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Failure Probability Analysis of Coherent Systems using Intuitionistic Fuzzy Fault Tree Approach

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Abstract
In scenarios where system component failure probabilities are insufficient or imprecise, classical fault tree analysis methods often fail in providing precise assessments of system reliability and safety. This limitation arises because classical fault tree analysis relies on precise data and probabilistic measures, which may not adequately capture inherent uncertainties and vagueness in real-world environments. Therefore, a general approach is required to reliably predict and diagnose system failures that incorporates the uncertainties inherent in failure data. The fuzzy sets effectively manage imprecise information and serve as a robust foundation to address ambiguity in real-world failure scenarios. Additionally, the use of intuitionistic fuzzy sets further enhances this framework by considering both membership and non-membership degrees, thereby providing a more comprehensive and precise reliability assessment. Therefore, in the current study, the intuitionistic fuzzy set theory is proposed to handle the inherent uncertainty in reliability assessments. Furthermore, triangular intuitionistic fuzzy numbers are used to represent the failure probabilities of the system components. The proposed method integrates intuitionistic fuzzy set theory with fault tree analysis to determine the reliability and failure probability of a rope on a crane, and the computed results are compared with the results of existing methods.

Keywords- Fault tree, Coherent system, Intuitionistic fuzzy sets, Fuzzy failure probability, Fuzzy reliability.

1. Introduction

Coherent systems are a fundamental concept in reliability engineering and are characterized by their structured arrangement of components that work together to perform a specific function. These systems are coherent in the sense that their overall performance improves as individual components perform better, and failure only occurs when certain combinations of component failures disrupt the system's operation. Coherent systems are typically made up of interconnected components, where the reliability of the entire system depends on the configuration and reliability of each individual component. Examples of coherent systems include series systems, in which all components must function for the system to succeed, and parallel systems, in which the system functions as long as at least one component is active. The primary feature of coherent systems is the monotonicity of their structure functions, which means that if any component's performance improves or remains the same, the overall system performance does not degrade. This characteristic allows for the analysis of system reliability through the relationships between

components, which can be mathematically modeled using fault tree and reliability block diagrams methods. Coherent systems are widely used in industries, such as aerospace, power systems, and telecommunications in which safety and reliability are critical. The complexity of these systems often requires sophisticated methods to assess failure probabilities, especially in the presence of uncertainty. Consequently, the establishment of such systems is fundamentally driven by system reliability engineering, which focuses on ensuring that systems perform their intended functions over their expected lifespans. Reliability is typically quantified as the probability of a system successfully completing a designated task under specified operating conditions for a defined period (National Research Council, 2012). This process involves comprehensive assessments, often conducted through routine inspections or by analyzing operational data collected during the system's use (Mo and Beckett, 2018). By integrating these advanced reliability assessment techniques with the complexity of coherent systems, industries can achieve more robust safety and performance outcomes.

However, traditional reliability analysis methodologies encounter significant limitations when confronted with imprecision or uncertainty (Zio, 2016). In real-world scenarios, the data available for reliability assessments are often incomplete, ambiguous, or subject to various uncertainties. This situation can arise from several factors, such as variability in material properties, environmental influences, and inconsistencies in operational practices (Sankararaman and Mahadevan, 2015). In these cases, the precise parameters required for traditional reliability models are either unavailable or unreliable. Probability theory, which supports classical reliability analysis, assumes a level of precision and completeness in data that is often not reflective of real-life conditions (Hennig et al., 2015). Therefore, probability theory requires clearly defined probabilities for different failure modes and system behaviors, which may not be ascertainable due to inherent uncertainties and the lack of comprehensive data. As a result, the probabilistic assessments produced may not fully capture true risk levels, leading to either overly optimistic or conservative reliability estimates (Domingues et al., 2017). Moreover, classical set theory, which forms the basis for many reliability assessment frameworks, is inadequate for handling situations characterized by significant imprecision. In classical set theory, an element either belongs to a set or is not, thus allowing for no partial membership or uncertainty. This binary approach fails to account for the nuanced and often vague nature of real-life data, where system parameters may not be precisely known and a significant degree of hesitation or doubt may exist.

