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A Situational Based Reliability Indices Estimation of ULT Freezer using Preventive Maintenance under Fuzzy Environment

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Abstract
Reliability of high demand machines is quite necessary and it can be maintained through proper and timely maintenance, Ultra-low temperature (ULT) freezer is one of those kinds of machines which are in high demand during covid-19 pandemic for the storage of vaccine. The rapid production of vaccines for the prevention of coronavirus disease 2019 (COVID-19) is a worldwide requirement. Now the next challenge is to store the vaccine in a ULT freezer. It's become really a big problem to store the vaccine which creates the demand of ULT freezer. The present paper investigates a situational based performance of the ULT freezer with the aim to predict the impact of different component failures as well as human errors on the final performance of the same. For the study, it is not possible to extract the parameters (failure rate and repair time) of the components that never failed before. Thus, to overcome this difficulty, here authors use the possibility theory. Authors present the available data in Right triangular fuzzy number with some tolerance as suggested by system analyst. The lambda-tau methodology and arithmetic operations on right triangular generalized fuzzy numbers (RTrFN) are used to find the various performance parameters namely MTTF, MTTR, MTBF, reliability, availability, maintainability (RAM) and ENOF, under fuzzy environment. The proposed model has been studied using possibility theory under working conditions, preventive maintenance as well as under the rest of conditions. This study reveals the most and least critical component of the ULT freezer which helps maintenance department to plan the maintenance strategy accordingly.

Keywords- ULT Freezer, Uncertainty, Reliability, Confidence level, Fuzzy number.

1. Introduction

Corona virus disease-2019 (COVID-19) is a disorder that develops in many forms across a wide variety of issues. The pandemic of COVID-19 is a social, medical and health concern. After the invention of the vaccine the next challenge is to store the vaccine at a place where it will not lose its efficacy. Scientists found ULT freezer as a solution of this problem because it maintains the temperature which is needed for vaccine. Many vaccines lose their efficacy when exposed to higher temperatures, and refrigeration does not compensate. It's really a worldwide phenomenon where the demand for ultra-cold storage is become so high. Vaccines are very temperature sensitive. If temperature is not maintained properly, then it will start losing their potential and immunization program will have no use. Everyone who is going to be potentially storing Covid-19 vaccine is looking to get their hands on something like ULT freezer.

1.1 Related Work

Many authors discussed the major challenges and opportunities against the vaccine like manufacturing, distribution, vaccination phases. Paltiel et al. (2021) investigate various parameters to deploy the best vaccine. They studied the effects of first and second dose of vaccine and found the relation between speed and efficacy for the betterment of vaccine. Wang et al. (2020) discussed the different types of vaccines and their compositions along with the advantages and potential drawbacks. It was observed that storage is also the major challenge faced in the vaccination process. Mills and Salisbury (2021) predicted that vaccination could initiate with the help of collaboration of government, manufacturing companies, health workers as well as public. In addition to this, cold storage is also a major factor for vaccine supplies. Jeyanathan et al. (2020) addressed the technical and logistical obstacles that would be faced in the phase of producing an effective vaccine as well as how COVID-19 vaccine strategies will change over the next few years. Rele (2021) provided an important overview of vaccine production processes and activities that can interrupt a pandemic response, and also applies to other evolving attempts to develop infectious diseases. Bell et al. (2001) indicated that issues with the storage of vaccines are frequent and are primarily due to insufficient management of cold storage systems or the use of freezer systems is insufficient, compact refrigerator/freezer units. They concluded that all the workers involved in vaccine handling should be educated to ensure that everyone knows the significance of the temperature scale and that everyone has access to the current authoritative guidelines on vaccine storage. The paper reveals the study of high reliability refrigerated storage structure systems. It was described in the research that doors and the structure of the door(s) and the place where the vaccines are stored are crucial provided the transient temperature raise during the time when the door opens for removing samples and storing the samples. Friend and Stone (2015) have provided a technical solution to support the cold chain of Ebola vaccines in Sierra Leone Guinea to meet the challenges in the resource environment on a time-limited basis. Rastegar et al. (2021) proposed an inventory-location mixed-integer framework for the delivery of influenza vaccines in developed countries after a pandemic. Song et al. (2021) provides suitable refrigerant-couples and corresponding enhancing methodologies in this study, and instead of COP, they analyse dynamic characteristics of operating stability and security for ultra-low temperature freezers. In addition to this, it also helps in strengthen the operating stability as well as security. It has been observed that along with preventive and regular maintenance of the system, system's components failure is an unavoidable event. These research papers motivate us to focus on the area of reliability for vaccine storage unit.

Since the data calculated to carry the research is uncertain and vague. There are no crisp failure rates and repair time. There is some tolerance power in the data. So, for this kind of the data, we can extend the classical set theory to fuzzy set theory to handle the uncertainty. Due to the imprecise available data, it is difficult for the analyst to find the performance of the system more accurately. So, the main concentration is to reduce the impreciseness of the data to get more accuracy. Many authors used the different techniques to handle these kinds of issues. Cai (1996) addressed various facts of the implementation of fuzzy methodology in system failure engineering, including fuzzy technique in fault diagnosis, in probist structures, in structural reliability, in application reliability, in human reliability, in safety engineering, in risk engineering, and in quality management, as well as profuse, posbist and postfust reliability theories. Garg (2017) analyzed the performance of cattle feed plant using soft computing based hybridized technique. Cheng and Mon (1993) used the fuzzy number over the confidence interval in place of probability and obtain the range value and the range of fuzzy reliability. The authors Sharma et al. (2008) suggested a methodological and organized approach incorporating both qualitative and quantitative risk in a system's stability measurement. In order to increase the stability and maintenance characteristics of the system, an in-depth qualitative review of the systems is carried out using Failure Mode and Effect Analysis (FMEA) by listing all potential failure modes, their causes and their effects on system performance. Mahmood et al. (2013) used fault tree analysis (FTA) to investigate a system's failure and found the overall reliability for the same. However, there are significant drawbacks to using standard FTA, such as addressing uncertainties, permitting the inclusion of language variables, and accounting for human mistake in failure logic models. As a result, fuzzy set theory has been developed to get about the drawback of traditional FTA. Goyal et al. (2022) used Rouben Ranking function to find the solution of a multi-level multi objective quadratic fractional programming model. Kumar et al. (2021) analyzed a linear and circular *k*-out-of-*n* type system and find out the hesitant and dual hesitant fuzzy using aggregation operator. The hesitant fuzzy of a system by aggregation and Weibull distribution is also calculated by Kumar and Ram (2018) in terms of fuzzy intervals. Awad and As'ad (2016) used an efficient maintenance plan as a core risk reduction mechanism that plays a crucial role in increasing the efficiency and availability of manufacturing plants. The author Garg (2013) suggested a technique in the form of a fuzzy membership function using the suggested confidence interval based on a fuzzy Lambda-Tau (CIBFLT) methodology for the study of a complex repairable industrial structures. Komal et al. (2010) used post-RAM general RAM index to identify the system on the basis of their unit performance. The methodology was applied to the press section (which is in series configuration) and a washing section (which is in series–parallel configuration), of a paper mill plant. They show that the results might be helpful for the system analyst for analyzing and optimizing the system's efficiency by effective repair techniques. Niwas and Garg (2018) define a mathematical model of a system on the basis of the Markov method and thus the different parameters, such as reliability, mean time for system failure, availability and the expected profit were extracted for the system. Garg (2018a) claimed different types of fuzzy numbers for handling uncertainties. On this basis, the expressions for reliability indices were obtained and concluded that these indices were reduced the range of prediction as compared to the existing approaches. Yusuf et al. (2019) deals with the stability simulation and assessment of a linear *2*-out-of-*4* successive method through online and offline maintenance work. The findings of this paper boost system efficiency are useful for prompt maintenance progress, decision-making, preparation and optimization. He et al. (2020) introduced an integrated approach to evaluate the reliability of a complex repairable closed-loop networks with uncertainties. Chandrawat and Joshi (2021) used triangular fuzzy numbers along with boundary condition to find the fuzzy solution of two immiscible fluid flow problems. Yang et al. (2017) obtains the two main reasons for the failure of the entire multicomponent repairable system as failure of one unit depends upon other units and the assembly effects of repair actions are difficult to calculate. They suggested a parametric mathematical model for the capture of information on failure dependency with general component repair activities. Garg (2018b) proposed an alternative approach for calculating the various arithmetic operations of the device using a sigmoidal number in a fuzzy environment. Dhiman and Garg (2016) developed the improved arithmetic operations on generalized trapezoidal fuzzy numbers. Its application is to consider the confidence level to the same degree in order to avoid the inexact results. Dhiman and Kumar (2021) analyzed the performance of a repairable industrial system under genuine human errors. It showed that human (experienced or inexperienced) is also a major part of structure. Fodor and Bede (2006) have sought to avoid this restriction in some of the extensions to trapezoidal fuzzy numbers based on theoretical arithmetic operations. The resulting fuzzy numbers are approximated to the right trapezoidal number, and the arithmetic procedures are redefined in ways that explicitly generate trapezoidal fuzzy numbers. Tanaka et al. (1983) suggested that the probability of failure should be included, viz. The fuzzy collection determines the probability space because it could be important to consider possible failure of the components, even though they have never failed before. Kowal and Torabi (2021) analyzed the high temperature engineering test reactor by failure mode and effect analysis (FMEA). They also developed a new FMEA-based screening technique and applied the same to find the relevant failure modes. It was concluded that the reliability may be not sufficient for long-term working. Some design modifications were suggested bases on the results. Knezevic and Odoom (2001) used Petri nets approach and found the reliability parameters using Lambda-Tau methodology. It was concluded that Petri nets is better for making minimum cut and path sets as compare to fault tree.

In order to explain clearly the focus of this study, authors primarily equate the structure of the selected model and solution process with those of closely previous research (see Table 1).

Table 1. Related work review.

