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Impact of Carbon Emission Policy on Fresh Food Supply Chain Model for Deteriorating Imperfect Quality Items

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Abstract

Carbon emissions can be decreased by adopting the carbon cap-and-alternate policy. The current study suggests a carbon buying and selling mechanism for things that are deteriorating or of poor quality while taking into account chilled logistics services in a fresh food supply chain. In addition to deliveries of perishables, suppliers also provide retailers with chilled logistics services and carbon emission certificates for excess inventory. The retail price, the cost of chilled strategies, and the contributions to various carbon trading options—such as internal carbon trade, external carbon exchange, and carbon exchange both internally and externally for the destruction of low-quality goods have all been evaluated in this paper. The store network members give estimating systems to new food, emanation permits and refrigerated planned operations administrations. We likewise uncover the connection between carbon purchasing and advancing and refrigerated strategies administrations and test out their joint effect on the provider retailer's helpful dating. Store network donors are also encouraged to participate in the carbon exchanging mechanism, which benefits from more sophisticated asset utilisation and more ruthless stockpile chains. The numerical examples have helped to validate the results. In the end, a thorough sensitivity analysis has been provided.

Keywords- Imperfect quality items, Carbon cap-and-change regulation, Fresh food supply chain, Chilled logistics services, Leader-follower recreation.

1. Introduction

It is essential to equip the centers like refrigerated warehouses and vehicles to keep the food fresh in a food supply chain. However, those centers are so steeply-priced that most effective big-scale organisations can own them. For that reason, there comes the idea that those businesses provide refrigerated logistics offerings to other companies. For instance, on October 28, 2015, China Daily reported that Amazon has begun offering "Amazon Logistics+" (found at https://www.Z-exp.com) in China. Amazon is adopting some of the characteristics of a third-party logistics (3PL) company while also fulfilling its logistics requirements by providing logistics services. Large businesses may act as chilled logistics suppliers for clean meal delivery chains like Americold (http://americold.com/). Therefore, the development of useful models is imperative, and this work aims to achieve that.

In comparison to traditional prescriptive legislation, carbon cap-and-trade programs have proven to be more effective in achieving environmental objectives (Benjaafar et al., 2012; Sodhi and Tang, 2018). The cap-and-alternate regulatory strategy is widely employed to reduce carbon emissions. Under this approach, providers offer excess carbon emission permits and chilled logistics services to merchants in exchange for unique carbon trading alternatives (Wang et al., 2019). In their study, Wang et al. (2019) examine a fresh food supply chain that operates under a carbon cap-and-exchange program, involving a large supplier and multiple small businesses. By implementing optimal operational changes and collaborating with other participants in their supply chain, these firms aim to effectively reduce their carbon emissions without significantly increasing costs. Two commonly used methods for allocating carbon permits within the carbon cap-and-exchange regulation are grandfathering and benchmarking. The grandfathering approach refers to the allocation of permits to a company based on its previous emissions. On the other hand, benchmarking involves the allocation of permits based on the surplus or shortage of permits due to emissions (Chang et al., 2017).

The supply chain for fresh food is a complicated network that faces numerous interrelated difficulties, including demand variability, manufacturing and process variability, time-to-market, traceability, transportation, and storage problems. To increase profit margins on fresh food items like dairy, meat, fish, prepared foods, vegetables, frozen dinners, and so forth, effective management of pricing and food handling procedures is crucial across the whole logistics process.

Researchers have developed several models to optimize the fresh food delivery chain. For example, Pal and Mahapatra (2017) developed a three-layer supply chain production-inventory model that considers the production cost and order lead time at each stage of the supply chain. Huang et al. (2018) developed a three-level food delivery chain Stackelberg game model to maximize individual profits.

Said another way, the fresh food delivery chain is an intricate system beset with difficulties. Selling fresh food can be profitable if pricing and food handling are properly managed. Models that businesses can use to optimize their fresh food delivery chains have been developed by researchers. In 2019, Kumar et al. (2019) created a model that assists companies in selecting the optimal price and terms of warranty for a new product. In simpler terms: Kumar et al. (2019) created a model to help businesses set prices and warranties for new products, considering different warranty lengths and future production changes. According to Meneghetti and Monti (2015), most of the energy consumption and carbon emissions in fast food chains can be attributed to refrigeration. In this study, we examine a carbon trading mechanism within the delivery chain, considering the provision of chilled logistics services. The research focuses on how issuers (companies that sell carbon emission permits) and stores can work together to use the issuer's unused assets, such as chilled logistics facilities and carbon emission permits, more effectively (Alamri et al., 2022). To maximize profits for all parties involved, the objective is to optimize retail pricing decisions, chilled logistics service costs, and the trading pace of emission permits along the supply chain.

2. Literature Review

2.1 Fresh Food Supply Chains

Fresh food delivery chains have garnered significant attention due to their distinctive features (Nahmias, 1982; Goyal and Giri, 2001; Van Der Vorst et al., 2009; Bakker et al., 2012).

Significant financial losses may occur when the seamless provision of meals depends on perishable goods and carbon emission costs. As such, these supply chains' modelling and optimisation constitute a theoretical as well as a practical endeavour.

Exponential decay is employed to characterize the tendencies of perishable fresh food (Ghare, 1963). Based on the findings of the experiments, several theories have been proposed regarding the rate of deterioration. For instance, Bhunia and Maiti (1999) consider it to be a constant. Mukhopadhyay et al. (2004) predicts an inverse relationship between exponential decay and the remaining shelf life of fresh food. Prasad and Mukherjee (2016) utilized a parameter Weibull distribution to model this decay.

Other researchers, such as Zhang et al. (2015), have developed models to help companies decide how much to invest in preservation technology to reduce food spoilage, similar to Dye (2013). Zhang et al. (2015) focused on a supply chain with one producer and one retailer, where the companies shared the cost of upgrading the technology. They did this by adapting Dye's model to a supply chain setting.

Modeling the demand for healthy meals using a constant demand price is possible (Raafat et al., 1991). A price-established demand can be enforced if customers are uncomfortable with the selling price (Abad, 1996; Zhu and Cetinkaya, 2015). According to Dobson et al. (2017), a desire for relying on the availability of meals may also be taken into account. High-quality food is referred to as a property of time and temperature since it can have a longer shelf life if stored at the optimum temperature (Wang and Li, 2012). Demand is also influenced by the quantity of food that is available (Ghiami et al., 2013). To forecast demand, certain researchers have created models that consider a variety of variables, including the promotional price, inventory levels, and expiration date (Feng et al., 2017). These models predict that the call will be contingent upon the promotional price, listed stocks, and expiration date (Mittal and Sarkar, 2023). The current study assumes that the meal supply deteriorates exponentially with an ordinary degradation charge and considers the pattern of deterministic name for this to be inversely associated with retail rate.

The carbon emission policy addresses the environmental effects of food production, distribution, and transit, which is in line with long-term sustainability goals for the fresh food supply chain. How to do it is as follows:

Minimising Environmental Impact: The transportation, refrigeration, and packaging processes involved in the supply chains for fresh food frequently result in the release of greenhouse gases. Organisations are forced to lower their carbon footprint by enacting carbon emission policies, which lessens the supply chain operations' negative environmental effects.