To address these issues, Zadeh (1965) introduced fuzzy set theory, refining the concept of classical set theory by allowing an assertive degree of membership in a set. Fuzzy set theory is an effective tool for handling situations involving imprecise data. Fuzzy set theory has been widely utilized to address issues where precise data are unavailable, thereby aiding decision-making from vague information (Purba et al., 2015; Yazdi and Zarei, 2018). Yazdi et al. (2017) integrated fuzzy set theory with fault tree (FT) by representing failure probabilities as trapezoidal fuzzy numbers and using the fuzzy extension principle to estimate the failure probability of a chemical storage tank. Many researchers (Abdelgawad and Fayek, 2011; Purba, 2014; Rajakarunakaran et al., 2015) explored various methodologies and applications of fuzzy logic to assess system reliability and risk, highlighting the importance of incorporating uncertainty into fault tree models. Combining fuzzy set theories with expert elicitation has also been employed by Yin et al. (2020) to assess the reliability of natural gas storage tank systems. Recently, Kumar et al. (2024) investigated the fuzzy reliability and availability of wind turbine systems by considering failure modes as triangular fuzzy numbers. Despite these advancements, there are instances in which we may feel that an object belongs to a specific set, yet uncertainty persists regarding the precise level of its membership. This hesitancy can complicate reliability assessment and necessitates more advanced methods to accurately capture and represent uncertainty.

Furthermore, fuzzy set theory, introduced by Zadeh (1965), allows for degrees of membership but does not address the degree of non-membership. To overcome this situation, Atanassov (1986) introduced intuitionistic fuzzy set (IFS) theory, which includes two components: membership and non-membership degrees. For example, consider classifying a student's performance as "excellent." In traditional fuzzy sets, a membership degree is assigned based on grades or achievement. However, this does not reflect uncertainty in the evaluation process. An IFS approach would assign a membership degree (e.g., 0.7), a non-membership degree (e.g., 0.1, indicating aspects that are not excellent), and a hesitation degree (e.g., 0.2, capturing the uncertainty in this assessment). This approach is particularly useful when evaluations are often subjective and not entirely based on clear criteria. The inclusion of hesitation helped better represent the evaluators' confidence levels, thus providing a more comprehensive and accurate reflection of students' performance. In many studies, intuitionistic fuzzy numbers have been used to represent the failure probabilities and failure rates of components in complex systems to determine reliability measures. Shu et al. (2006) defined arithmetic operations for triangular intuitionistic fuzzy numbers and integrated FT approach to determine the reliability of printed circuit board assemblies. Later, Kumar (2014) modeled the failure probabilities of components in printed circuit board assemblies using various intuitionistic fuzzy numbers and applied the weakest *t*-norm-based approximate arithmetic operations to compute the top event's failure probability. Chachra et al. (2023) presented a new method by integrating IFS with the UGF process to estimate the fuzzy reliability and availability of a multistate wireless communication system. Many researchers evaluated the reliability of systems using vague or intuitionistic fuzzy sets, rather than traditional fuzzy sets, integrating FT (Kumar, 2014; Wang et al., 2013). Moreover, numerous studies (Niroomand, 2018; Parvathi and Malathi, 2012; Sahoo et al., 2022) supported the development and application of the IFS theory. Recently, researchers (Chachra et al., 2024; Kumar and Singh, 2022; Khajuria, 2023; Kumar et al., 2023; Kumar and Singh, 2023, 2024) evaluated the reliability of complex systems. In addition to IFS, several other types of fuzzy sets have also been developed. Hesitant fuzzy sets were introduced by Torra and Narukawa (2009), dual hesitant fuzzy sets by Zhu et al. (2012), and Pythagorean fuzzy sets by Yager (2013). These developments have expanded the scope and application of fuzzy set theory in various fields. This study introduces a method for determining the failure probability of a coherent system, using triangular intuitionistic fuzzy numbers to characterize the failure probabilities of its basic components. To demonstrate the effectiveness of the method, it was applied to evaluate the fuzzy reliability, failure probability, and critical component of a rope on a crane. The proposed methodology was also validated and compared with results from previous studies.