References	Objectives	Application area	Modeling method	Solution algorithm
Wang et al. (2020)	Addressed that how to alleviate challenges in vaccine distribution and administration	Possible modification required in the early development process	Vaccine-stabilizing techniques and the use of complex mucosal immune response-inducing non-invasive routes of administration.	This paper identified already tested COVID-19 vaccines and offered an in-depth look into the different types of vaccines, their formulations, benefits and possible limitations.
Bell et al. (2001)	To estimate whether the draft manual improved storage and handling practices.	Find the best ways to improve vaccine storage and handling	χ^2 analysis and Student's *t* tests to compare the administrative characteristics and quality assurance practices	Estimates of immunization sites found to have a suboptimally stored vaccine at a single point in time
Garg (2018a)	Behavior, performance and sensitivity analysis of the system have been investigated at different levels of confidence	These reliability indices are reduced range of prediction	Different types of numbers for uncertainties.	Gamma, normal, Cauchy and triangular fuzzy numbers
Li et al. (2018)	Effective and accurate method is proposed	Comparison of proposed method with Monte Carlo simulation	PMS-MMDD model (Phased Mission System-Multi-state multi-valued decision diagram model for phased mission system**)**	Markov renewal equation
Komal et al. (2010)	RAM analysis	Design modifications	Genetic Algorithms based Lambda–Tau (GABLT)	Lambda–Tau methodology
Dhiman and Garg (2016)	Behavior analysis of structure	For more realistic decision	Improved arithmetic operations under the fuzzy environment	Lambda–Tau methodology, arithmetic operations of generalized trapezoidal (triangular) fuzzy numbers
Dhiman and Kumar (2021)	RAM assessment	Strategies to improve system performance and cost reduction for maintenance	Improved arithmetic operations under the fuzzy environment and human errors	Lambda–Tau methodology, arithmetic operations of generalized trapezoidal fuzzy numbers
Tanaka et al. (1983)	Possibility of system failure		Possibility theory	fuzzy fault-tree model
Rastegar et al. (2021)	Vaccine distribution	Vaccine distribution in developing countries during the pandemic	Inventory-location mixed-integer linear programming	objective function to distribute vaccines
This paper	Performance prediction and ranking of components of ULT freezer	Future maintenance plans under preventive maintenance and regular maintenance	Possibility measure method, Right triangular fuzzy numbers and its arithmetic operations	Lambda-Tau methodology and Ranking

1.2 Motivation

Authors have critically analyse the work done by different researcher in past and identified the following research gap.

In comparison to the papers described above, this paper formulates a mixed configuration model that is series-parallel system (using AND-OR logic gates) based on major components and human error/operator.

- To the best of our understanding, this is the first work in which author estimate the performance, behavior and components ranking of the vaccine storage ULT freezer in a fuzzy environment by considering regular and preventive maintenance.
- From literature, it has been observed that the analysis of a ULT freezer has not been done by any of the authors in past. Keeping this in mind, here authors investigated the performance of ULT freezer, with the help of uncertain available data, for finding the various reliability indices of the same.

2. Problem Description

In order to provide the description of the analyzed problem, in this section the detailed outline is given in three subsections i.e., Ultra-low temperature freezer structure, regular maintenance and preventive maintenance, and fault occurs in the structure.

2.1 Ultra-Low Temperature Freezer Structure

It is not like the standard freezer that we are using in homes. It has been designed for the most rigorous temperature requirements and qualification needs. This freezer fulfills the needs of scientific and industrial applications, where temperatures lie between -50^{0}C to - 80^{0}C.

The ULT freezer contains the following major components which play an important role in it working.

Power unit: It is having two measure components namely electricity and battery. Electricity is first step to run the whole system. The Ultra freezer needs continuous supply of the electricity for its working. There is a backup battery to compensate the electricity. Battery is a standby unit for electricity. In case of power failure, battery is called upon automatically. The power unit fails if electricity fault occurs and battery fails.

Operator: Human plays an important role in handling the ULT freezer. Human operates the system to open and close the doors. Human also monitor the temperature and working of the system.

Outer door/Access door: The main entry door, which is often insulated with polyurethane, ensures that it stays cool, is blocked by an external closure usually fitted with a safety key. The lock does not activate until the door is locked. The first thing you need to do is rotate the latch to the open position.

Inner door: The Ultra freezer includes inside shelves. Each shelf is enclosed with an inner door to keep the temperature as standardized as possible on the shelves where no vaccine is withdrawn or stored. The purpose of the inner door in an ultra-low freezer is to protect the contents stored on the shelves behind each inner door (referred to as compartments). One inner door will be open for the user to enter items contained within the container, the other inner doors are used to shield the contents of other compartments from being exposed to warm ambient air.

Gasket: Sealing between doors is necessary in order to preserve the internal temperature of the cabin and reduce the temperature of the samples. The main use of gasket is to avoid humid, damp air in to the storage area.

Freezer chamber: The freezer condenses the coolant, which heats it, and then moves the heated steam through the coils in the back. The heat radiates from the coils to the kitchen, and the temperature of the coolant decreases. This decreases the temperature of the remaining coolant, allowing the heat to flow from the freezer compartment to the coolant.

Aeration grill: The condenser plays a crucial role in keeping the ULT freezer cool. It helps to transfer heat from the inside out. When dust accumulates on it, heat cannot be moved.

The systematic diagram of ULT freezer is given in Figure 1.

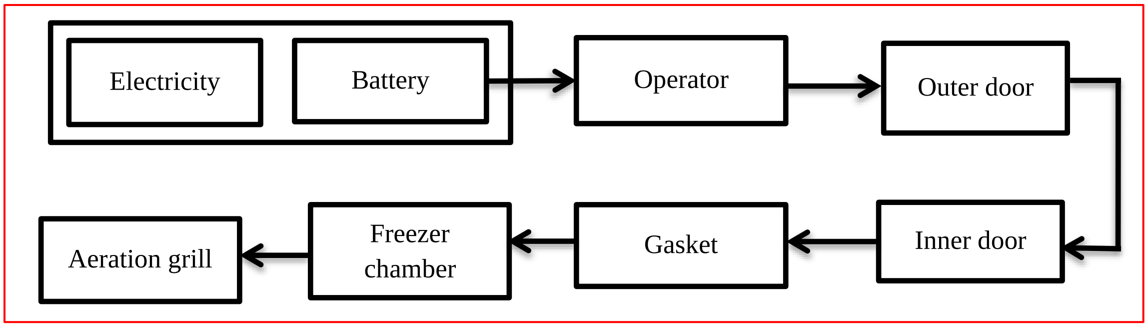

Figure 1. Systematic diagram of ULT freezer.

2.2 Regular and Preventive Maintenance

Whenever the system is taken into the use, some regular maintenance and preventive maintenance is suggested, which includes inspecticleaningean as directed in the user manual. It is very important to follow these guidelines to keep the unit/system working. It results in the long-run performance of the system. Using a ULT freezer is not like using a refrigerator or standard freezer. Some best practices for preventive maintenance are listed below.

- Minimize the duration of door openings.
- Minimize the frequency of door openings.
- Have a system of inventory management.
- Pay attention to what the unit is telling you.
- Make use of the onboard capabilities.
- Only open the inner doors that you need to open.

Beside the above preventive maintenance, one must take care some best practices for regular maintenance as follows.

Defrost the freezer once a year or if ice builds up. Clean the condenser filter(s) in every 2 to 3 months. Check the gaskets inside the door periodically for punctures or tears. Fix up the ice regularly around the gasket. The lid gasket should be cleaned a minimum of once per month. Remove all products and place it in another freezer. Allow the frost to melt and become loose. Allow the freezer to operate empty overnight before reloading the product. Clean the condenser every six months, more so if the field of the laboratory is dusty. Replace the battery as suggested by the system warning or, if appropriate, by person status assessment. Check the battery contacts regularly. Although not needed, daily battery replacement is recommended to ensure proper battery status in the event of power failure.

2.3 Faults in ULT Freezer

Besides, the preventive maintenance is taken into consideration there exist some fault/failure occurs which lead to complete failure of the ULT freezer. Situational based faults are considered to extract the failure data. Some of the possible failure are listed below.

Power unit failure: Due to sudden power supply interruption the system will operate through the battery, which is in standby mode. The system will be in failed state if main power supply as well as standby battery is failed.

Inner door loose/Outer door loose: Due to frequent opening of the door and maximum duration of opening of the door, the door may become loose and it effects the desired mechanism of the ULT freezer.

Human error: Human operator is also one of the important parts of mechanism. In the system, mainly the doors are operated by the human operator to put the vaccine inside to store and take out the vaccine outside for use. During this process some genuine human mistake occurs that also considered as fault in the system.

Gasket: Gasket can be found punctured/tear during the process. It needs to be repair or changed for the proper mechanism/working. It prevents the chamber from cooling which results in the system failure.

Freezer chamber: It is the part where the cooling is required. It may stop cooling due to improper functioning or the desired temperature is not maintained. It results in the failure of entire system.

Aeration grill: There may be fault in this unit if it is not cleaned properly.

The working of ULT freezer, with the help of AND-OR logic gates, is shown in Figure 2.

3. Assumptions

The following assumptions are taken into consideration throughout this work.

- The effectiveness of standby components is the same as active components.
- The system works with the same efficiency after repair.
- After failure immediate repair is always available.
- Human/operator is a part of the considered system (refer to section 2.3).

4. Notations

In the following Table 2, the notations are defined with their usual meaning. The same is used throughout the paper.

Table 2. Notations.