Encouraging Sustainable Practices: The policy pushes for the adoption of sustainable practices that give energy efficiency, renewable energy sources, and emission reduction techniques top priority among those involved in the fresh food supply chain. In addition to lowering carbon emissions, this move towards sustainability minimises waste, preserves natural resources, and safeguards ecosystems.

2.2 Supply Chain Operation under Carbon Cap-and-Trade Regulation

The importance of environmental sustainability in supply chain operations has been widely recognized by (Marconi et al., 2017). Government programs to reduce emissions and promote environmental sustainability have forced supply chains to change their operations. This study is based on the carbon cap-and-trade system, which Sarkis and Zhu (2018) and Benjaafar et al. (2012) have discussed. The two most common and well-studied carbon reduction solutions proposed by Benjaafar et al. (2012) are the carbon tax and the carbon cap-and-trade system.

Research on the operational decisions made in supply chains under the carbon cap-and-trade system has been growing rapidly. This legislation addresses manufacturing and inventory issues (Chen et al., 2013; Du et al., 2016). Other research examines distribution decisions, such as warehouse location, network design, and transportation modes, in order to balance transportation costs, facility investments, and carbon emissions (Marufuzzaman et al., 2014; Qiu et al., 2017). Supply chain coordination has also been studied as a way to reduce costs and emissions (Toptal and Cetinkaya, 2017; Xu et al., 2017; Mittal and Sharma, 2021).

The format of this paper is as follows. The assumptions and notation are introduced in section 2. The model development and associated supply chain objective functions are shown in section 3. The procedure for resolving the three potential carbon trading schemes is explained in section 4. Numerical examples and solutions are presented in section 5. Sensitivity analysis is carried out in section 6. Section 7 concludes by summarizing the results and outlining potential directions for future study.

3. Research Gaps in Literature

The extant literature on inventory management in the context of supply chains for fresh food predominantly concentrates on conventional factors like order quantity optimisation, demand forecasting, and shelf-life considerations. Nonetheless, there is a clear research vacuum concerning our knowledge of the complex interactions between carbon emission regulations and the handling of items of declining or subpar quality in the fresh food supply chain. The growing emphasis on sustainability and environmental responsibility in the modern business environment makes this research gap especially important.

There is a lack of thorough research that examines the direct implications of carbon emissions on inventory management, particularly in the context of fresh food supply chains dealing with deteriorating imperfect quality items, despite the growing awareness of the effects of carbon emissions on the environment and the subsequent development of policies to mitigate these effects. Because fresh food is perishable and has quality flaws, it presents special challenges that call for a sophisticated understanding of the relationship between inventory management techniques and carbon emission policies.

Although some research addresses more general sustainability concerns in supply chain management, little is known about the complexities of handling subpar products and their associated perishability in the context of carbon emission regulations. Developing robust and sustainable supply chain models can be greatly aided by having a thorough grasp of how carbon emission regulations affect inventory management.

The lack of a comprehensive investigation into the effects of carbon emission regulations on the inventory management of fresh food supply chains handling deteriorating imperfect quality items can therefore be summed up as the research gap. A concentrated study in this field would not only improve our comprehension of the difficulties and possibilities but also offer insightful information to practitioners and legislators who are working to harmonise supply chain operations with environmental sustainability objectives.

4. Assumptions and Notations

This section introduces the assumptions and notation for the deteriorating imperfect quality items in a fresh food delivery chain.

4.1 Assumptions

(i) Inside the one-provider, one-retailer chain of sparkling meals, a pioneer admirer may be looking. The leader who performs a dominant role is the provider (Wang et al., 2018).
(ii) Remarkable rot of shimmering dinners happens. The weakening charge is a predictable.
(iii) A two-section levy settlement covers the cost of payments for refrigeration scheduled activities. The variable cost is based on the quantity requested and the fee for coordinated refrigeration operations contributions (Wang et al., 2018).
(iv) The lingering cost of unused fossil fuel byproduct grants is zero.
(v) The expense of outflow licenses exchanged in the inventory network doesn't surpass that inside the carbon market (Wang et al., 2018).
(vi) There might be a negative relationship between's commercial center call for rate and the retail expense.

4.2 Notations

The following notations from Table 1 will be used throughout the model.

Table 1. Notations, decision variables and functions.

Parameters	
v	potential market size (units/year)
c	production cost ($/units)
T	the length of sale period (years)
a	the price sensitivity coefficient of consumer
w	wholesale price ($/units)
F	the fixed price for chilled logistics services ($/units)
u	fresh food rate of degradation (per year)
χ	the refergeration facilities carbon emission ratio
s	outer trade price ($/units)
α	percentage of defective items in the lot size Q
t_1	screening time (years)
p_s	salvage value per unit for defective units ($/units)
C_s	permit for the supplier's leftover carbon emission
C_r	retailer's carbon cap
Decision variables	
p	retail price $(0 \le p \le \frac{v}{a})$ ($/units)
r	the price of carbon emission permits traded within the supply chain $(0 \le r \le s)$ ($/unit)
k	the price of refrigerated logistics services, $k \ge 0$ (denoted as logistics price hereinafter) ($/unit)
Functional Values	
$D(p)$	market demand rate, which is negatively correlated with retail price (units/year)
$Q(t)$	the food quantity at time t, which is a function of time (units)
$G(Q(0))$	the fee of refrigerated logistics services charged under a Two-part tariff contract and G= F + kQ(0) ($/unit)
E(Q(0))	the emissions of the retailer caused by refrigerated facilities, which are proportional to order quantity, and E = χ Q(0) (units)
$R_s(E)$	the supplier's revenue of carbon trading ($/unit/year)
$P_r(E)$	the retailer's cost of carbon trading ($/unit/year)
$\lambda(u)$	deterioration coefficient, which is positively correlated with deterioration rate
$\phi(\lambda)$	the critical threshold of the retailer's carbon cap

5. Model Formulation

A fresh food inventory network under the carbon cap-and-trade scheme consists of a supplier (S) and a retailer (R). Both supply chain members are allocated a certain amount of carbon emissions permits, known as carbon caps. The carbon caps are assigned based on the 'grandfathering' principle, which means that companies are allocated permits based on their historical emissions**.** The supplier not only delivers fresh food, but also helps with the refrigerated operations. The retailer covers the cost of the services, and the store bears the liability for any emissions that arise from their use of chilled facilities. As a result, the association's discharge licences are overloaded, and the shop's emissions may potentially exceed its carbon cap. Then, carbon buying for and advancing may also get up inside the conveying chain.