The article is structured as follows: Section 2 covers the basic definitions of IFSs and their properties. The basic events of the fault tree and the results of the failure probability analysis for the considered system are discussed in Section 3. The obtained results are discussed and compared with existing approaches in Section 4. Section 5 offers a comprehensive summary of the critical component analysis of the system. Finally, Section 6 summarizes the paper and highlights potential directions for further research.

2. Preliminaries

This section presents a review of the key definitions and concepts that are integral to the current study.

2.1 Intuitionistic Fuzzy Set

Let Ω be a domain of discourse. An intuitionistic fuzzy set $\widetilde{\mathbb{A}}$ in Ω is defined as $\widetilde{\mathbb{A}} = \{z, \mu_{\widetilde{\mathbb{A}}}(z), \upsilon_{\widetilde{\mathbb{A}}}(z) | z \in \Omega\}$, where, $\mu_{\widetilde{\mathbb{A}}}(z)$ and $\upsilon_{\widetilde{\mathbb{A}}}(z)$ are functions from Ω to the interval [0, 1], satisfying $0 \leq \mu_{\widetilde{\mathbb{A}}}(z) + \upsilon_{\widetilde{\mathbb{A}}}(z) \leq 1$ for all $z \in \Omega$. Here, $\mu_{\widetilde{\mathbb{A}}}(z)$ represents the degree of membership of the element z, while $\upsilon_{\widetilde{\mathbb{A}}}(z)$ denotes the degree of non-membership. The hesitancy degree $\tau(z)$ is given by $\tau(z) = 1 - \mu_{\widetilde{\mathbb{A}}}(z) - \upsilon_{\widetilde{\mathbb{A}}}(z)$.

2.2 Convex Intuitionistic Fuzzy Set (Garg, 2016)

An IFS $\widetilde{\mathbb{A}}$ defined on Ω is convex if and only if

(a) The membership function $\mu_{\widetilde{\mathbb{A}}}(z)$ of $\widetilde{\mathbb{A}}$ is fuzzy convex, i.e.,

$$\mu_{\widetilde{\mathbb{A}}}(kz_1 + (1-k)z_2) \geq \min(\mu_{\widetilde{\mathbb{A}}}(z_1), \mu_{\widetilde{\mathbb{A}}}(z_2)) \quad \text{for all } z_1, z_1 \in \Omega,\ k \in [0,1].$$

(b) The non-membership function $\upsilon_{\widetilde{\mathbb{A}}}(z)$ of $\widetilde{\mathbb{A}}$ is fuzzy concave, i.e.,

$$\upsilon_{\widetilde{\mathbb{A}}}(kz_1 + (1-k)z_2) \leq \max(\upsilon_{\widetilde{\mathbb{A}}}(z_1), \upsilon_{\widetilde{\mathbb{A}}}(z_2)) \quad \text{for all } z_1, z_1 \in \Omega,\ k \in [0,1].$$

2.3 Intuitionistic Fuzzy Number (Garg, 2016)

An intuitionistic fuzzy set $\widetilde{\mathbb{A}} = \{z, \mu_{\widetilde{\mathbb{A}}}(z), \upsilon_{\widetilde{\mathbb{A}}}(z) | z \in \Omega\}$ is known as an intuitionistic fuzzy number (IFN) (see **Figure 1**) if it satisfies the following conditions:

(i) There exists an $z \in \mathbb{R}$ such that $\mu_{\widetilde{\mathbb{A}}}(z) = 1$ and $\upsilon_{\widetilde{\mathbb{A}}}(z) = 0$, where z represents the mean value of $\mu_{\widetilde{\mathbb{A}}}(z)$.

(ii) The functions $\mu_{\widetilde{\mathbb{A}}}(z)$ and $\upsilon_{\widetilde{\mathbb{A}}}(z)$ are piecewise continuous and map from $\mathbb{R}$ to $[0,1]$.

(iii) The support of $\widetilde{\mathbb{A}}$, denoted as $S(\widetilde{\mathbb{A}}) = \{z \in \Omega | \ \mu_{\widetilde{\mathbb{A}}}(z) > 0\}$, is bounded.

(iv) $\widetilde{\mathbb{A}}$ is IF-normal, meaning there exist $z_1, z_2 \in \Omega$ such that $\mu_{\widetilde{\mathbb{A}}}(z_1) = 1$ and $\upsilon_{\widetilde{\mathbb{A}}}(z_2) = 1$.