Notation	Meaning	Notation	Meaning
ω	Confidence level	$\lambda_{Component}$	Failure rate of each component
$\omega_{\min}$	Minimum of all ω	$\tau_{Component}$	Repair time of ith component
λ_{ULT}	Failure rate of ULT freezer	*EL*	Electricity
τ_{ULT}	Repair time of ULT freezer	*B*	Battery
R_{ULT}	Reliability of ULT freezer	*OD*	Outer door
MTTF	Mean time to failures	*ID*	Inner door
MTTR	Mean time to repair	*O*	Operator
MTBF	Mean time between failures	*G*	Gasket
A_{ULT}	Availability of the structure system	*FC*	Freezer chamber
ENOF	Expected number of failures	*AG*	Aeration grill
M_{ULT}	Maintainability of ULT freezer		

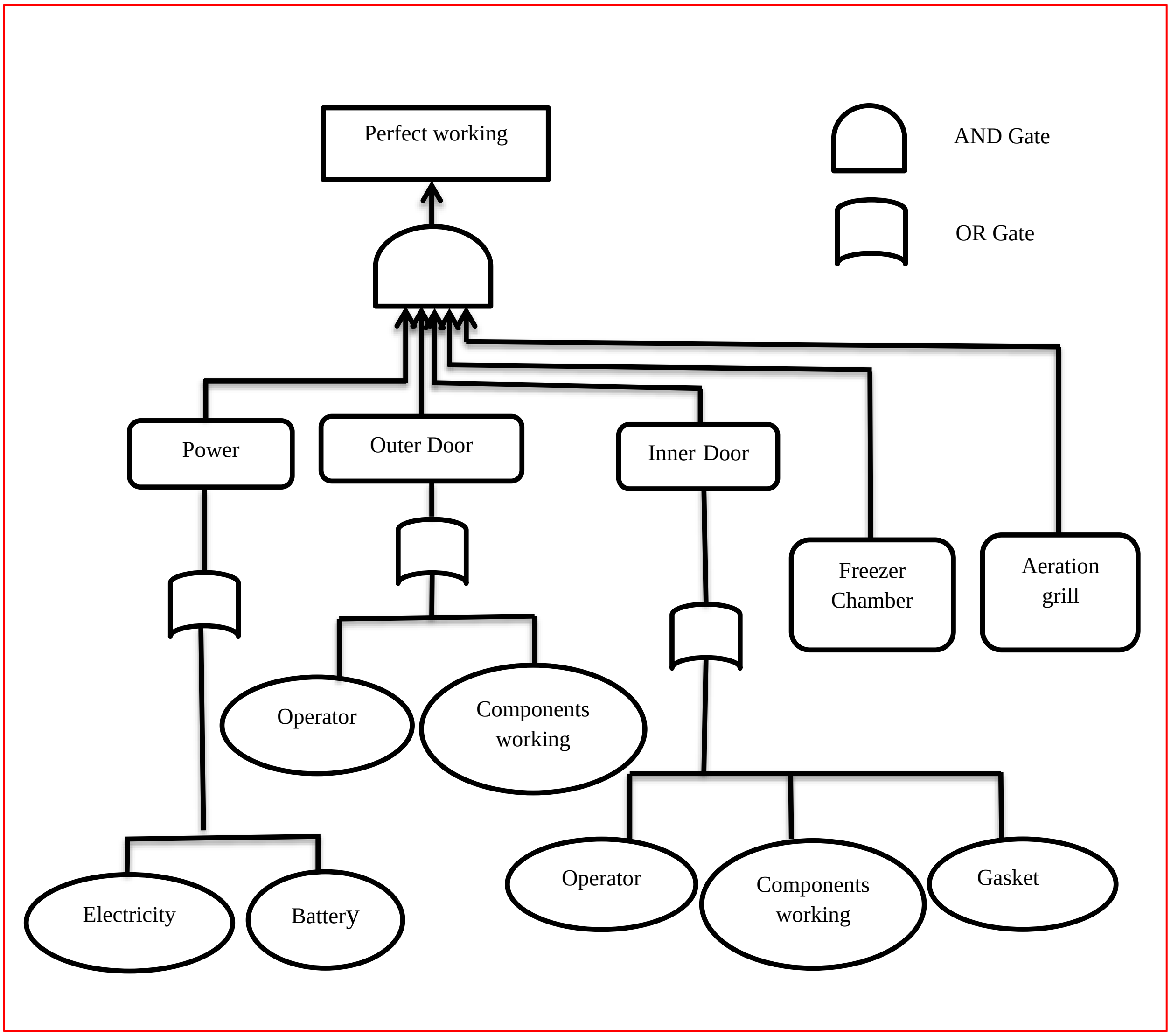

Figure 2. Configuration of ULT freezer in series-parallel combination.

5. Proposed Methodology

5.1 Tolerance

It is defined as the difference between high limit and low limit of a parameter's numerical value. There are two types of tolerance, namely unilateral and bilateral, with respect to an uncertain information. Unilateral tolerance is one sided tolerance either in positive (see Figure 3) or in negative (see Figure 4) from crisp value. On the other side, Bilateral tolerance is a two-sided tolerance, both in positive and negative from crisp value, as given in Figure 5.

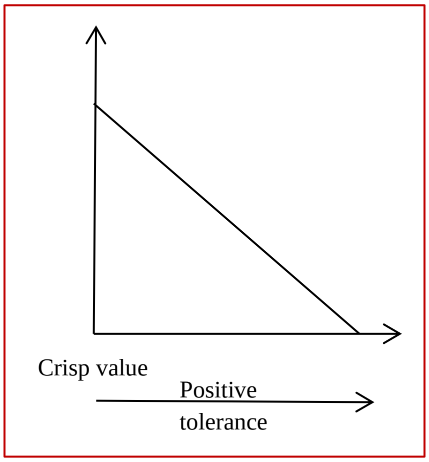

Figure 3. Positive side tolerance.

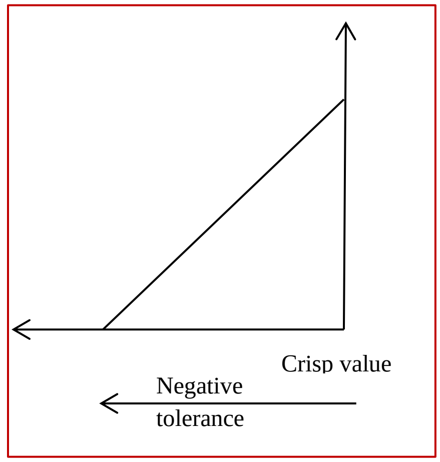

Figure 4. Negative side tolerance.

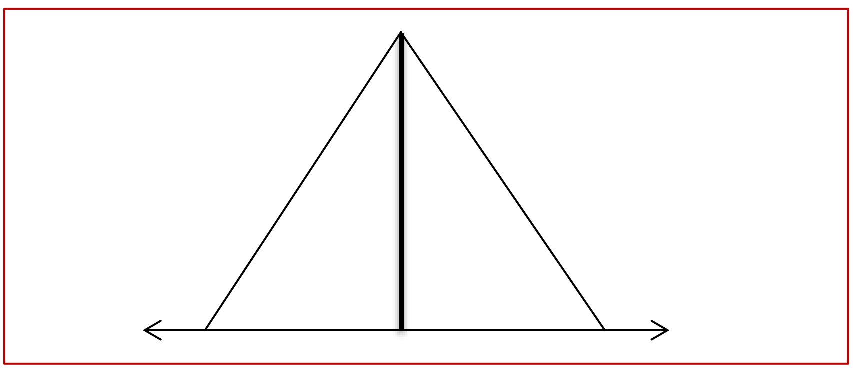

Figure 5. Both positive and negative side tolerance.

5.2 Right Triangular Generalized Fuzzy Number

Right triangular generalized fuzzy number (RTrFN) can be defined on the basis of tolerance and confidence level. A fuzzy number $F=\left(\delta_1,\delta_2;\omega\right)$ is said to be a right triangular generalized fuzzy number if its membership function is given in equation (1). The Graphical representation of a right triangular generalized fuzzy number is shown in Figure 6.

$$\mu_F=\begin{cases}\omega & ;x=\delta_1\\ \left(\dfrac{\delta_2-x}{\delta_2-\delta_1}\right) & ;\delta_1<x\le\delta_2\\ 0 & ;otherwise\end{cases} \tag{1}$$

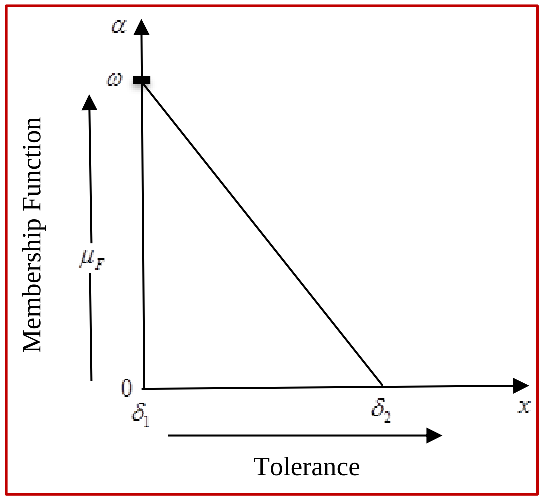

Figure 6. Right triangular generalized fuzzy number.

5.3 Alpha-Cut

The alpha cut for the right triangular generalized fuzzy number is given in equation (2) and same is represented in Figure 7.

$$F_\alpha = [F_\alpha^L, F_\alpha^R] = \left[\alpha, \delta_2 - \frac{\alpha}{\omega}(\delta_2 - \delta_1); \alpha \in (0, \omega]\right] \tag{2}$$

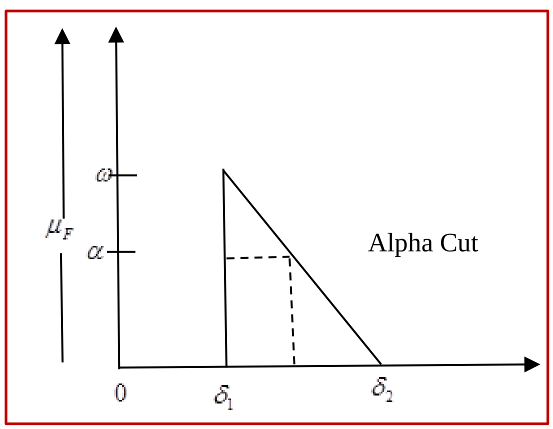

Figure 7. Pictorial representation of alpha-cut.

