incurs higher costs compared to the favourable attributes of case 2. The comprehensive assessment highlights that case 2 is the most fitting option, striking a balance between environmental sustainability and economic efficiency, thus making it the preferred choice for enhancing the overall performance of the supply chain.

Furthermore, sensitivity analysis also supports case 2 which makes it easier for the manager to choose the best case over the others, as case 2 is the most cost-effective and has the least impact on the environment. This study confirms the long-term advantages of case 2 in enhancing overall supply chain efficiency by offering vital decision-making insights that enable companies to choose the most economically and ecologically sound course of action.

7. Conclusion and Future Scope

The findings from this study have attempted to investigate the implications of inferior quality items in a lot, focusing on how this affects various costs and retailer ordering practices. The research seems to be primarily concerned with the possible advantages of including defective items and how this could impact the dynamics of the whole supply chain. The study's conclusions imply that including defective goods may increase lot size while simultaneously reducing item cost, holding cost, setup cost, transportation cost and emission tax expenses. The research also emphasizes how retailer ordering policies can be positively influenced by taking into account both carbon emissions and defective goods. Crucially, the incorporation of trade credit outcomes unveils an additional category of impact on the comprehensive cost framework and procurement choices, thus enhancing the complex understanding of the processes involved in inventory control concerning defective items. This paper supports its conclusions with mathematical analysis by presenting a model that might be built by combining a supply chain model with learning under a credit policy involving trade credit. The proposed approach is meant to help companies effectively include defective products into their supply chain operations while considering the impact of trade credit on buying decisions. All things considered, this paper looks at the possible benefits of integrating faulty goods in batch sizes and their effects on the supply chain. Together with the additional dimension of trade credit, it emphasises the need of factor in carbon emissions and faulty products in carbon emissions and design of retailer buying methods. Incorporating these parameters into their decision-making process helps companies to maximise their supply chain, cut expenses, and keep quality standards generally intact. The results of the paper offer companies looking for a whole strategy for supply chain management optimisation important new perspectives. The paper offers several paths for more research. First, one can investigate several approaches to treat goods with low quality. This could call for hiring personnel or acquiring tools for repair, shipping broken items to a repair facility instead of selling them for less, or doing on-site repairs. These choices may increase the expected total profit even if they would cost more in terms of labour, transportation and maintenance.

Lead time should be included into manufacturing, transportation, and setup schedules to help the current item be better. This approach could provide valuable insights on how variations in lead time influence the general profitability and cost of the supply chain. Future studies can offer more sophisticated solutions including these extra components that managers can choose to maximise their operations linked to supply chains. Furthermore, a possible expansion could let the inventory management paradigm experience partial backlogging or shortages, including learning effects will also help to clarify the dynamics influencing the total cost and profit in the framework of defective products, including multi-echelon supply chains, incorporating dynamic pricing based on carbon credits, or by using the model beyond the manufacturing industries and one more possibility by integrating real-time data to trace down carbon emission effect. These additions offer useful insights for managerial decision-making and help to clarify supply chain optimisation.

Conflict of Interest

There is no conflict of interest to declare for this publication.

AI Disclosure

During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

Acknowledgments

We would like to express our heartfelt gratitude to all who have contributed to the successful completion of this research work. We would also like to thank Amity Institute of Applied Sciences, Amity University Uttar Pradesh, for their constant support. Also, we sincerely thanks to the reviewers for their valuable feedback and constructive comments, which helps to the improve the quality of this work.

References

Aastha, Pareek, S., Cárdenas-Barrón, L.E., & Mittal, M. (2020). Impact of imperfect quality items on inventory management for two warehouses with shortages. *International Journal of Mathematical, Engineering and Management Sciences*, *5*(5), 869-885. https://doi.org/10.33889/ijmems.2020.5.5.067.

Ada, N. (2022). Sustainable supplier selection in agri-food supply chain management. *International Journal of Mathematical, Engineering and Management Sciences*, *7*(1), 115-130. https://doi.org/10.33889/ijmems.2022.7.1.008.

Ahmad, N., Jaysawal, M.K., Sangal, I., Kumar, S., & Alam, K. (2023). Carbon tax and inflationary conditions under learning effects: a green EOQ inventory model. *Macromolecular Symposia*, *407*(1), 2200117. https://doi.org/10.1002/masy.202200117.

Aljazzar, S.M., Jaber, M.Y., & Moussawi-Haidar, L. (2017). Coordination of a three-level supply chain (supplier–manufacturer–retailer) with permissible delay in payments and price discounts. *Applied Mathematical Modelling*, *48*, 289-302. https://doi.org/10.1016/j.apm.2017.04.011.

Bazan, E., Jaber, M.Y., & Zanoni, S. (2015). Supply chain models with greenhouse gases emissions, energy usage and different coordination decisions. *Applied Mathematical Modelling*, *39*(17), 5131-5151. https://doi.org/10.1016/j.apm.2015.03.044.

Bhardwaj, S., Mittal, M., & Chaudhary, R. (2023). Managing carbon foot prints in supply chain for imperfect quality items. *Journal of Reliability and Statistical Studies*, *16*(1), 117-136.

Chaudhary, R., Mittal, M., & Jayaswal, M.K. (2023). A sustainable inventory model for defective items under fuzzy environment. *Decision Analytics Journal*, *7*, 100207. https://doi.org/10.1016/j.dajour.2023.100207.