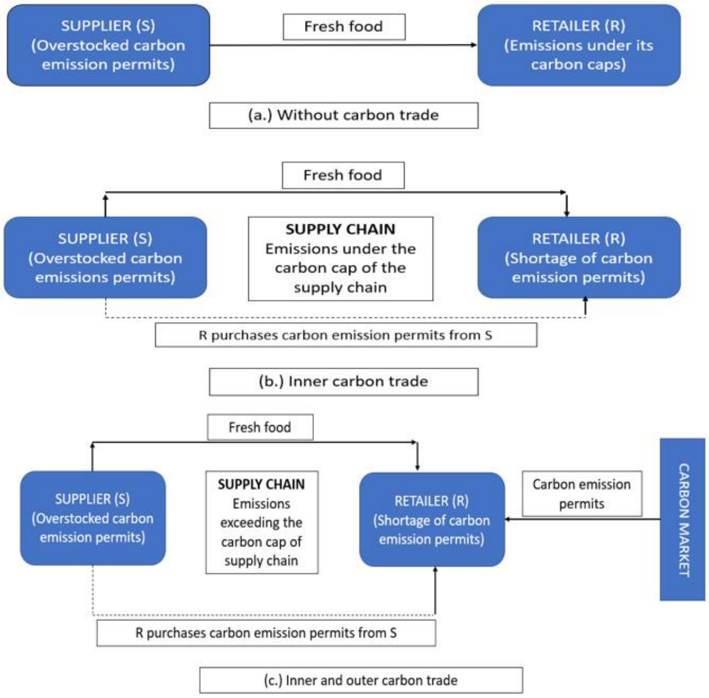

Figure 1. A fresh food supply chain with a carbon trading mechanism.

In Figure 1, the method for purchasing and selling carbon is shown. The merchant offers three options for purchasing and selling carbon.

(i) The shop maintains emissions within its carbon limit with this function, and no carbon trading or purchase occurs.

(ii) Internal carbon exchange: In this scenario, a store buys additional emission permits from a dealer when its own expire.

(iii) Trade in inner and outer carbon: Under this option, the store consumes even the company's emission permits before purchasing further permits on the carbon market.

Table 2. The emission constraints, the supplier's carbon sales and the retailer's carbon price in each choice.

-	**Emission Constraints**	$R_s(E)$	$P_r(E)$
Option 1	$E \leq C_r$	0	0
Option 2	$C_r < E \leq C_r + C_s$	$(E - C_r)r$	$(E - C_r)r$
Option 3	$E > C_r + C_s$	$C_s r$	$C_s r + (E - C_r - C_s)s$

I(t) be the inventory level at time t,

$$\frac{dI(t)}{dt} + uI(t) = -D, (0 \le t \le T).$$

The solution of the above differential equation along the boundary condition, $t = 0,\ \ I(t) = Q$,

$$\Rightarrow I(t) = Qe^{-uT} + \frac{D}{u}[e^{-uT} - 1].$$

After the screening process, the number of defective items at time t_1, is αQ.

Hence, effective inventory level during $t_1 \le t \le T$,

$$\Rightarrow I(t) = Qe^{-uT} + \frac{D}{u}[e^{-uT} - 1] - \alpha Q, \qquad t_1 \le t \le T.$$

At $t = T, I(T) = 0$ gives order quantity which follows as,

$$Q = \frac{D(e^{uT}-1)}{u(1-\alpha e^{uT})} \Rightarrow Q = \frac{\lambda TD}{(1-\alpha e^{uT})}$$

From the Table 2, there are carbon bargains for the backer ($R_s(E)$) and the shop ($P_r(E)$). Under option 1, outflows are no longer more than the retailer's carbon allowance (signified as discharge limitation 1). As a result, there is no growth in carbon trading, resulting in no carbon compensation for the guarantor and no carbon expense for the retailer. In need 2 (also known as discharge urgent 2), emissions are over the retailer's carbon cap but below the transport chain's cap. In this manner, the retailer purchases $(E - C_r)$ discharge allows in from the guarantor to fulfill its interest and incurs expenses in accordance with the quantity of allows in offered by the retailer $(E - C_r)$ and the internal substitution rate (r). With relation to option 3, discharges exceed the carbon cap on the convey chain (meant as discharge imperative 3). The supplier's permits are then handed out, and they enable them to generate carbon revenue that rises to the level established by their carbon cap (C_s) and internal exchange rate (r) as discussed in the Figure 2. In any event, the business must apply for extra permits at the carbon commercial center due to a permit shortfall with a required amount of $(E - C_s - C_r)$. In this method, the shop's typical carbon value includes both the payment made to the carbon market $((E - C_s - C_r) * s)$ and the supplier $(C_s * r)$.

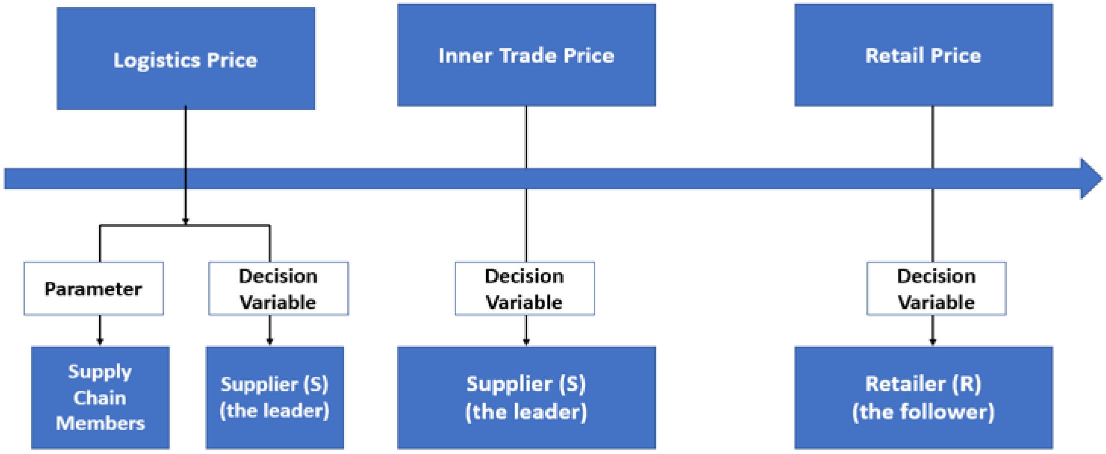

Figure 2. The decision sequence of supply chain members in the carbon trading mechanism.

5.1 Objective Functions

The supplier and retailer's profitability are the primary objectives.

The profit of the supplier is revealed by

$$\Pi_s = wQ(t) - cQ(t) + G + R_s(E) \tag{1}$$

where, the supplier's sales income is the first term, the cost of manufacturing is the second, the revenue from the refrigerated logistics services is the third, and the carbon revenue is the fourth.

The profit for the retailer is determined by

$$\Pi_r = pTD + p_s\alpha Q - wQ(t) - G - P_r(E) \tag{2}$$

where, the first term is the retailer's sales income, the second is the sale of goods of subpar quality, the third is the cost of procurement, the fourth is the cost of chilled logistics services, and the last term is the cost of carbon.

6. Solution Procedure

Within the leader-follower game, the selection process of supply chain contributors are demonstrated in the diagram. Initially, the logistics fee is determined. Subsequently, the supplier makes a decision regarding the optimal internal trade rate. Finally, the retailer optimizes its retail fee. In this analysis, we are examining two scenarios in relation to the cost of logistics. The participants in the supply chain have reached an agreement on a negotiated logistics charge, thus, scenario 1 represents a straightforward situation where the logistics cost serves as a parameter. In scenario 2, we assume that the supplier will maximize profits by optimizing the logistical fee. The model in scenario 2 is established using the solution obtained from scenario 1 as an intermediary variable, thus, scenario 2 is fully explored based on Scenario 1.