5.4 Fuzzy Arithmetic Operations

The practical use of fuzzified operations is shown to be straightforward, requiring no more computation than in the classical tolerance analysis when dealing with error intervals (see Appendix).

5.5 Ranking

It is one of the system's performance measures which tells about the most critical component, in term of effect on system's performance, of a system. Ranking is an extensive approach to classify the sensitive part of the structure. Tanaka et al. (1983) used this approach to find the rank of different component of a system in terms of their effect on overall performance of the system. In the present paper, this approach has been extended for ULT freezer through right triangular generalized fuzzy numbers.

λ_{ULT} is defined as a right triangular generalized fuzzy number as given in equation (3).

$$\lambda_{ULT} = \lambda_{ULT}\left(\lambda_{Component}, \tau_{Component}\right) = \left(\lambda_x, \lambda_y, \lambda_z\right) \tag{3}$$

Here height pair for confidence level for $\alpha = \omega_{\min}$ is $\left(\lambda_x, \lambda_y\right)$, and for $\alpha < \omega_{\min}$ is $\left(\lambda_y, \lambda_z\right)$.

$\lambda_{Component} = (\lambda_{c1}, \lambda_{c2}, \lambda_{c3})$, is defined as the failure rate of component in right triangular generalized fuzzy number.

$$V\left(\lambda_{ULT}, \lambda_{Component}\right) = \left(\lambda_x - \lambda_{c1}, \lambda_y - \lambda_{c2}, \lambda_z - \lambda_{c3}\right) \tag{4}$$

This expression gives the improvement magnitude for the structure.

$$R\left(\lambda_{ULT}, \lambda_{Component1}\right) > R\left(\lambda_{ULT}, \lambda_{Component2}\right) \tag{5}$$

6. Computational Analysis

6.1 Data Collection

The first step for behavior analysis is to collect the data. The required data is extracted from the collected data and arranged in the desired form. As operator plays a vital role in handling the system. So, genuine human mistake is also considered in working of ULT freezer. Operator may or may not be perfect in the allotted task. The Table 3 defines the two categories of operators along with their failure rates and repair times. This data is taken as a source data from Dhiman and Kumar (2021).

Table 3. Human mistakes (errors).

Human Performance	Failure Rate, λ (per hour)	Repair Rates (per hours)
Experienced (E_1)	0.125	0.04
Inexperienced (E_2)	0.500	0.10
Average (E_1+E_2)	0.3125	0.07

In addition to human, the remaining major components of the structure are given in Table 4. The parameters of the structure are extracted from the user manual of Ultra-Low Temperature Freezers Installation and Operation(*https://centers.njit.edu/york/sites/york/files/TSX%20600A_Ultra%20Low%20Temp%20Freezer_Operating%20manual.pdf)*.

Table 4. Crisp values of the parameters of components.

Components	Notations	Failure rate (λ) (in hrs.)	Repair time(τ) (in hrs.)
Electricity	*EL*	0.0013	1
Battery	*B*	0.578×10^{-4}	1
Outer door	*OD*	0.0013	1
Inner door	*ID*	0.0013	1
Operator	*O*	0.3125	14.28
Gasket	*G*	0.0013	0.25
Freezer chamber	*FC*	0.0001	12
Aeration grill	*AG*	0.0004	0.5

6.2 Fuzzified Data

In this step, the data is fuzzified by taking *15%* unilateral tolerance on right hand side as shown in Figure 8 and Figure 9. With the help of tolerance, here authors convert the crisp value to interval value or range. In Figure 8, conversion of failure rate is given and in Figure 9 conversion of repair time is given.

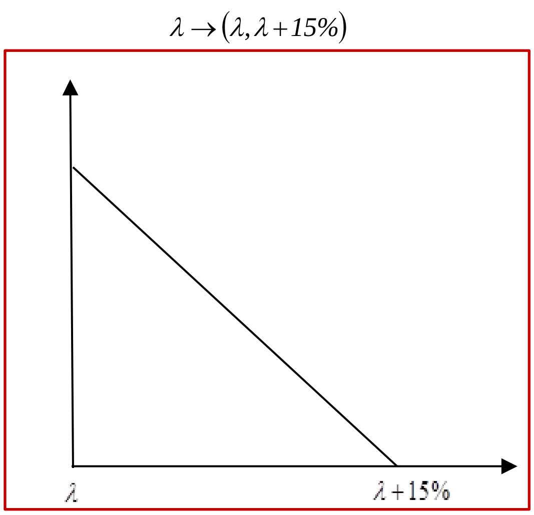

Figure 8. $15\%^{+}$ tolerance of failure rate (λ).

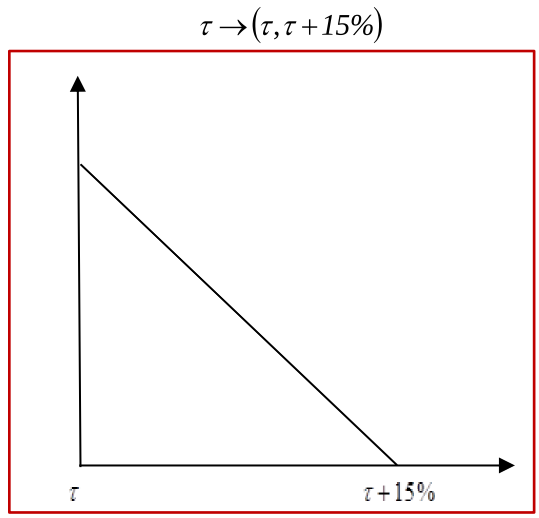

Figure 9. $15\%^{+}$ tolerance of repair time (τ).

By using the above conversion, the crisp parameters are converted into fuzzy parameters and allocated some weightage as given in Table 5.

Table 5. Fuzzified values of the parameters of components.

Failure rate (λ) (in hrs.)	**Repair time (τ) (in hrs.)**	**Weightage $\omega_{\min}$**
λ_{EL} = (0.0013, 0.001495, 0.001495; 0.65)	τ_{EL} = (1, 1.15, 1.15; 0.65)	0.65
λ_{B} = (5.78*10^{-5}, 0.000067, 0.000067;0.65)	τ_{B} = (1, 1.15, 1.15; 0.65)	0.65
λ_{OD} = (0.0013, 0.001469, 0.001495; 0.75)	τ_{OD} = (1, 1.13, 1.15; 0.75)	0.75
λ_{ID} = (0.0013, 0.001469, 0.001495; 0.75)	τ_{ID} = (1, 1.13, 1.15; 0.75)	0.75
λ_{O} = (0.3125, 0.350586, 0.359375; 0.8)	τ_{O} = (14.28, 16.0204, 16.422; 0.8)	0.8
λ_{G} = (0.0013, 0.001459, 0.001495; 0.8)	τ_{G} = (0.25, 0.2805, 0.2875; 0.8)	0.8
λ_{FC} = (0.0001,0.000111, 0.000115; 0.9)	τ_{FC} = (12, 13.3, 13.8; 0.9)	0.9
λ_{AG} = (0.0004, 0.000444, 0.00046; 0.9)	τ_{AG} = (0.5, 0.5542, 0.575; 0.9)	0.9

6.3 Transformation of the Data to Same Level of Confidence

After fuzzifying the data, it is necessary to generalize the data to preserve the flatness. Generalization helps to convert all the fuzzy values up to same satisfaction level. This process is given as,

$$\omega_{\min} = min(0.65, 0.65, 0.75, 0.75, 0.8, 0.8, 0.9, 0.9) = 0.65 \tag{6}$$

The generalize values of all the parameters are given in Table 6. In this table λ is transformed to λ^* to preserve the flatness in the data. All the parameters are transformed with same level of confidence. Now we can apply the arithmetic operations on it.

Table 6. Values of the parameters of components to same level of confidence.

Failure rate (λ) (in hrs.)	**Repair time (τ) (in hrs.)**	**Weightage $\omega_{\min}$**
λ_{EL} = (0.0013, 0.001495, 0.001495; 0.65)	τ_{EL} = (1, 1.15, 1.15; 0.65)	0.65
λ_{B} = (5.78*10^{-5}, 0.000067, 0.000067; 0.65)	τ_{B} = (1, 1.15, 1.15; 0.65)	0.65
λ^*_{OD} = (0.0013, 0.001469, 0.001495; 0.65)	τ^*_{OD} = (1, 1.13, 1.15; 0.65)	0.65
λ^*_{ID} = (0.0013, 0.001469, 0.001495; 0.65)	τ^*_{ID} = (1, 1.13, 1.15; 0.65)	0.65
λ^*_{O} = (0.3125, 0.350586, 0.359375; 0.65)	τ^*_{O} = (14.28, 16.0204, 16.422; 0.65)	0.65
λ^*_{G} = (0.0013, 0.001459, 0.001495; 0.65)	τ^*_{G} = (0.25, 0.2805, 0.2875; 0.65)	0.65
λ^*_{FC} = (0.0001, 0.000111, 0.000115; 0.65)	τ^*_{FC} = (12, 13.3, 13.8; 0.65)	0.65
λ^*_{AG} = (0.0004, 0.000444, 0.00046; 0.65)	τ^*_{AG} = (0.5, 0.5542, 0.575; 0.65)	0.65

7. Solution Methodology

7.1 AND-OR Logic Gates Expressions

The expressions for failure rate and repair times are defined with the help of AND-OR logic gates as shown in Table 7.

Table 7. Lambda-tau expressions.