Gilotra, M., Pareek, S., Mittal, M., & Dhaka, V. (2020). Effect of carbon emission and human errors on a two-echelon supply chain under permissible delay in payments. *International Journal of Mathematical, Engineering and Management Sciences*, *5*(2), 225-236. https://doi.org/10.33889/ijmems.2020.5.2.018.

Gurtu, A., Jaber, M.Y., & Searcy, C. (2015). Impact of fuel price and emissions on inventory policies. *Applied Mathematical Modelling*, *39*(3-4), 1202-1216. https://doi.org/10.1016/j.apm.2014.08.001.

Hill, R.M. (1997). The single-vendor single-buyer integrated production-inventory model with a generalised policy. *European Journal of Operational Research*, *97*(3), 493-499.

Jaber, M.Y., & Osman, I.H. (2006). Coordinating a two-level supply chain with delay in payments and profit sharing. *Computers & Industrial Engineering*, *50*(4), 385-400. https://doi.org/10.1016/j.cie.2005.08.004.

Jaber, M.Y., Glock, C.H., & El Saadany, A.M. (2013). Supply chain coordination with emissions reduction incentives. *International Journal of Production Research*, *51*(1), 69-82. https://doi.org/10.1080/00207543.2011.651656.

Jaggi, C.K., Khanna, A., & Mittal, M. (2011). Credit financing for deteriorating imperfect-quality items under inflationary conditions. *International Journal of Services Operations and Informatics*, *6*(4), 292-309.

Jayaswal, M., Sangal, I., Mittal, M., & Malik, S. (2019a). Effects of learning on retailer ordering policy for imperfect quality items with trade credit financing. *Uncertain Supply Chain Management*, *7*(1), 49-62. https://doi.org/10.5267/j.uscm.2018.5.003.

Jayaswal, M.K., Mittal, M., & Sangal, I. (2021). Ordering policies for deteriorating imperfect quality items with trade-credit financing under learning effect. *International Journal of System Assurance Engineering and Management*, *12*, 112-125. https://doi.org/10.1007/s13198-020-01038-y.

Jayaswal, M.K., Sangal, I., & Mittal, M. (2019b). Learning effect on stock-policies with Imperfect quality and deteriorating Items under trade credit. In *2019 Amity International Conference on Artificial Intelligence* (pp. 499-504). IEEE. Dubai, United Arab Emirates.

Lesmono, D., Limansyah, T., & Loedy, N. (2020). A joint return policy for a multi-item perishable inventory model with deterministic demands, return and all-units discount. *International Journal of Mathematical, Engineering and Management Sciences*, *5*(3), 416-431. https://doi.org/10.33889/ijmems.2020.5.3.035.

Pimsap, P., & Srisodaphol, W. (2022). Economic order quantity model of imperfect items using single sampling plan for attributes. *Journal of Applied Science and Engineering*, *25*(6), 1217-1225.

Raiya, G., & Mittal, M. (2024). Optimal lot size and backordered quantity under carbon tax. *International Journal of Applied and Computational Mathematics*, *10*(4), 136. https://doi.org/10.1007/s40819-024-01703-x.

Sarkar, B., Saren, S., & Cárdenas-Barrón, L.E. (2015). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Annals of Operations Research*, *229*, 677-702. https://doi.org/10.1007/s10479-014-1745-9.

Sarkar, S., Tiwari, S., & Giri, B.C. (2022). Impact of uncertain demand and lead-time reduction on two-echelon supply chain. *Annals of Operations Research*, *315*(2), 2027-2055. https://doi.org/10.1007/s10479-021-04105-0.

Singh, S., & Chaudhary, R. (2023). Effect of inflation on EOQ model with multivariate demand and partial backlogging and carbon tax policy. *Journal of Future Sustainability*, *3*(1), 35-58. https://doi.org/10.5267/j.jfs.2022.11.004.

Tiwari, S., Cárdenas-Barrón, L.E., Malik, A.I., & Jaggi, C.K. (2022). Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit. *Computers & Operations Research*, *138*, 105617. https://doi.org/10.1016/j.cor.2021.105617.

Tiwari, S., Daryanto, Y., & Wee, H.M. (2018). Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. *Journal of Cleaner Production*, *192*, 281-292. https://doi.org/10.1016/j.jclepro.2018.04.261.

Wahab, M.I.M., Mamun, S.M.H., & Ongkunaruk, P. (2011). EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. *International Journal of Production Economics*, *134*(1), 151-158.

Yadav, R., Pareek, S., & Mittal, M. (2018). Supply chain models with imperfect quality items when end demand is sensitive to price and marketing expenditure. *RAIRO-Operations Research-Recherche Opérationnelle*, *52*(3), 725-742. https://doi.org/10.1051/ro/2018011.

Yadav, R., Pareek, S., Mittal, M., & Jayaswal, M.K. (2022). Two-level supply chain models with imperfect quality items when demand influences price and marketing promotion. *Journal of Management Analytics*, *9*(4), 480-495. https://doi.org/10.1080/23270012.2021.1944350.

Yadav, S., Khanna, A., & Jaggi, C.K. (2021). Sustainable preservation strategies with deterioration management and environment sensitive demand. *International Journal of Mathematical, Engineering and Management Sciences*, *6*(4), 1089-1099. https://doi.org/10.33889/ijmems.2021.6.4.064.