We are using the notion of reverse enlistment as a game idea in order to fix this pioneer devotee game. Being the devotee, the retailer first accepts the perceived decision factors from the provider and obtains its central reaction component in order to increase the pay. Next, the supplier resolves the issue of benefit expansion and simplifies its selection criteria based on the shop's acknowledged response. Not too long after, the retailer's preferred variable is acquired. In two scenarios, decisions about the arrangements are made segment by segment.

The agreed-upon logistics rate is a parameter in scenario 1, which is indicated by the superscript ‘*n*’. The superscript ‘*o*’ denotes the situation where the dealer assists in reducing the logistical fee.

6.1 Scenario 1: Basic Scenario

In this section, we use the logistics price to calculate the equilibrium solutions for three alternatives in scenario 1. The three alternatives are denoted by the subscripts 1-3, respectively.

6.1.1 Solution in Option 1

If we substitute the order quantity values, carbon revenue, and carbon cost into the objective function Equations (1) and (2), we can express the profit of the supplier and retailer in option 1 as follows,

$$\Pi_{s1}^n = (w-c)\frac{\lambda T(v-ap_1^n)}{1-\alpha e^{uT}} + [k_1^n \frac{\lambda T(v-ap_1^n)}{1-\alpha e^{uT}}) + F] \tag{3}$$

$$\Pi_{r1}^n = p_1^n T(v-ap_1^n) + p_s\alpha\frac{\lambda T(v-ap_1^n)}{1-\alpha e^{uT}} - w\frac{\lambda T(v-ap_1^n)}{1-\alpha e^{uT}} - [k_1^n \frac{\lambda T(v-ap_1^n)}{1-\alpha e^{uT}} + F] \tag{4}$$

Taking the first partial derivative of Equation (4) w.r.t p_1^{n*} and equating it to zero, we obtain the optimal retail price,

$$p_1^{n*} = \frac{v}{2a} + \frac{\lambda(w+k_1^n-\alpha p_s)}{2(1-\alpha e^{uT})} \tag{5}$$

Option 1 does not entail any carbon trading, hence the inner trade price is not taken into consideration when making a choice. When the value of of p_1^n from Equation (5) is substituted into Equations (3) and (4), the supplier's and the retailer's profits are as follows,

$$\Pi_{s1}^n = \frac{a\lambda T(w+k_1^n-c)}{2(1-\alpha e^{uT})}\left[\frac{v}{a} - \frac{\lambda(w+k_1^n-\alpha p_s)}{1-\alpha e^{uT}}\right] + F \tag{6}$$

$$\Pi_{r1}^n = \frac{aT}{4}\left[\frac{v}{a} - \frac{\lambda(w+k_1^n-\alpha p_s)}{1-\alpha e^{uT}}\right]^2 - F \tag{7}$$

6.1.2 Solution in Option 2

Option 2 uses the following formulas to describe the earnings of the retailer and the supplier:

$$\Pi_{s2}^n = (w-c)\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} + \left[k_2^n\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} + F\right] + \left[\chi\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} - C_r\right]r_2^n \tag{8}$$

$$\begin{aligned}\Pi_{r2}^n = {} & p_2^n T(v-ap_2^n) + p_s\alpha\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} - w\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} - \left[k_2^n\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} + F\right] - \\ & \left[\chi\frac{\lambda T(v-ap_2^n)}{1-\alpha e^{uT}} - C_r\right]r_2^n\end{aligned} \tag{9}$$

Differentiating Equation (9) w.r.t p_2^n and we get the retailer's ideal response function by setting the derivative to zero,

$$p_2^{n*}(r_2^n) = \frac{v}{2a} + \frac{\lambda(w+k_2^n+\chi r_2^n-\alpha p_s)}{2(1-\alpha e^{uT})} \tag{10}$$

Now plug Equation (10) to Equation (8),we obtain the profit of the supplier as a function of r_2^n,

$$\Pi_{s2}^n = \frac{a\lambda T(w+k_2^n+\chi r_n^n-c)}{2(1-\alpha e^{uT})}\left[\frac{v}{a} - \frac{\lambda(w+k_2^n+\chi r_2^n-\alpha p_s)}{1-\alpha e^{uT}}\right] + F \tag{11}$$

To find the best trade price, we maximize Equation (11),

$$r_2^{n*} = \frac{v(1-\alpha e^{uT})}{2a\lambda\chi} + \frac{c+\alpha p_s}{2\chi} - \frac{w+k_2^n}{\chi} - \frac{C_r(1-\alpha e^{uT})^2}{a\lambda^2 T\chi^2} \tag{12}$$

If we substitute Equation (12) into Equation (10), we get the optimal retail price,

$$p_2^{n*} = \frac{3v}{4a} + \frac{\lambda(c-\alpha p_s)}{4(1-\alpha e^{uT})} - \frac{C_r(1-\alpha e^{uT})}{2a\lambda T\chi} \tag{13}$$

Equations (8) and (9) are used to determine the profitability of the supply chain members given the optimal retail price and optimal inner trade price,

$$\begin{aligned}\Pi_{s2}^n = {} & \frac{aT}{8}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} + \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right]\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} - \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right] - \\ & C_r\left[\frac{v(1-\alpha e^{uT})}{2a\lambda\chi} + \frac{(c+\alpha p_s)}{2\chi} - \frac{(w+k_2^n)}{2\chi} - \frac{C_r(1-\alpha e^{uT})^2}{a\lambda^2 T\chi^2}\right]\end{aligned} \tag{14}$$

$$\begin{aligned}\Pi_{r2}^n = {} & \frac{aT}{16}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} + \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right]^2 + \\ & C_r\left[\frac{v(1-\alpha e^{uT})}{2a\lambda\chi} + \frac{(c+\alpha p_s)}{2\chi} - \frac{(w+k_2^n)}{2\chi} - \frac{C_r(1-\alpha e^{uT})^2}{a\lambda^2 T\chi^2}\right]\end{aligned} \tag{15}$$

6.1.3 Solution in Option 3

In case of option 3, the profits of the supply chain members are,

$$\Pi_{s3}^{n} = (w-c)\frac{\lambda T(v-ap_3^n)}{1-\alpha e^{uT}} + [k_1^n \frac{\lambda T(v-ap_3^n)}{1-\alpha e^{uT}} + F] + C_s r_3^n \tag{16}$$

$$\Pi_{r3}^{n} = p_3^n T(v-ap_3^n) + p_s \alpha \frac{\lambda T(v-ap_3^n)}{1-\alpha e^{uT}} - w\frac{\lambda T(v-ap_3^n)}{1-\alpha e^{uT}} - [k_3^n \frac{\lambda T(v-ap_3^n)}{1-\alpha e^{uT}} + F] - [[\chi \frac{\lambda T(v-ap_3^n)}{1-\alpha e^{uT}} - C_s - C_r]s + C_s r_3^n] \tag{17}$$

The ideal retail price is initially determined by using the same solution process as before,

$$p_3^{n*} = \frac{v}{2a} + \frac{\lambda(w+K_3^n+\chi s-\alpha p_s)}{2(1-\alpha e^{uT})} \tag{18}$$