Logic Gate	**AND**		**OR**	
	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expressions	$\prod_{j=1}^{n}\lambda_j\left[\sum_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\tau_j\right]$	$\dfrac{\prod_{i=1}^{n}\tau_j}{\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\tau_i\right]}$	$\sum_{i=1}^{n}\lambda_i$	$\dfrac{\sum_{i=1}^{n}\lambda_i\tau_i}{\sum_{i=1}^{n}\lambda_i}$

Based on the expressions (Table 7), failure rate and repair time of the ULT freezer is defined as given in equation (7-8).

$$\lambda_{ULT} = \lambda_{EL}\lambda_B\cdot(\tau_{EL}+\tau_B) + \lambda^*_O\lambda^*_{OD}\cdot(\tau^*_O+\tau^*_{OD}) + \lambda^*_O\lambda^*_{ID}\lambda_G\cdot(\tau^*_O\lambda^*_G+\tau^*_{ID}\lambda^*_G+\tau^*_{ID}\tau^*_O) + \lambda^*_{FC}+\lambda^*_{AG} \tag{7}$$

$$\tau_{ULT} = \frac{\lambda_{EL}\lambda_B\tau_{EL}\tau_B + \lambda^*_O\lambda^*_{OD}\tau^*_O\tau^*_{OD} + \lambda^*_O\lambda^*_{ID}\lambda^*_G\tau^*_O\tau^*_{ID}\tau^*_G + \lambda^*_{FC}\tau^*_{FC} + \lambda^*_{AG}\tau^*_{AG}}{\lambda_{ULT}} \tag{8}$$

By using the arithmetic operations given in Appendix, equation (7-8) can be obtained and their values are shown in Table 8.

Table 8. Fuzzy values of lambda and tau.

Fuzzy values of lambda and tau	
λ_{ULT}	(0.0067, 0.0094, 0.0100)
τ_{ULT}	(0.7203, 1.1755, 1 7914)

8. Results and Discussion

In this section author have discussed the performance of the ULT freezer. With the aid of Table 8, the various reliability measures, e.g., MTTF, MTTR, MTBF, reliability, availability and ENOF, of ULT freezer are obtained. In Table 9, the notations and the numerical values of MTTF, MTTR and MTBF are given. These values are represented as right triangular fuzzy number with *0.65* confidence level and *15%* tolerance.

Table 9. Expressions for Reliability parameters and their fuzzy values.

Parameters	**Notation**	**Values**
MTTF	$\frac{1}{\lambda_{SS}}$	[100.0000 106.3830 149.2537]
MTTR	$\frac{1}{\mu_{SS}} = \tau_{SS}$	[0.7203 1.1755 1.7914]
MTBF	MTTF+MTTR$= \frac{1}{\lambda_{SS}} + \tau_{SS}$	[100.7203 107.5585 151.0451]

The following Table 10 gives the alpha cuts of the parameters namely MTTF, MTTR and MTBF.

Table 10. Alpha cut range of MTTF, MTTR and MTBF.

Confidence level (α)	**MTTF**		**MTTR**		**MTBF**	
	Left $\alpha - cut$	**Right** $\alpha - cut$	**Left** $\alpha - cut$	**Right** $\alpha - cut$	**Left** $\alpha - cut$	**Right** $\alpha - cut$
0	100	149.2537	0.7203	1.7914	100.7203	151.0451
0.05	100	145.956	0.7203	1.744023	100.7203	147.7
0.1	100	142.6582	0.7203	1.696646	100.7203	144.3549
0.15	100	139.3605	0.7203	1.649269	100.7203	141.0097
0.2	100	136.0627	0.7203	1.601892	100.7203	137.6646
0.25	100	132.765	0.7203	1.554515	100.7203	134.3195
0.3	100	129.4672	0.7203	1.507138	100.7203	130.9744
0.35	100	126.1695	0.7203	1.459762	100.7203	127.6292
0.4	100	122.8717	0.7203	1.412385	100.7203	124.2841
0.45	100	119.574	0.7203	1.365008	100.7203	120.939
0.5	100	116.2762	0.7203	1.317631	100.7203	117.5939
0.55	100	112.9785	0.7203	1.270254	100.7203	114.2487
0.6	100	109.6807	0.7203	1.222877	100.7203	110.9036
0.65	100	106.383	0.7203	1.1755	100.7203	107.5585

The fuzzy values of the parameters MTTF (Figure 10), MTTR (Figure 11) and MTBF (Figure 12) are represented as right triangular fuzzy numbers up to maximum confidence level *0.65*. The pictorial representation of fuzzy values for reliability indices MTTF, MTTR, MTBF as right triangular generalized fuzzy numbers with *15%* tolerance is given below in Figure 10, Figure 11 and Figure 12 respectively.

Using the expressions from Table 11, the fuzzy values of reliability, availability, maintainability and ENOF indices has been calculated and is given in Table 12. These values are represented as fuzzy right triangular number with *0.65* confidence level and *15%* tolerance.

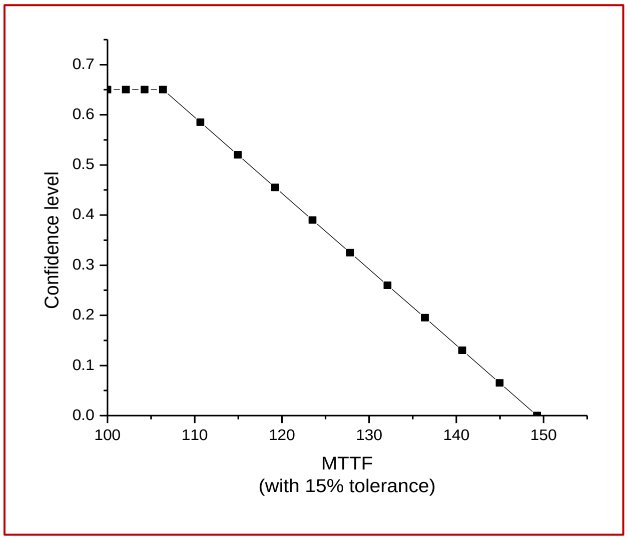

Figure 10. MTTF of ULT Freezer.

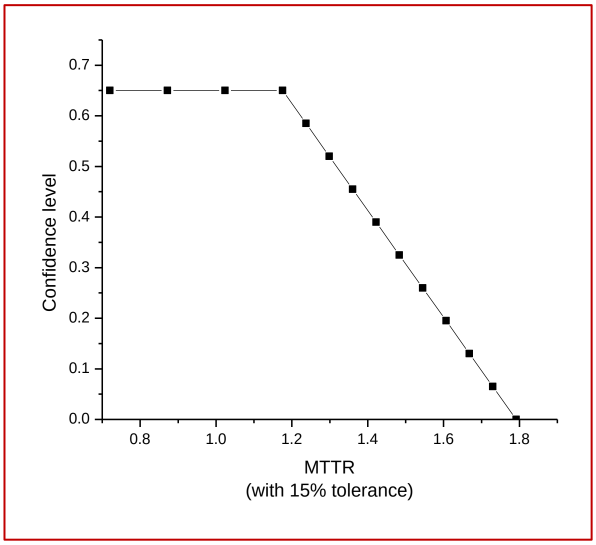

Figure 11. MTTR of ULT Freezer.

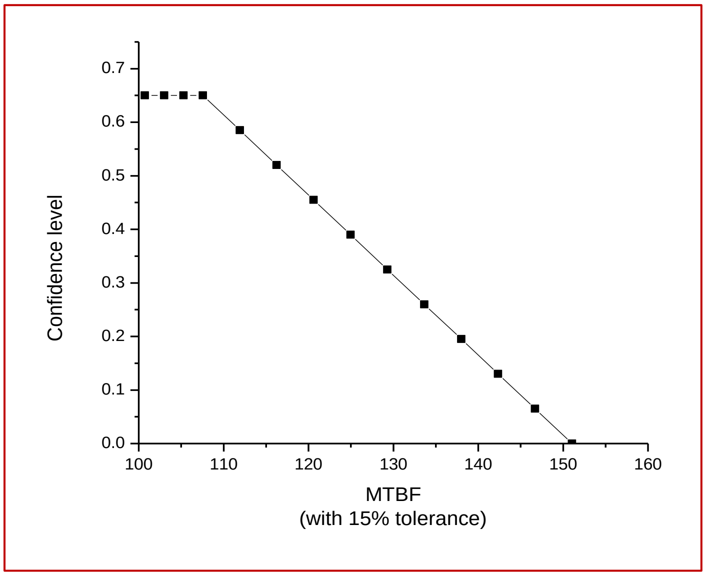

Figure 12. MTBF of ULT Freezer.

Table 11. Expressions for reliability parameters.

R_{ULT}	$e^{-(\lambda_{ULT})t}$
A_{ULT}	$\dfrac{\mu_{ULT}}{\mu_{ULT}+\lambda_{ULT}}+\dfrac{\lambda_{ULT}}{\mu_{ULT}+\lambda_{ULT}}e^{-(\mu_{ULT}+\lambda_{ULT})\cdot t}$
M_{ULT}	$1-e^{-(\mu_{ULT})t}$
ENOF	$\dfrac{\mu_{ULT}\cdot\lambda_{ULT}\cdot t}{\mu_{ULT}+\lambda_{ULT}}+\dfrac{\lambda_{ULT}{}^{2}}{(\mu_{ULT}+\lambda_{ULT})^{2}}\left[1-e^{-(\mu_{ULT}+\lambda_{ULT})*t}\right]$

Table 12. Fuzzified values of reliability, availability and ENOF.