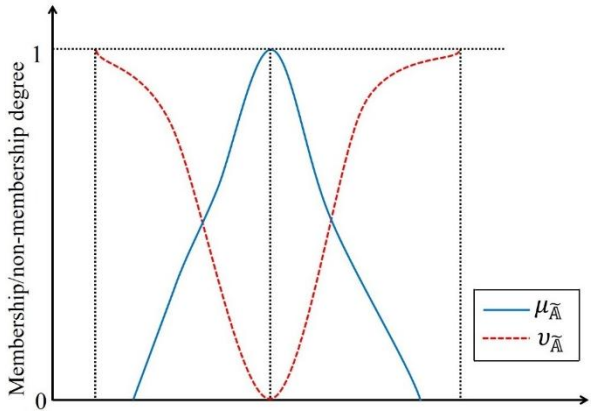

Figure 1. Intuitionistic fuzzy number.

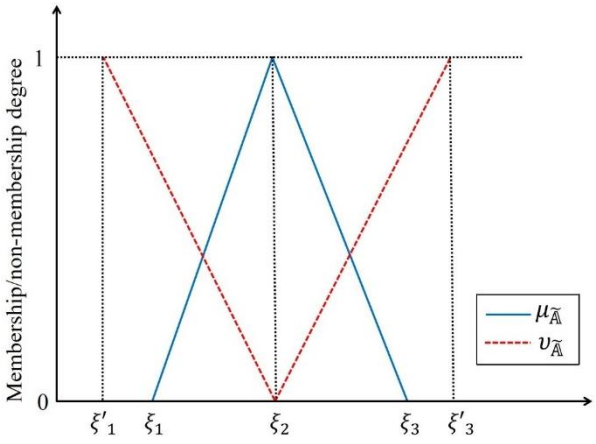

Figure 2. Triangular intuitionistic fuzzy number.

2.4 Trapezoidal Intuitionistic Fuzzy Number (Garg, 2016)

A trapezoidal intuitionistic fuzzy number (TrIFN) is an IFS in $\mathbb{R}$ characterized by a membership function $\mu_{\widetilde{\mathbb{A}}}(z)$ and a non-membership function $\upsilon_{\widetilde{\mathbb{A}}}(z)$ defined as

$$\mu_{\widetilde{\mathbb{A}}}(z) = \begin{cases} \frac{z-\xi_1}{\xi_2-\xi_1}, & \xi_1 \le z \le \xi_2 \\ 1, & \xi_2 \le z \le \xi_3 \\ \frac{\xi_4-z}{\xi_4-\xi_3}, & \xi_3 \le z \le z_4 \end{cases}$$

$$\upsilon_{\widetilde{\mathbb{A}}}(z) = \begin{cases} \frac{z-\xi_1'}{\xi_2'-\xi_1}, & \xi_1' \le z \le \xi_2' \\ 1, & \xi_2' \le z \le \xi_3' \\ \frac{\xi_4'-z}{\xi_4'-\xi_3'}, & \xi_3' \le z \le \xi_4' \end{cases}$$

where, $\xi_1' \le \xi_1 \le \xi_2' \le \xi_2 \le \xi_3 \le \xi_3' \le \xi_4 \le \xi_4'$ and $\xi_1',\ \xi_1,\ \xi_2',\ \xi_2, \xi_3,\ \xi_3',\ \xi_4,\ \xi_4' \in \mathbb{R}$. This TrIFN is denoted by $\widetilde{\mathbb{A}} = (\xi_1, \xi_2, \xi_3,\ \xi_4;\ \xi_1',\ \xi_2',\ \xi_3',\ \xi_4')$. It includes the following specific cases:

(a) If $\xi_3 = \xi_3' = \xi_2 = \xi_2'$ then TrIFN defined above represents a triangular intuitionistic fuzzy number (see **Figure 2**).

(b) If $\xi_1 = \xi_1', \xi_2 = \xi_2', \xi_3 = \xi_3', \xi_4 = \xi_4'$ then TrIFN defined above also represents a trapezoidal fuzzy number $\widetilde{\mathbb{A}} = (\xi_1, \xi_2, \xi_3,\ \xi_4)$.