The supplier's task of maximising profit becomes,

$$\max_{0\le r_3^n \le s} \Pi_{s3}^{n} = \frac{aT\lambda(w+k_3^n-c)}{2(1-\alpha e^{uT})}[\frac{v}{a} - \frac{\lambda(w+k_3^n+\chi s-\alpha p_s)}{(1-\alpha e^{uT})}] + C_s r_3^n + F \tag{19}$$

Because Π_{s3}^{n} is a monotone increasing function of r_3^n, the optimal inner trade price is the upper bound of Π_{s3}^{n},

$$r_3^{n*} = s \tag{20}$$

After knowing the ideal inner trade price and retail price, Equations (16) and (17) are used to calculate the revenues of supply chain participants,

$$\Pi_{s3}^{n} = \frac{aT\lambda(w+k_3^n-c)}{2(1-\alpha e^{uT})}[\frac{v}{a} - \frac{\lambda(w+k_3^n+\chi s-\alpha p_s)}{(1-\alpha e^{uT})}] + C_s s + F \tag{21}$$

$$\Pi_{r3}^{n} = \frac{aT\lambda(w+k_3^n-c)}{4}[\frac{v}{a} - \frac{\lambda(w+k_3^n+\chi s-\alpha p_s)}{(1-\alpha e^{uT})}]^2 + C_r s - F \tag{22}$$

6.1.4 Limitations on the Three Possible Solutions

Ideas 1 and 2 further support the findings. Each proposition is denoted by an overline and underline, representing the upper and lower bounds. All assertions are verified in the appendix. The definitions of the capabilities E(Q(0)) and D(p) illustrate the relationships between outflow limitations, request total, and retail rate. The retailer's structural amount must adhere to the outflow restrictions. Additionally, if the retail charge is specified accurately, the requested amount can be stored as permissible. Consequently, considering emission constraints 1-3, suggestion 1 provides the precise levels of the retail charges for the three choices.

***Proposition* 1**: The retail pricing range's top and lower bounds are as follows:

(i) In option 1, $\overline{p_1^n} = \frac{v}{a}$ and $\underline{p_1^n} = \frac{v}{a} - \frac{C_r(1-\alpha e^{uT})}{a\chi T\lambda}$.

(ii) In option 2, $\overline{p_2^n} = \frac{v}{a} - \frac{C_r(1-\alpha e^{uT})}{a\chi T\lambda}$ and $\underline{p_2^n} = \frac{v}{a} - \frac{(C_r+C_s)(1-\alpha e^{uT})}{a\chi T\lambda}$.

(iii) In option 3, $\overline{p_3^n} = \frac{v}{a} - \frac{(C_r+C_s)(1-\alpha e^{uT})}{a\chi T\lambda}$ and $\underline{p_3^n} = 0$.

From condition (5), (18) and suggestion 1, the strategies cost in the choices 1 and 3 are restricted to specific reaches because of their connection with retail cost. Additionally, condition (12) suggests that the supposition that internal exchange cost doesn't surpass outer exchange cost forces limitations on the coordinated factors costs three choices are summed up in recommendation 2.

***Proposition* 2:** Following are the logistics pricing range's top and lower bounds:

(i) In option 1, $\overline{k_1^n} = \frac{v(1-\alpha e^{uT})}{a\lambda} - w + \alpha p_s$ and $\underline{k_1^n} = \frac{v}{a\lambda} - \frac{2C_r(1-\alpha e^{uT})^2}{a\chi T\lambda^2} - w - +\alpha p_s$.

(ii) In option 2, $\overline{k_2^n} = \frac{v(1-\alpha e^{uT})}{2a\lambda} + \frac{c+\alpha p_s}{2} - \frac{C_r(1-\alpha e^{uT})^2}{a\chi T\lambda^2} - w$ and $\underline{k_2^n} = \frac{v(1-\alpha e^{uT})}{2a\lambda} + \frac{c+\alpha p_s}{2} - \frac{C_r(1-\alpha e^{uT})^2}{a\chi T\lambda^2} - w - \chi s$.

(iii) In option 3, $\overline{k_3^n} = \frac{v(1-\alpha e^{uT})}{a\lambda} - \frac{2(C_r+C_s)(1-\alpha e^{uT})^2}{a\chi T\lambda^2} - w - \chi s + \alpha p_s$ and $\underline{k_3^n} = 0$.

Because scenario 1 is used as a stepping stone in solving the model in scenario 2, propositions 1 and 2 also apply to scenario 2's answers.

6.2 Scenario 2: The Logistics Price is Optimised by the Supplier

In scenario 2, the logistics price is optimized to maximize the supplier's profit. The answers for the three possibilities will be drawn from the outcomes of scenario 1. The three alternatives are indicated by the subscripts 1-3.

6.2.1 Solution in Option 1

In order to maximize the supplier's profit, we proceed by differentiating Equation (6) with respect to k_1^n and subsequently setting the derivative to zero.

The unconstrained optimisation problem's best logistics price ($k_1^\#$) is therefore determined to be,

$$k_1^\# = \frac{v(1-\alpha e^{uT})}{2a\lambda} + \frac{c+\alpha p_s}{2} - w \tag{23}$$

Whether $k_1^\#$ falls inside the range of k_1^n in Proposition 2 must be ascertained. It is clear that $\overline{k_1^n} > k_1^\#$ holds. Also, the ideal logistics price will be determined using the crucial threshold of the retailer's carbon cap in Option1, $\Phi(\lambda)$), which is stated in the notations. About the investigation of the connection between $k_1^\#$ and $\underline{k_1^n}$, this results in,

$$\phi(\lambda) = \frac{a\lambda T\chi}{4(1-\alpha e^{uT})^2}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c+\alpha p_s)\right].$$

There are two circumstances in terms of the relationship bewteen $\phi(\lambda)$ and C_r.

***Case* 1:** If the retailer's carbon emissions exceed the critical threshold,

$$C_r > \phi(\lambda) \tag{24}$$

then, $k_1^\# > k_1^n$. Thus, $k_1^\#$ is within the feasible region meaning that,

$$k_1^{o*} = k_1^\# = \frac{v(1-\alpha e^{uT})}{2a\lambda} + \frac{c+\alpha p_s}{2} - w \tag{25}$$

Therfore, based on the Equations (5) and (25), the optimal retail price is obtained,

$$p_1^{o*} = \frac{v}{2a} + \frac{\lambda(w+k_1^n-\alpha p_s)}{2(1-\alpha e^{uT})} = \frac{3v}{4a} + \frac{\lambda(c-\alpha p_s)}{4(1-\alpha e^{uT})} \tag{26}$$

when we know the best logistics price and retail price, the profits of the supply chain members are as follows:

$$\Pi_{s1}^{o} = \frac{aT}{8}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})}\right]^2 + F \tag{27}$$

$$\Pi_{r1}^{o} = \frac{aT}{16}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})}\right]^2 - F \tag{28}$$

***Case* 2:** If the retailer's carbon emissions cap is less than the critical threshold,

$$C_r \leq \phi(\lambda) \tag{29}$$

then, $k_1^{\#} \leq \underline{k_1^n}$. Thus, $k_1^{\#}$ is beyond the feasible region. As a result, the lower limit represents the best logistical price,

$$k_1^{o*} = \underline{k_1^n} = \frac{v(1-\alpha e^{uT})}{a\lambda} - \frac{2C_r((1-\alpha e^{uT})^2)}{a\lambda^2\chi T} - w + \alpha p_s \tag{30}$$