Time	Fuzzy Reliability	Fuzzy Availability	Fuzzy Maintainability	Fuzzy ENOF
0	[1 1 1]	[0.4040 1.0000 2.4754]	[0 0 0]	[0.0265 0.0930 0.2461]
1	[0.9900 0.9906 0.9933]	[0.4004 0.9937 2.4677]	[0.7505 0.5729 0.4278]	[0.0265 0.0931 0.2462]
2	[0.9802 0.9814 0.9867]	[0.3995 0.9911 2.4634]	[0.9378 0.8176 0.6725]	[0.0265 0.0931 0.2463]
3	[0.9704 0.9722 0.9801]	[0.3993 0.9899 2.4609]	[0.9845 0.9221 0.8126]	[0.0265 0.0931 0.2463]
4	[0.9608 0.9631 0.9736]	[0.3992 0.9894 2.4595]	[0.9961 0.9667 0.8928]	[0.0265 0.0931 0.2463]
5	[0.9512 0.9541 0.9671]	[0.3992 0.9892 2.4587]	[0.9990 0.9858 0.9386]	[0.0265 0.0931 0.2464]
6	[0.9418 0.9452 0.9606]	[0.3992 0.9892 2.4582]	[0.9998 0.9939 0.9649]	[0.0265 0.0931 0.2464]
7	[0.9324 0.9363 0.9542]	[0.3992 0.9891 2.4579]	[0.9999 0.9974 0.9799]	[0.0265 0.0931 0.2464]
8	[0.9231 0.9276 0.9478]	[0.3992 0.9891 2.4578]	[1.0000 0.9989 0.9885]	[0.0265 0.0931 0.2464]
9	[0.9231 0.9276 0.9478]	[0.3992 0.9891 2.4577]	[1.0000 0.9995 0.9934]	[0.0265 0.0931 0.2464]
10	[0.9048 0.9103 0.9352]	[0.3992 0.9891 2.4577]	[1.0000 0.9998 0.9962]	[0.0265 0.0931 0.2464]

The numerical value of reliability, availability and ENOF with alpha-cuts is shown in following Table 13.

Table 13. Alpha cut range of reliability, availability, maintainability and ENOF.

Confidence level (α)	Reliability		Availability		Maintainability		ENOF	
	Left $\alpha-cut$	Right $\alpha-cut$	Left $\alpha-cut$	Right $\alpha-cut$	Left $\alpha-cut$	Right $\alpha-cut$	Left $\alpha-cut$	Right $\alpha-cut$
0	0.9048	0.9352	0.3992	2.4577	1	0.9962	0.0265	0.2464
0.05	0.9048	0.933285	0.3992	2.344731	1	0.996477	0.0265	0.234608
0.1	0.9048	0.931369	0.3992	2.231762	1	0.996754	0.0265	0.222815
0.15	0.9048	0.929454	0.3992	2.118792	1	0.997031	0.0265	0.211023
0.2	0.9048	0.927538	0.3992	2.005823	1	0.997308	0.0265	0.199231
0.25	0.9048	0.925623	0.3992	1.892854	1	0.997585	0.0265	0.187438
0.3	0.9048	0.923708	0.3992	1.779885	1	0.997862	0.0265	0.175646
0.35	0.9048	0.921792	0.3992	1.666915	1	0.998138	0.0265	0.163854
0.4	0.9048	0.919877	0.3992	1.553946	1	0.998415	0.0265	0.152062
0.45	0.9048	0.917962	0.3992	1.440977	1	0.998692	0.0265	0.140269
0.5	0.9048	0.916046	0.3992	1.328008	1	0.998969	0.0265	0.128477
0.55	0.9048	0.914131	0.3992	1.215038	1	0.999246	0.0265	0.116685
0.6	0.9048	0.912215	0.3992	1.102069	1	0.999523	0.0265	0.104892
0.65	0.9048	0.9103	0.3992	0.9891	1	0.9998	0.0265	0.0931

The fuzzy values of the parameters namely reliability, availability, maintainability and ENOF are represented as right triangular fuzzy numbers up to maximum confidence level *0.65* in Figure 13, Figure 14, Figure 15 and Figure 16 respectively.

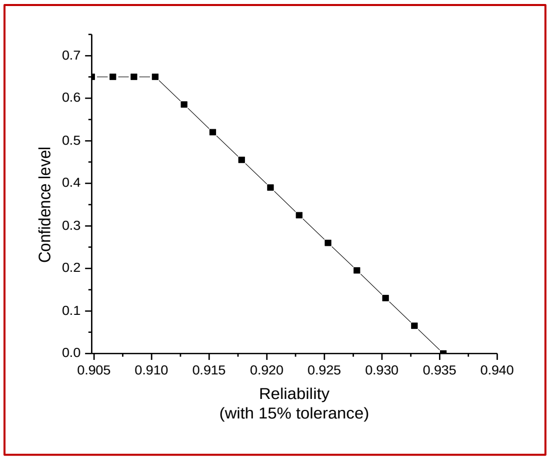

Figure 13. Reliability of ULT Freezer.

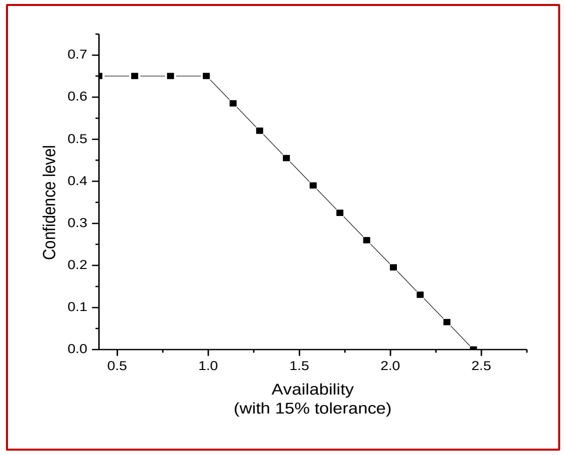

Figure 14. Availability of ULT Freezer.

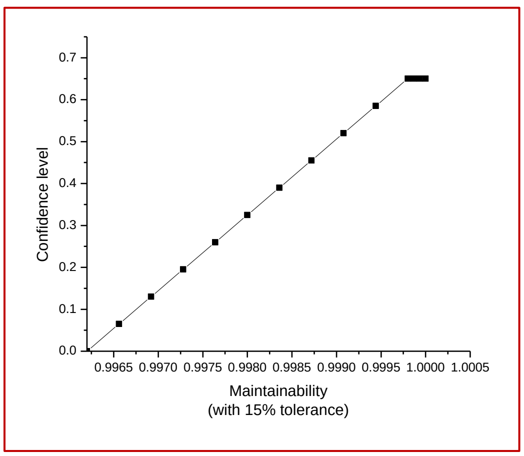

Figure 15. Maintainability of ULT Freezer.

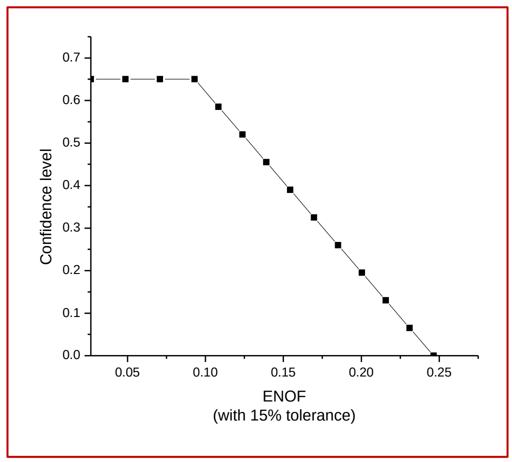

Figure 16. ENOF of ULT Freezer.

After finding the various reliability parameters, now authors proceed to find out the ranking using section 4.5. In the following Table 14, rankings of components are given.

Table 14. Ranking to find the critical component of the system.

λ_{ULT}	(0.0067, 0.0094, 0.0100)	$R(\lambda_{ULT}, \lambda_{UNIT})$		**Rank**
λ_{EL}	(0.0013, 0.001495, 0.001495; 0.65)	$R(\lambda_{ULT}, \lambda_{EL})$	0.0218	3
λ_{B}	(5.78*10^{-5}, 0.000067, 0.000067; 0.65)	$R(\lambda_{ULT}, \lambda_{B})$	0.0258	1
λ_{OD}	(0.0013, 0.001469, 0.001495; 0.65)	$R(\lambda_{ULT}, \lambda_{OD})$	0.0218	3
λ_{ID}	(0.0013,0.001469,0.001495;0.65)	$R(\lambda_{ULT}, \lambda_{ID})$	0.0218	3
λ_{O}	(0.3125, 0.350586, 0.359375; 0.65)	$R(\lambda_{ULT}, \lambda_{O})$	-0.9964	4
λ_{G}	(0.0013, 0.001459, 0.001495; 0.65)	$R(\lambda_{ULT}, \lambda_{G})$	0.0218	3
λ_{FC}	(0.0001, 0.000111, 0.000115; 0.65)	$R(\lambda_{ULT}, \lambda_{FC})$	0.0258	1
λ_{AG}	(0.0004, 0.000444, 0.00046; 0.65)	$R(\lambda_{ULT}, \lambda_{AG})$	0.0248	2

From Table 14, it has been observed that the component freezer chamber and battery have highest rank 1 and on the other side the component human/operator have lowest rank 4. It shows that elimination of battery and freezer chamber are most critical components of the systems while human/operator is the least critical component of the system. The expert may plan the maintenance strategy according to the rank of the system.

8. Conclusions

This section presents conclusion of the paper related to the performance of the ULT freezer. To keep the vaccine, perform its intended function, it is required to store it in the ultra-low temperature freezer. In this paper, the challenge occurs of vaccine storage is discussed. The temperature of the vaccine is in a given range with some tolerance power. For this, a reliable unit is required to keep the vaccine. Here in this research the authors predict the future performance of the system based upon the available data and assuming that preventing maintenance and regular maintenance is taken. The performance measures have been obtained in the form of various parameters namely MTTF, MTTR, MTBF, RAM analysis and ENOF. Also, authors present a suitable membership function to represent the data in fuzzy environment; it helps to overcome the issue of uncertainty. All the parameters are investigated with *15%* unilateral tolerance and *0.65* confidence level.

The reliability parameters MTTF, MTTR, MTBF have been calculated (refer Table 9) and in Table 10, the alpha cuts are given for these parameters up to $\alpha = 0.65$. In Table 12, the fuzzy values of ENOF and RAM are given at different time (t) units. In Table 13, alpha cuts are given for these parameters up to $\alpha = 0.65$. Table 14 shows the ranks of the different of the system. The higher the rank, the more important the part is having. Lowest of rank, the least important aspect would be. In the entire framework scheme, the portion with the highest ranking requires more attention than the other components. It can be seen through the obtained results that the most sensitive part of ULT freezer is freezer chamber.