(c) If $\xi_1 = \xi_1', \xi_2 = \xi_2' = \xi_3 = \xi_3', \xi_4 = \xi_4'$ then TrIFN defined above represents a triangular fuzzy number $\widetilde{\mathbb{A}} = (\xi_1, \xi_2, \xi_3)$.

(d) If $\xi_1 = \xi_1' = \xi_2 = \xi_2' = \xi_3 = \xi_3' = \xi_4 = \xi_4'$ then the TrIFN represents the crisp interval $[\xi_1,\ \xi_4]$.

(e) If $\xi_1 = \xi_1' = \xi_2 = \xi_2' = \xi_3 = \xi_3' = \xi_4 = \xi_4' = \lambda$ then TrIFN represents the real number λ.

2.5 Properties of Triangular Intuitionistic Fuzzy Numbers (Chachra et al., 2023)

Let $\widetilde{\mathbb{A}} = (\xi_1, \xi_2, \xi_3;\ \xi_1',\ \xi_2',\ \xi_3')$ and $\widetilde{\mathbb{B}} = (\zeta_1, \zeta_2, \zeta_3,\ \zeta_1',\ \zeta_2',\ \zeta_3')$ be two triangular intuitionistic fuzzy numbers then their addition, multiplication and scalar multiplication are defined as follows:

(i) Addition

$$\widetilde{\mathbb{A}} \oplus \widetilde{\mathbb{B}} = (\xi_1 + \zeta_1, \xi_2 + \zeta_2, \xi_3 + \zeta_3; \xi_1' + \zeta_1', \xi_2 + \zeta_2, \xi_3' + \zeta_3').$$

(ii) Subtraction

$$\widetilde{\mathbb{A}} \ominus \widetilde{\mathbb{B}} = (\xi_1 - \zeta_1, \xi_2 - \zeta_2, \xi_3 - \zeta_3; \xi_1' - \zeta_1', \xi_2 - \zeta_2, \xi_3' - \zeta_3').$$

(iii) Multiplication

$$\widetilde{\mathbb{A}} \otimes \widetilde{\mathbb{B}} = (\xi_1\zeta_1,\ \xi_2\zeta_2, \xi_3\zeta_3;\ \xi_1'\zeta_1',\ \xi_2\zeta_2,\ \xi_3'\zeta_3').$$

(iv) Scalar multiplication

The scalar multiple $\kappa\widetilde{\mathbb{A}}$ can be defined as:

$$\kappa\widetilde{\mathbb{A}} = (\kappa\xi_1, \kappa\xi_2, \kappa\xi_3;\ \kappa\xi_1', \kappa\xi_2, \kappa_3') \text{ if } \kappa > 0,$$
$$\text{and } \kappa\widetilde{\mathbb{A}} = (\kappa\xi_3, \kappa\xi_2, \kappa\xi_1;\ \kappa\xi_3', \kappa\xi_2, \kappa_1') \text{ if } \kappa < 0.$$

2.6 (α, β)-Cut of Triangular Intuitionistic Fuzzy Number (Garg, 2016)

Let $\widetilde{\mathbb{A}} = (\xi_1, \xi_2, \xi_3;\ \xi_1',\ \xi_2, \xi_3')$ be a triangular intuitionistic fuzzy number then the α-cut set of $\widetilde{\mathbb{A}}$ is defined as

$\widetilde{\mathbb{A}}_\alpha = \{z \in \mathbb{R} | \mu_{\widetilde{\mathbb{A}}}(z) \geq \alpha\} = \left[\widetilde{\mathbb{A}}_\alpha^L, \widetilde{\mathbb{A}}_\alpha^U\right]$, where, $0 \leq \alpha \leq 1$,

and $\widetilde{\mathbb{A}}_\alpha^L = \min\{z \in \mathbb{R} | \mu_{\widetilde{\mathbb{A}}}(z) \geq \alpha\}$ and $\widetilde{\mathbb{A}}_\alpha^U = \max\{z \in \mathbb{R} | \mu_{\widetilde{\mathbb{A}}}(z) \geq \alpha\}$.