Therefore, substituting the Equation (30) into Equation (5), the optimal retail price is obtained,

$$p_1^{o*} = \frac{v}{2a} + \frac{\lambda(w+k_1^n-\alpha p_s)}{2(1-\alpha e^{uT})} = \frac{v}{a} - \frac{C_r(1-\alpha e^{uT})}{a\lambda T\chi} \tag{31}$$

Since the retail price and ideal logistics price are determined, the supply chain participants' earnings may be calculated as follows:

$$\Pi_{s1}^{o} = \frac{C_r}{\lambda\chi}\left[\frac{v}{a}(1-\alpha e^{uT}) - \lambda(c-\alpha p_s) - \frac{2C_r(1-\alpha e^{uT})^2}{a\lambda T\chi}\right] + F \tag{32}$$

$$\Pi_{r1}^{o} = \frac{C_r^2(1-\alpha e^{uT})^2}{a\lambda^2 T\chi^2} - F \tag{33}$$

6.2.2 Solution in Option 2

Since, it is concluded from Equation (14) that Π_{s2}^n is a monotonic increasing function of k_2^n, thus, its upper bound is the best logistical price,

$$k_2^{o*} = \overline{k_2^n} = \frac{v(1-\alpha e^{uT})}{2a\lambda} + \frac{(c+\alpha p_s)}{2} - \frac{C_r(1-\alpha e^{uT})^2}{a\lambda^2 T\chi} - w \tag{34}$$

As a result, the best inner trade is determined using Equations (12) and (34),

$$r_2^{o*} = \frac{v(1-\alpha e^{uT})}{2a\lambda\chi} + \frac{(c+\alpha p_s)}{2\chi} - \frac{w+k_2^n}{\chi} - \frac{C_r(1-\alpha e^{uT})^2}{a\lambda^2 T\chi^2} = 0 \tag{35}$$

As indicated by condition (13), p_2^{n*} isn't related with k_2^n. Subsequently, the ideal retail cost in situation 2 should continue as before as that in situation 1. Now, it actually needs further examination because of the requirements on p_2^n in Suggestion 1.

The basic edge of the retailer's carbon cap in choice 2 are $\phi_2^l(\lambda)$ is the lower basic limit and $\phi_2^h(\lambda)$ is the higher basic edge respectively.

Through examining the relationship bewteen p_2^{n*} what's more, the limitations on p_2^n, we determine the edges that,

$$\phi_2^h = \frac{a\lambda T\chi}{2(1-\alpha e^{uT})^2}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s)\right],$$

$$\phi_2^l = \frac{a\lambda T\chi}{2(1-\alpha e^{uT})^2}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s)\right] - 2C_s.$$

There are three possible scenarios in terms of the interactions between C_r, ϕ_2^h and ϕ_2^l.

***Case* 1:** If the retailer's carbon emissions quota falls between the minimum and maximum values,

$$\phi_2^l < C_r < \phi_2^h \tag{36}$$

then, $\underline{p_2^n} < p_2^{n*} < \overline{p_2^n}$. Therefore, the statement that the ideal selling price is achievable means,

$$p_2^{o*} = \frac{3v}{4a} + \frac{\lambda(c-\alpha p_s)}{4(1-\alpha e^{uT})} - \frac{C_r(1-\alpha e^{uT})}{2a\lambda T\chi} \tag{37}$$

Next, the supply chain's members' earnings are as follows,

$$\Pi_{s2}^o = \frac{aT}{8}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} + \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right]\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} - \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right] + F \tag{38}$$

$$\Pi_{r2}^o = \frac{aT}{16}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} + \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right]^2 - F \tag{39}$$

***Case* 2:** If the retailer emits more carbon than permitted,

$$C_r \geq \phi_2^h \tag{40}$$

then, $p_2^{n*} \geq \overline{p_2^n}$ means that p_2^{n*} is outside the realm of possibility. The best retail price is thus its higher limit,

$$p_2^{o*} = \overline{p_2^n} = \frac{v}{a} - \frac{C_r(1-\alpha e^{uT})}{a\lambda T\chi} \tag{41}$$

The profit of the supply chain participants is then:

$$\Pi_{s2}^o = \frac{C_r}{2\chi\lambda}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s) - \frac{2C_r(1-\alpha e^{uT})^2}{a\lambda T\chi}\right] + F \tag{42}$$

$$\Pi_{r2}^o = \frac{C_r}{2\chi\lambda}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s)\right] - F \tag{43}$$

***Case* 3:** If the retailer's carbon emissions quota drops below the lower critical threshold,

$$C_r \leq \phi_2^l \tag{44}$$

then, $p_2^{n*} \leq \underline{p_2^n}$ indicating that p_2^{n*} is outside the realm of possibility. Thus, its lower bound is the ideal selling price,

$$p_2^{o*} = \underline{p_2^n} = \frac{v}{a} - \frac{(C_r+C_s)(1-\alpha e^{uT})}{a\lambda T\chi} \tag{45}$$

The profit of the supply chain participants is then:

$$\Pi_{s2}^o = \frac{(C_s+C_r)}{2\chi\lambda}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s) - \frac{2C_r(1-\alpha e^{uT})}{a\lambda T\chi}\right] + F \tag{46}$$

$$\Pi_{r2}^o = \frac{(C_s+C_r)}{2\chi\lambda}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s) - \frac{2C_s(1-\alpha e^{uT})}{a\lambda T\chi}\right] - F \tag{47}$$

6.2.3 Solution in Option 3

In order to increase the supplier's profit in Option 3, we differentiate the Equation (21) w.r.t k_3^n and put the derivative's value at 0.

As a result, the following is the unconstrained optimisation issue $k_3^{\#}$ logistics price,

$$k_3^{\#} = \frac{v(1-\alpha e^{uT})}{2a\lambda} - \frac{\chi s}{2} + \frac{(c+\alpha p_s)}{2} - w \tag{48}$$

According to Equation (20), r_3^{n*} is not correlated with $k_3^{\#}$. Therefore, it is derived that,

$$r_3^{o*} = r_3^{n*} = s \tag{49}$$

Now, next we have to determine whether $k_3^{\#}$ is within the range given in proposition 2.

Therefore, based on the relationship bewteen $k_3^{\#}$ and $\overline{k_3^n}$, the critical threshold if the retailer carbon cap in option 3 is,

$$\phi_3 = \frac{a\lambda T\chi}{4}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda\chi s - \lambda(c-\alpha p_s)\right] - C_s.$$

There are two possible scenarios in terms of the interaction between C_r and ϕ_3.