In order to prevent the breakdown of the complete system, the maintenance team must plan the repair scheme according to the component ranking. On the other hand, redundancy of the sensitive parts can be added at the time of manufacturing of the system to prevent the failure of the parts. It also helps to increase the performance of the system.

The proposed study focus on finding the reliability indices of the ULT freezer. In future, this study can be extended to find the other parameters like efficiency and cost-effectiveness of the ULT freezer. Also, the model can be analyzed by taking different type of fuzzy numbers like trapezoidal fuzzy number. The proposed technique can also be applied on the other repairable industrial systems for finding different performance measures related to the same.

The limitation of the proposed model is the assumption taken for the confidence level. If one takes different confidence level than it may give different results.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

The different operation used in the present paper are defined as follow.

1. Fuzzy arithmetic operations

Take $F_1=(\delta_1,\delta_2;\omega_1)$ and $F_2=(\eta_1,\eta_2;\omega_2)$ recognizes two right triangular generalized fuzzy numbers, with memberships μ_{F_1} and μ_{F_2} can be written as the following membership functions.

- Membership functions of $F_1=(\delta_1,\delta_2;\omega_1)$ and $F_2=(\eta_1,\eta_2;\omega_2)$

$$\mu_{F_1}=\begin{cases}\omega_1; x=\delta_1\\ \omega_1\left(\dfrac{\delta_2-x}{\delta_2-\delta_1}\right); \delta_1<x\le\delta_2\\ 0; otherwise\end{cases} \tag{1}$$

$$\mu_{F_2}=\begin{cases}\omega_2; x=\eta_1\\ \omega_2\left(\dfrac{\eta_2-x}{\eta_2-\eta_1}\right); \eta_1<x\le\eta_2\\ 0; otherwise\end{cases} \tag{2}$$

$$\delta_1,\delta_2,\eta_1,\eta_2\in R \text{ and } 0\le\omega_1,\omega_2\le 1$$

The fuzzy numbers can be transformed to right triangular generalized fuzzy numbers.

Transformation for the desired confidence level.

$F_1\to F_1^*$ and $F_2\to F_2^*$

Minimum satisfaction level= Intersection of ω_1 and ω_2. It is given by $\omega_{\min}$,

$$\omega_{\min}=\min(\omega_1,\omega_2) \tag{3}$$

$$F_1^*=(\delta_1,\delta_2^*,\delta_2;\omega_1)=(\delta_1,\delta_2,\delta_2;\omega_1)=F_1;\ F_2^*=(\eta_1,\eta_2^*,\eta_2;\omega_2)$$

where, $\delta_1^*=\delta_1+\omega_{\min}\left(\dfrac{\delta_2-\delta_1}{\omega_1}\right)$, $\eta_2^*=\eta_1+\omega_{\min}\left(\dfrac{\eta_2-\eta_1}{\omega_2}\right)$ (4)

1.1 Operation 1

Scalar multiplication a fuzzy number

Multiplication of right triangular generalized fuzzy number $F_1=(\delta_1,\delta_2;\omega_1)$ with a scalar generates a right triangular generalized fuzzy number.

$$K=kF_1=(K_1,K_2;\omega_1); k\ge 0 \tag{5}$$

where, $\left.\begin{matrix}K_1=k\delta_1\\ K_2=k\delta_2\end{matrix}\right\}$

$$K^{*} = kF_1 = (K_1^{*}; K_2^{*}; \omega_1); k \le 0, k = -n(say) \tag{6}$$

where $\left.\begin{aligned} K_1^{*} &= k\delta_2 \\ K_2^{*} &= k\delta_1 \end{aligned}\right\}$

1.2 Operation 2

Addition of two numbers

Addition of two right triangular generalized fuzzy numbers $F_1 = (\delta_1, \delta_2; \omega_1)$ and $F_2 = (\eta_1, \eta_2; \omega_2)$ confidence levels generates a right triangular generalized fuzzy number, $A = F_1 + F_2 = (A_1, A_2, A_3; \omega_{\min})$ where

$$\left.\begin{aligned} A_1 &= \delta_1 + \eta_1 \\ A_2 &= \delta_2 + \left(\eta_1 + \omega_{\min}\left(\frac{\eta_2 - \eta_1}{\omega_2}\right)\right) \\ A_3 &= \delta_2 + \eta_2 \end{aligned}\right\} \tag{7}$$

1.3 Operation 3

Subtraction of two numbers

Subtraction of two right triangular generalized fuzzy numbers $F_1 = (\delta_1, \delta_2; \omega_1)$ and $F_2 = (\eta_1, \eta_2; \omega_2)$ with two different confidence levels generates a right triangular generalized fuzzy number.

$$S = F_1 - F_2 = F_1 + (-F_2) = F_1 + (-1)F_2 = (S_1, S_2, S_3; \omega_{\min}) \tag{8}$$

where, $\left.\begin{aligned} S_1 &= \delta_1 - \eta_2 \\ S_2 &= \delta_2 - \left(\eta_1 + \omega_{\min}\left(\frac{\eta_2 - \eta_1}{\omega_2}\right)\right) \\ S_3 &= \delta_2 - \eta_1 \end{aligned}\right\}.$

1.4 Operation 4

Multiplication of two numbers

Multiplication of two right triangular generalized fuzzy numbers $F_1 = (\delta_1, \delta_2; \omega_1)$ and $F_2 = (\eta_1, \eta_2; \omega_2)$ with two different confidence levels generates a right triangular generalized fuzzy number.

$$M = F_1 * F_2 = (M_1, M_2, M_3; \omega_{\min}) \tag{9}$$

where, $\left.\begin{aligned} M_1 &= \delta_1 * \eta_1 \\ M_2 &= \delta_2 * \left(\eta_1 + \omega_{\min}\left(\frac{\eta_2 - \eta_1}{\omega_2}\right)\right) \\ M_3 &= \delta_2 * \eta_2 \end{aligned}\right\}.$

1.5 Operation 5

Division of two numbers

Division of two right triangular generalized fuzzy numbers $F_1 = (\delta_1, \delta_2; \omega_1)$ and $F_2 = (\eta_1, \eta_2; \omega_2)$ with two different confidence levels generates a right triangular generalized fuzzy number.

$$D = F_1 / F_2 = (D_1, D_2, D_3; \omega_{\min}) \tag{10}$$

where, $$\left. \begin{aligned} D_1 &= \frac{\delta_1}{\eta_2} \\ D_2 &= \frac{\delta_2}{\left(\eta_1 + \omega_{\min}\left(\frac{\eta_2 - \eta_1}{\omega_2}\right)\right)} \\ D_3 &= \frac{\delta_2}{\eta_1} \end{aligned} \right\}$$

1.6 Operation 6

Complement of a number

Complement of a right triangular generalized fuzzy number $F_1 = (\delta_1, \delta_2; \omega_1)$ is

$$C = 1 - (F_1, F_2) = (C_1, C_2; \omega_1) \tag{11}$$

where, $$\left. \begin{aligned} C_1 &= 1 - \delta_2 \\ C_2 &= 1 - \delta_1 \end{aligned} \right\}$$

2. Cases for Operations

Let F_1 and F_2 are two right triangular generalized fuzzy numbers (RTrFNs) on which the operations to be applied. Based upon the confidence level, three cases arise and which are discussed in this section.

2.1 Case 1

Let F_1 and F_2 be two right triangular generalized fuzzy numbers with same satisfaction level as shown in Figure 17 and Figure 18 respectively.

$F_1 = (\delta_1, \delta_2; \omega_1)$ and $F_2 = (\eta_1, \eta_2; \omega_2)$. It is the case when level of satisfaction for F_1 and F_2 are equal *i.e.* $\omega_{F_1} = \omega_{F_2} = \omega$.

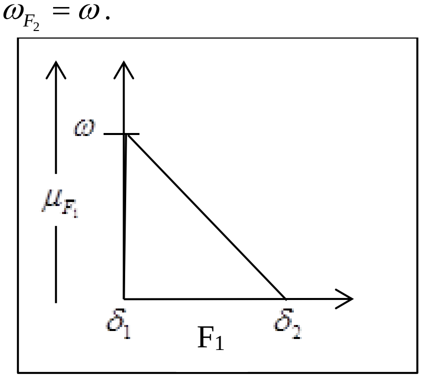

Figure 17. Right triangular Generalized Fuzzy Number (RTrGFN) F_1.

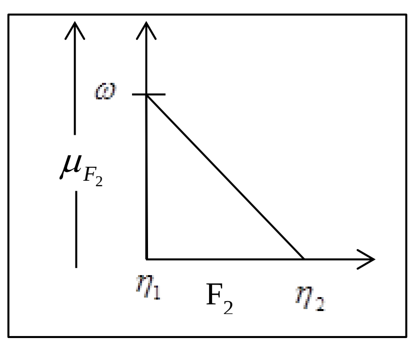

Figure 18. Right triangular Generalized Fuzzy Number (RTrGFN) F_2.