Also, the β-cut set of $\widetilde{\mathbb{A}}$ can be expressed as

$\widetilde{\mathbb{A}}_\beta = \{z \in \mathbb{R} | \mu_{\widetilde{\mathbb{A}}}(z) \leq \beta\} = \left[\widetilde{\mathbb{A}}_\beta^L, \widetilde{\mathbb{A}}_\beta^U\right]$, where $0 \leq \beta \leq 1$,

and $\widetilde{\mathbb{A}}_\beta^L = \min\{z \in \mathbb{R} | \mu_{\widetilde{\mathbb{A}}}(z) \leq \beta\}$ and $\widetilde{\mathbb{A}}_\beta^U = \max\{z \in \mathbb{R} | \mu_{\widetilde{\mathbb{A}}}(z) \leq \beta\}$.

2.7 Evaluation of Top Event Probability (Hong and Lee, 2009)

Let p_i, be the failure probability of i^{th}(where , $i = 1,2,3, \ldots, n$) basic events then the failure probability p_{AND} in the "AND" gate and p_{OR} in the "OR" gate of the top event can be defined as

$p_{AND} = \prod_{i=1}^{n} p_i$ and $p_{OR} = 1 - \prod_{i=1}^{n}(1 - p_i)$ (1)

Table 1. System fault tree events and their descriptions (Huang et al., 2004).

Event	Description	Event	Description
Top event	Broken	D_1	Drawback of materials
A	Overloading	D_2	Drawback of machining
B	Hoisting objects aslant	D_3	Insufficient inspection
B_1	Dragging	E	Unsuitable diameter
B_2	Hoisting objects alternately	F	Drawback of use
C	Inadequate strength	F_1	Inadequate overhauling
D	Drawback of manufacturing	F_2	Arriving at limit of failure

Table 2. Failure probabilities of every basic event of the system fault tree (Huang et al., 2004).

Basic events	Failure probability	Triangular fuzzy number	Triangular intuitionistic fuzzy number
A	0.05	(0.045, 0.05, 0.055)	(0.042, 0.045, 0.05, 0.055. 0.058)
B_1	0.03	(0.028, 0.03, 0.032)	(0.026, 0.028, 0.03, 0.032, 0.034)
B_2	0.004	(0.003, 0.004, 0.005)	(0.002, 0.003, 0.004, 0.005, 0.006)
D_1	0.002	(0.001, 0.002, 0.003)	(0.0009,0.001, 0.002, 0.003, 0.004)
D_2	0.001	(0.0009, 0.001, 0.002)	(0.0008, 0.0009, 0.001, 0.002, 0.003)
D_3	0.003	(0.002, 0.003, 0.004)	(0.001, 0.002, 0.003, 0.004, 0.005)
E	0.005	(0.004, 0.005, 0.006)	(0.003, 0.004, 0.005, 0.006, 0.007)
F_1	0.02	(0.015, 0.02, 0.025)	(0.012,0.015, 0.02, 0.025, 0.027)
F_2	0.5	(0.45, 0.5, 0.55)	(0.42, 0.45, 0.5, 0.55, 0.58)

3. Failure Probability Analysis of a Rope on a Crane

The failure probability analysis of a rope on a crane is essential for ensuring operational safety and reliability. This process involves collecting data on the rope's usage, material properties, and past maintenance records. Failure probability analysis of a rope on a crane centre around the top event: rope

breakage, which can result from various underlying causes. One major contributor is material drawbacks (D_1), where issues such as overloading (A), or hoisting objects at an angle (B), can intensify the rope's weaknesses. Further contributing factors include insufficient machining quality (D_2), and inadequate inspections (D_3), both of which elevate the probability of failure. Specific actions, such as dragging the rope (B_1), or alternating hoisting patterns (B_2), can also introduce additional stresses, especially if the rope's diameter is unsuitable (E). Inadequate strength (C), plays a significant role, as does poor manufacturing quality (D), and improper usage (F). The latter category includes insufficient overhauls (F_1), and the rope reaching its failure limit without timely replacement (F_2). Considered system fault tree events and their descriptions are described in **Table 1**. **Table 2** provides the failure probabilities corresponding to basic events of the proposed system.