Case* 1*: If a retailer's carbon emissions allowance is below the critical threshold,

$$C_r < \phi_3 \tag{50}$$

then, $k_3^{\#} < \overline{k_3^n}$, indicates that $k_3^{o*} = k_3^{\#}$. Thus, the optimal retail price is obtained,

$$p_3^{o*} = \frac{v}{2a} + \frac{\lambda(w+k_3^n+\chi s-\alpha p_s)}{2(1-\alpha e^{uT})} = \frac{3v}{4a} + \frac{\lambda(c-\alpha p_s+\chi s)}{4(1-\alpha e^{uT})} \tag{51}$$

Then, the profits of the supply chain members are:

$$\Pi_{s3}^{o} = \frac{aT}{8}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} - \frac{\lambda\chi s}{(1-\alpha e^{uT})}\right] + C_s s + F \tag{52}$$

$$\Pi_{r3}^{o} = \frac{aT}{16}\left[\frac{v}{a} - \frac{\lambda(c-\alpha p_s)}{(1-\alpha e^{uT})} - \frac{\lambda\chi s}{(1-\alpha e^{uT})}\right]^2 + C_r s - F \tag{53}$$

***Case* 2:** If the retailer's carbon footprint goes beyond the critical threshold,

$$C_r \geq \phi_3 \tag{54}$$

then, $k_3^{\#} \geq \overline{k_3^n}$ indicated that $k_3^{\#}$ is outside the realm of possibility. The best logistics price is thus its upper bound,

$$k_3^{o*} = \overline{k_3^n} = \frac{v(1-\alpha p_s)}{a\lambda} - \frac{2(C_r+C_s)(1-\alpha e^{uT})^2}{a\lambda^2 T\chi} - w - \chi s - \alpha p_s \tag{55}$$

The optimal retail price is given by,

$$p_3^{o*} = \frac{v}{a} - \frac{(C_r+C_s)(1-\alpha e^{uT})}{a\lambda T\chi} \tag{56}$$

By knowing the values of $k_3^{o*}, k_3^{o*}, p_3^{o*}$, the profits of the supply chain members are,

$$\Pi_{s3}^{o} = \frac{(C_r+C_s)}{\lambda\chi}\left[\frac{v(1-\alpha e^{uT})}{a} - \lambda(c-\alpha p_s) - \lambda\chi s - \frac{2(C_r+C_s)(1-\alpha e^{uT})^2}{a\lambda T\chi}\right] + C_s s + F \tag{57}$$

$$\Pi_{r3}^{o} = \frac{(C_r+C_s)^2(1-\alpha e^{uT})^2}{a\lambda^2 T\chi^2} + C_r s - F \tag{58}$$

7. Numerical Example

The model has been validated with the following data: $v = 10$ (units/year), $a = 1$, $c = 2$ (\$/units), $T = 100$ (year), $F = 10$ (\$/units), $\chi = 0.2$, $s = 2$ (\$/units), $C_s = 30$, $p_s = 2.5$ (\$/units), deterioration coefficient $u = 0.02$ (per year) and percentage defective variable $\alpha = 0.02$.

***Case* 1:** In option 1, the critical level is surpassed when the retailer carbon cap $C_r = 50 > \phi_1$. The optimal value of the logistics price is k_1^{o*} =2.9313 (\$/unit) and the optimized price of the retail price is $p_1^{o*} = 8.0532$ (\$/unit) . By substituting the optimal values of the logistics price and retail price, we obtain the quantity of the order $Q_1^{o*} = 220$ (units). Consequently, the earnings for the supplier and the retailer are $\Pi_{s1}^{o} = 768.0198$ (\$/year) and $\Pi_{r1}^{o} = 369.0099$ (\$/year).

***Case* 2:** In option 1, the critical level is surpassed when the retailer carbon cap C_r=35 $\leq \phi_1$.. The optimal value of the logistics price is k_1^{o*} =3.6445 (\$/unit) and the optimized price of the retail price is p_1^{o*}=8.4578 (\$/unit). By substituting the optimal values of the logistics price and retail price, we obtain the quantity of the order $Q_1^{o*} = 175$ (units). Consequently, the earnings for the supplier and the retailer are $\Pi_{s1}^{o} = 735.2788$ (\$/year) and $\Pi_{r1}^{o} = 227.8421$ (\$/year).

***Case* 3:** In option 2, when the retailer carbon cap falls within the range $\phi_2^l < C_r = 50 < \phi_2^h$, the optimal value of the logistics price is k_2^{o*} =0.9898 (\$/unit) and the optimized price of the retail price is $p_2^{o*} = 6.9516$ (\$/unit). By substituting the optimal values of the logistics price and retail price, we obtain the quantity of the order $Q_2^{o*} = 345.9118$ (units). Consequently, the earnings for the supplier and the retailer are $\Pi_{s2}^{o} = 525.3238$ (\$/year) and $\Pi_{r2}^{o} = 919.2732$ (\$/year).

***Case* 4:** In option 2, the greater critical threshold is surpassed when the retailer carbon cap exceeds it $C_r = 70 \geq \phi_2^h$. The optimal value of the logistics price is k_2^{o*}=0.2131 (\$/unit) and the optimized price of the retail price is $p_2^{o*} = 6.9156$ (\$/unit). By substituting the optimal values of the logistics price and retail price, we obtain the quantity of the order $Q_2^{o*} = 350$ (units).

***Case* 5:** In option 2, the critical threshold is surpassed when the retailer carbon cap $C_r = 20 \leq \phi_2^l$. The optimal value of the logistics price is $k_2^{o*} = 2.1547$ (\$/unit), and the optimized price of the retail price is $p_2^{o*} = 7.7968$ (\$/unit). By substituting the optimal values of the logistics price and retail price, we obtain the quantity of the order $Q_2^{o*} = 250$ (units). Consequently, the supplier's and the retailer's profits amount to $\Pi_{s2}^{o} = 673.6738$ (\$/year) and $\Pi_{r2}^{o} = 556.5955$ (\$/year).

***Case* 6:** In option 3, the critical threshold is surpassed when the retailer carbon cap $C_r = 10 < \phi_3$. The optimal value of the logistics price is $k_3^{o*} = 2.7313$ (\$/unit), and the optimized price of the retail price is $p_3^{o*} = 8.1667$ (\$/unit). By substituting the optimal values of the logistics price and retail price, we obtain the quantity of the order $Q_3^{o*} = 208$ (units). Consequently, the supplier's earnings are $\Pi_{s3}^{o} = 742.2303$ (\$/year) and the retailer's earnings are $\Pi_{r3}^{o} = 346.1151$ (\$/year).

***Case* 7:** In option 3, the critical threshold is surpassed when the retailer carbon cap, $C_r = 20 < \phi_3$ The optimal value of the logistics price is k_3^{o*}=2.0795 (\$/unit), and the optimized price of the retail price is $p_3^{o*} = 7.7968$ (\$/unit). By substituting the optimal values of the logistics price and retail price, we obtain the order quantity $Q_3^{o*} = 250$ (units). Consequently, the supplier's earnings are $\Pi_{s3}^{o} = 714.8773$ (\$/year) and the retailer's earnings are $\Pi_{r3}^{o} = 515.3919$ (\$/year).

8. Sensitivity Analysis

Sensitivity analysis for case 1-7 is performed to study the impact of the percentage imperfect quality items on the food quantity (Q), the retail price (p), logistics price (k), trade price (r) and the profits of the supplier (Π_s) and the retailer (Π_r). Results are summarised in Tables 3-9, respectively.

Table 3. For case-1: $C_r > \phi_1$.