The corresponding membership functions of F_1 and F_2 are given as:

$$\mu_{F_1}=\begin{cases}\omega; x=\delta_1\\ \omega\cdot\left(\dfrac{\delta_2-x}{\delta_2-\delta_1}\right); x\in(\delta_1,\delta_2)\\ 0; otherwise\end{cases} \text{ and } \mu_{F_2}=\begin{cases}\omega; x=\eta_1\\ \omega\cdot\left(\dfrac{\eta_2-x}{\eta_2-\eta_1}\right); x\in(\eta_1,\eta_2)\\ 0; otherwise\end{cases} \tag{12}$$

The following arithmetic operations can be carried out:

- Addition of two numbers

$$A=F_1+F_2=(\delta_1+\eta_1,\delta_2+\eta_2;\omega) \tag{13}$$

- Subtraction of two numbers:

$$S=F_1-F_2=(\delta_1-\eta_2,\delta_2-\eta_1;\omega) \tag{14}$$

- Multiplication of two numbers:

$$M=F_1*F_2=(\delta_1*\eta_1,\delta_2*\eta_2;\omega) \tag{15}$$

- Division of two numbers:

$$D=F_1/F_2=(\delta_1/\eta_2,\delta_2/\eta_1;\omega) \tag{16}$$

- Alpha cut of Right Triangular Generalized Fuzzy Number (RTrGFN)

Let $F=(\delta_1,\delta_2;\omega)$ be any number, then alpha cut of F is given

$$A_\alpha=\left[A_\alpha^L,A_\alpha^R\right]=\left(\delta_1,\delta_2-\frac{\alpha}{\omega}(\delta_2-\delta_1)\right) \tag{17}$$

2.2 Case 2

$\omega_1\neq\omega_2$ $\quad\omega_1<\omega_2;\omega_{\min}=\min(\omega_1,\omega_2)=\omega_1$.

F_1 and F_2 are two Right Triangular Generalized Fuzzy Numbers (RTrFN) as shown in Figure 19 and Figure 20 respectively. Therefore, F2 (see Figure 20) can be modified as F2* (see Figure 21) according to the minimum weight from F_1 and F_2.

$$F_2^*=(\eta_1^*,\eta_2^*,\eta_3^*;\omega_1) \tag{18}$$

where,

$$\eta_1^*=\eta_1,\eta_2^*=\eta_1+\omega_{\min}\left(\frac{\eta_2-\eta_1}{\omega_2}\right),\eta_3^*=\eta_2$$

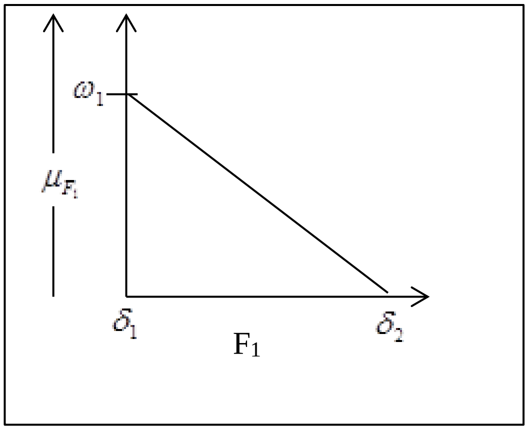

Figure 19. F_1 with maximum level ω_1.

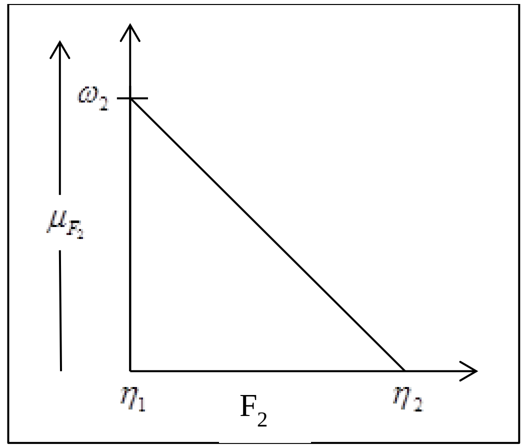

Figure 20. F_2 with maximum level ω_2.

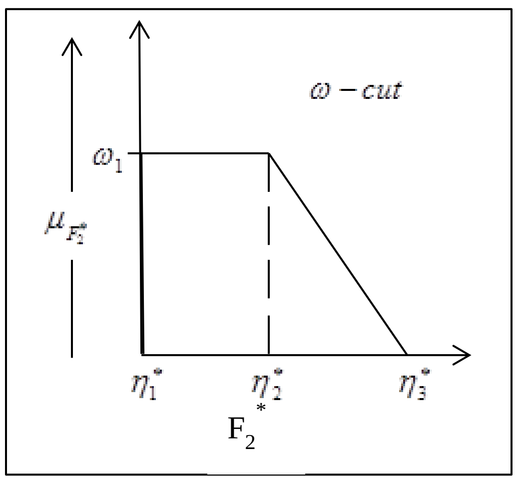

Figure 21. Generalization of F_2 up to same level of F_1 and transformation of F_2 to F_2^*.

Also, F_1 can be transformed by using [26] and also defined by equation (12) as

$$F_1 = (\delta_1, \delta_2, \delta_2; \omega_1) \tag{19}$$

After the transformation of F_1 and F_2, their membership functions are given by

$$\mu_{F_1} = \begin{cases} \omega_1; x = \delta_1 \\ \omega_1 \cdot \left(\dfrac{\delta_2 - x}{\delta_2 - \delta_1}\right); (\delta_1, \delta_2] \\ 0; otherwise \end{cases} \text{ and } \mu_{F_2}^* = \begin{cases} \omega_1; (\eta_1^*, \eta_2^*) \\ \omega_1 \cdot \left(\dfrac{\eta_3^* - x}{\eta_3^* - \eta_2^*}\right); (\eta_2^*, \eta_3^*] \\ 0; otherwise \end{cases} \tag{20}$$

The following arithmetic operations can be carried out:

- Addition of two numbers

$$A = F_1 + F_2^* = \left(\delta_1 + \eta_1^*, \delta_2 + \eta_2^*, \delta_2 + \eta_3^*; \omega_1\right) \tag{21}$$

- Subtraction of two numbers:

$$S = F_1 - F_2^* = \left(\delta_1 - \eta_3^*, \delta_2 - \eta_2^*, \delta_2 - \eta_1^*; \omega_1\right) \tag{22}$$

- Multiplication of two numbers:

$$M = F_1 * F_2^* = \left(\delta_1 * \eta_1^*, \delta_2 * \eta_2^*, \delta_2 * \eta_3^*; \omega_1\right) \tag{23}$$

- Division of two numbers:

$$D = F_1 / F_2^* = \left(\delta_1 / \eta_3^*, \delta_2 / \eta_2^*, \delta_2 / \eta_1^*; \omega_1\right) \tag{24}$$

2.3 Case 3

If the two generalized fuzzy numbers F_1 (see Figure 22) and F_2 (see Figure 23) are obtained after $\omega-$cut the level (see Figure 23). The membership function of F_1^* and F_2^* are given by equations (25-26).

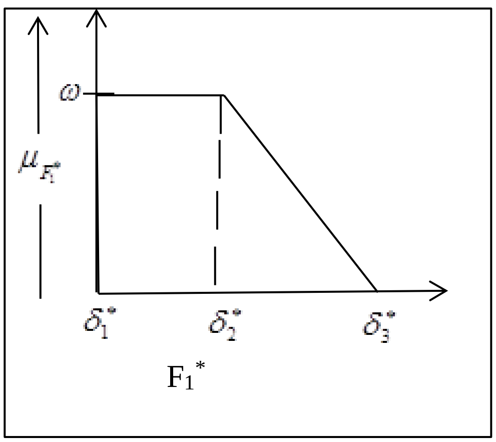

Figure 22. Transformed Generalized Fuzzy Number F_1 after $\omega - cut$.

$$\mu_{F_1}^* = \begin{cases} \omega; & (\delta_1^*, \delta_2^*) \\ \omega\left(\dfrac{\delta_3^* - x}{\delta_3^* - \delta_2^*}\right); & (\delta_2^*, \delta_3^*] \\ 0; & otherwise \end{cases} \tag{25}$$

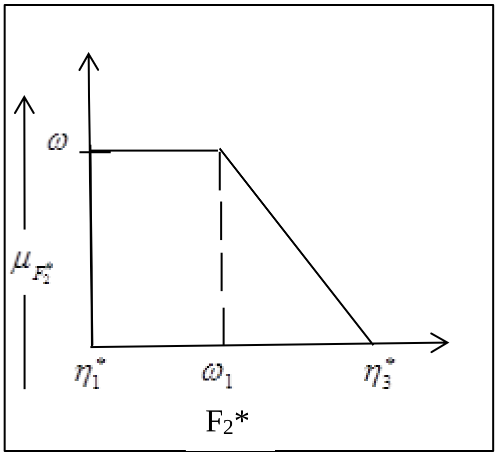

Figure 23. Transformed Generalized Fuzzy Number F_2 after $\omega - cut$.

$$\mu_{F_2}^* = \begin{cases} \omega; (\eta_1^*, \eta_2^*) \\ \omega\left(\dfrac{\eta_3^* - x}{\eta_3^* - \eta_2^*}\right); (\eta_2^*, \eta_3^*] \\ 0; otherwise \end{cases} \tag{26}$$

The following arithmetic operations can be carried out:

- Addition of two numbers:

$$A = F_1^* + F_2^* = \left(\delta_1^* + \eta_1^*, \delta_2^* + \eta_2^*, \delta_3^* + \eta_3^*; \omega\right) \tag{27}$$

- Subtraction of two numbers:

$$S = F_1^* - F_2^* = \left(\delta_1^* - \eta_3^*, \delta_2^* - \eta_2^*, \delta_3^* - \eta_1^*; \omega\right) \tag{28}$$

- Multiplication of two numbers:

$$M = F_1^* * F_2^* = \left(\delta_1^* * \eta_1^*, \delta_2^* * \eta_2^*, \delta_3^* * \eta_3^*; \omega\right) \tag{29}$$

- Division of two numbers:

$$D = F_1^* / F_2^* = \left(\delta_1^* / \eta_3^*, \delta_2^* / \eta_2^*, \delta_3^* / \eta_1^*; \omega\right) \tag{30}$$

- Alpha cut of Right Triangular Generalized Fuzzy Number (RTrGFN)

Let $F = \left(\delta_1^*, \delta_2^*, \delta_3^*; \omega\right)$ be any number, and then alpha cut of F is given by

$$A_\alpha = \left[A_\alpha^L, A_\alpha^R\right] = \left(\delta_1^*, \delta_3^* - \frac{\alpha}{\omega}\left(\delta_3^* - \delta_2^*\right)\right) \tag{31}$$

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