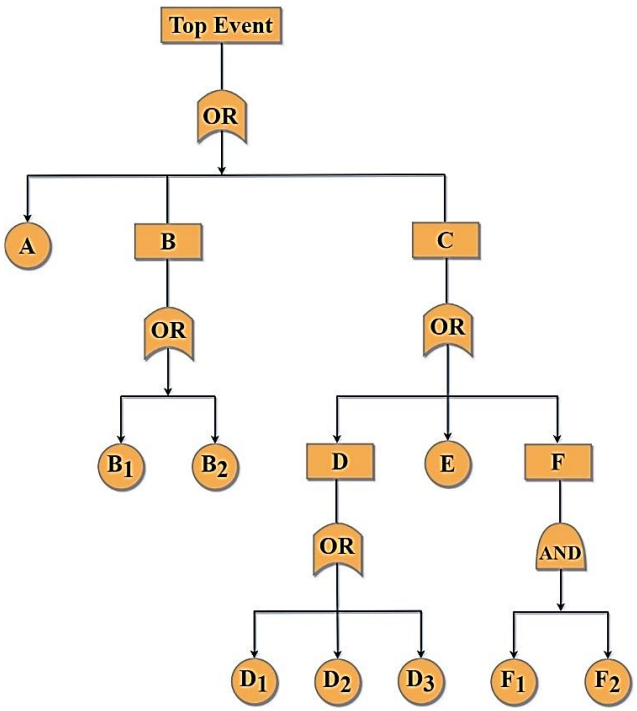

Figure 3. Fault tree of a failure of the rope on a crane.

Given the fault tree and using Equation (1) the failure probability p_{Top} of the top event can be calculated as

$p_{\text{Top}} = 1 - (1 - p_A) \times (1 - p_B) \times (1 - p_C)$.

The failure probability of event B is

$p_B = 1 - (1 - p_{B1}) \times (1 - p_{B2})$.

The failure probability of event C is given by

$p_C = 1 - (1 - p_D) \times (1 - p_E) \times (1 - p_F)$.

The failure probability of event D is

$p_D = 1 - (1 - p_{D1}) \times (1 - p_{D2}) \times (1 - p_{D3})$.

The failure probability of event F is
$p_F = 1 - (1 - p_{F1}) \times (1 - p_{F2})$.

Substituting p_B, p_C, p_D, and p_F in the p_{Top} formula, we get
$p_{\text{Top}} = 1 - (1 - p_A) \times (1 - p_{B1}) \times (1 - p_{B2}) \times (1 - p_{D1}) \times (1 - p_{D2}) \times (1 - p_{D3}) \times (1 - p_A) \times (1 - p_{B1}) \times (1 - p_{B2})$ (2)

4. Findings and Discussion

In this study, the failure probability and reliability of a rope on a crane are determined using classical reliability, fuzzy fault tree (FFT), and the proposed approach. Using the data from **Table 2** (Column 2) and applying Equation (2), the failure probability and reliability of the considered system are obtained to be 0.55520 and 0.44480, respectively. These values are listed in **Tables 3** and **4** and shown in **Figures 4** and **6**. The traditional method does not consider data uncertainty; thus, it is suitable only for situations in which the data are precise and certain. In situations where data involve uncertainties, the FFT approach serves as an effective approach for addressing them. Therefore, by applying the FFT approach, which incorporates the uncertainty associated with system parameters, and using the data from **Table 2** (Column 3) and Equation (2), the failure probability and reliability of the rope on a crane are obtained to be (0.50257, 0.55520, 0.60661) and (0.39339, 0.44480, 0.49743), respectively. The same results are displayed in **Tables 3** and **4** and depicted in **Figures 4** and **6**. The results demonstrate the effectiveness of the FFT approach as it captures inherent uncertainties and provides a more robust reliability assessment. In addition, FFT approach, while effective for deterministic evaluations, struggle to address the imprecise and vague information frequently encountered in complex systems. To address the limitations of both the crisp and FFT approaches in the intuitionistic fuzzy fault tree (IFFT) approach, the failure probabilities of all the basic events are considered as triangular intuitionistic fuzzy numbers. Now, by applying the proposed IFFT approach and using the data as shown in **Table 2** and Equation (2), the fuzzy failure probability of the rope on a crane is (0.46941, 0.50257, 0.55520, 0.60661, 0.63734). Moreover, the fuzzy reliability of the rope on a crane is (0.36266,0.39339,0.44480,0.49743,0.53059). The outcomes and comparisons between fuzzy failure probabilities and reliability using the proposed and existing approaches are listed in **Tables 3** and **4** and shown in **Figures 4**, **5**, **6**, and **7**. Therefore, from **Tables 3** and **4** and **Figures 4**, **5**, and **7**, it can be observed that the classic probability reliability approach cannot handle uncertain information in reliability calculations. In addition, fuzzy information in reliability calculations cannot be managed by the FFT approach. By adopting the proposed approach, the uncertainty in reliability (interval of reliability) was found to be in a more general way (see **Figures 5** and **7**). Therefore, the proposed approach is a generalization of the conventional probability reliability and FFT approaches.