α	Q	p	k	r	Π_s	Π_r
0.05	229	8.0527	2.8033	0	768.4235	369.2118
0.04	226	8.0528	2.8460	0	768.2855	369.1427
0.03	223	8.0530	2.8887	0	768.1509	369.0755
0.02	220	8.0532	2.9313	0	768.0198	369.0099
0.01	218	8.0533	2.9740	0	767.8919	368.9459

Table 4. For case-2: $C_r \le \phi_1$.

α	Q	p	k	r	Π_s	Π_r
0.05	175	8.5157	3.5888	0	725.5408	210.3111
0.04	175	8.4964	3.6082	0	728.9359	216.0802
0.03	175	8.4771	3.6268	0	732.1819	221.9238
0.02	175	8.4578	3.6445	0	735.2788	227.8421
0.01	175	8.4385	3.6613	0	738.2265	233.8348

Table 5. For case-3: $\phi_2^l < C_r < \phi_2^h$.

α	Q	p	k	r	Π_s	Π_r
0.05	345	6.8990	1.0366	0	540.4712	951.6452
0.04	342	6.8867	1.0292	0	533.8698	959.2839
0.03	339	6.8744	1.0212	0	527.2148	966.9649
0.02	337	6.8620	1.0127	0	520.5059	974.6879
0.01	335	6.8497	1.0036	0	513.7424	982.4526

Table 6. For case-4: $C_r \ge \phi_2^h$.

α	Q	p	k	r	Π_s	Π_r
0.05	350	7.0314	0.2855	0	284.9187	1146.287
0.04	350	6.9928	0.2622	0	276.7756	1161.156
0.03	350	6.9542	0.2381	0	268.3342	1176.035
0.02	350	6.9156	0.2131	0	259.5947	1191.023
0.01	350	6.8770	0.1873	0	250.5568	1205.921

Table 7. For case-5: $C_r \leq \phi_2^l$.

α	Q	p	k	r	Π_s	Π_r
0.05	250	7.8796	2.0839	0	655.9849	546.0620
0.04	250	7.8520	2.1078	0	661.9421	549.6645
0.03	250	7.8244	2.1314	0	667.8384	553.1756
0.02	250	7.7968	2.1547	0	673.6738	556.5955
0.01	250	7.7693	2.1778	0	679.4484	559.9240

Table 8. For case-6: $C_r < \phi_3$.

α	Q	p	k	r	Π_s	Π_r
0.05	215	8.1706	2.6033	2	739.3660	344.6830
0.04	213	8.1692	2.6460	2	740.3446	345.1723
0.03	210	8.1679	2.6887	2	741.2991	345.6495
0.02	208	8.1667	2.7313	2	742.2303	346.1151
0.01	205	8.1654	2.7740	2	743.1391	346.5695

Table 9. For case-7: $C_r < \phi_3$.

α	Q	p	k	r	Π_s	Π_r
0.05	250	7.8796	2.1097	2	722.4325	479.6144
0.04	250	7.8520	2.1009	2	720.2184	491.3881
0.03	250	7.8244	2.0908	2	717.7001	503.3139
0.02	250	7.7968	2.0795	2	717.7001	515.3919
0.01	250	7.7693	2.0670	2	711.750	527.6221

8.1 Observations from the Tables

Table 3 indicates that there might not be any impact at the inner trade rate (r) as the proportion of imperfect quality items decreases, the most appropriate order quantity (q) decreases, the retail fee (p) and the logistics fee (k) increase slightly. However, both the retailer's and the supplier's profit drops significantly. It suggests that the revenue is directly impacted by the percentage of items with imperfect quality.

From Table 4, it is evident that the retail price (p) decreases slightly, the logistics price (k) increases, the proportion of imperfect quality objects decreases, the choicest order amount (q) stays constant, the inner exchange charge (r) is unaffected, but the supplier's and retailer's profits both rise significantly.

Table 5 shows that while there is no change at the inner alternate fee (r), the retailer's income will increase and the supplier's profit will decrease as the share of items with imperfect quality drops, the finest order quantity (q) drops, the retail fee (p) and the logistics rate (k) increases slightly.

Table 6 demonstrates, how the percentage of items with imperfect quality drops, the leading order quantity (q) remains unchanged, the retail charge (p) drops slightly, the logistics cost (k) rises, the internal exchange rate (r) remains unchanged, but the supplier's earnings drop and the retailer's profit rises significantly.

Table 7 shows that while there may be no change at the inner trade rate (r), the profit for both the supplier and the retailer will increase significantly as the proportion of items with imperfect quality decreases, the greatest order amount (q) remains constant, the retail rate (p) decreases slightly, and the logistics charge (k) increases.

Table 8 indicates that the most efficient order quantity (q), the retail rate (p), the logistics price increases, the inner alternate price is equal to the market rate, and the proportion of imperfect quality items decreases. The provider's and retailer's profits, on the other hand, each increase significantly.

Table 9 signifies that while the first order quantity (q), the retail fees (p), the logistics fee, and the share of imperfect quality items will all decline slightly as the share of imperfect quality items declines, the retailer's income will rise significantly and the supplier's profit will decline.

As a result, we deduced the following from the tables:

(i) The supplier is unwilling to trade carbon if the retailer possesses an adequate number of carbon permits. However, the supplier prefers to trade carbon both inside and outside the company if the retailer is very close to reaching its carbon limit.

(ii) Regardless of the quantity of carbon permits it possesses, the retailer is eager to trade carbon. The store benefits most from internal and external carbon trading when its emissions are medium to high.

(iii) The retailer and supplier's carbon trading transactions demonstrate their competing interests. Should the retailer be significantly impacted by the supplier's dominant position, they will be compelled to select a less favourable carbon trading option.

9. Conclusion

Under the carbon cap-and-trade policy, this study examines a carbon trading instrument that considers the contributions of refrigerated planned operations to the breakdown of flawed quality items in a new food store network. The store network offers three carbon trading options (a) internal carbon trade, (b) inward and external carbon trade, and (c) carbon trading without carbon change. Each option considers the cost of emission permits traded within the network of stores and the retail charge, which is linked to the speed of coordinated refrigerated operations services. The implications of poor quality, the crumbling rate, and the retailer's carbon cap have been investigated and understood.

Key findings on the carbon trading system:

(i) Internal carbon trade strengthens the beneficial relationship between the supplier and the retailer when only internal carbon change occurs. However, when inward and external carbon substitutes occur simultaneously, internal carbon trade has negative effects on the retailer's relationship with the vendor.

(ii) Carbon trading and coordinated refrigeration operations often strengthen the link between the supplier and the retailer. However, the supplier's profitable refrigeration techniques promote carbon trading along the supply chain. Conversely, carbon trading encourages the retailer to choose the supplier's contributions for chilled coordinated reasons, regardless of the quality of the service.

(iii) Increasing the use of discharge permits and cooled workplaces highlights the benefits of the carbon trading system, adding intensity to supply chains.

Conflict of Interest

With regard to the research that is presented in this paper, the authors declare that they have no conflicts of interest. The research was carried out with complete transparency and integrity, devoid of any outside influences or biases that might have jeopardised the validity and objectivity of the results. Funding or other support from organisations or entities that might have a stake in the research's conclusions was not given to the authors.

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