5. Critical Component Analysis

Critical component analysis is a technique used to identify and prioritize the most essential elements in a system, process, or structure by focusing on the components that have the greatest impact on the overall system performance, efficiency, and reliability. This method is widely applied in engineering to assess risks, perform system analysis, and improve decision-making. This involves evaluating components based on criteria such as importance, probability of failure, and potential consequences. Here, the aforesaid method is applied to determine the most influential basic event in the considered system. Let the intuitionistic fuzzy number $f_{T_i} = (p_T^i,\ q_T^i,\ r_T^i,\ s_T^i,\ t_T^i)$ represent the failure probability of the system top event deleting the failure probability of the i^{th} basic event and $f_T = (p_T,\ q_T,\ r_T,\ s_T,\ t_T)$ represents the failure probability of top events of system. Also, D denotes the difference between f_T and f_{T_i}. Then D is defined as follows:
$D(f_T, f_{T_i}) = (p_T - p_T^i) + (q_T - q_T^i) + (r_T - r_T^i) + (s_T - s_T^i) + (t'_T - t_T^i)$ (3)

The larger of D represents the i^{th} bottom event has greater influence on fault tree. The fuzzy failure probability of the rope on a crane is (0.46941, 0.50257, 0.55520, 0.60661, 0.63734). Next, by deleting i^{th} basic event, the failure probability of the top event is determined, as shown in **Table 5** (Column 2). Subsequently, applying Equation (3) the D values for each component are determined and listed in **Table 5**. These values are ranked from largest to smallest to determine the most influential basic failure event for system reliability. The basic event with the highest D is F_2, indicating (Arriving at limit of failure) it is the most critical basic event in the system based on this analysis.

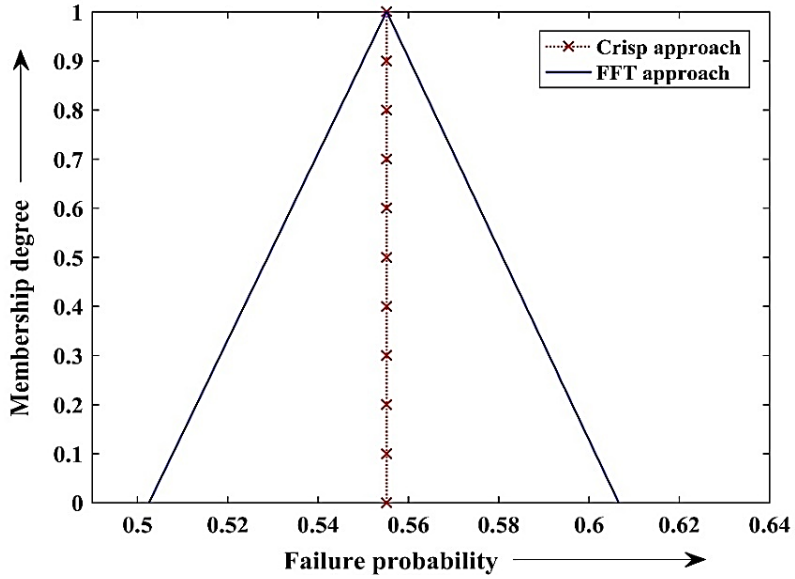

Figure 4. Failure probability of a rope on a crane.

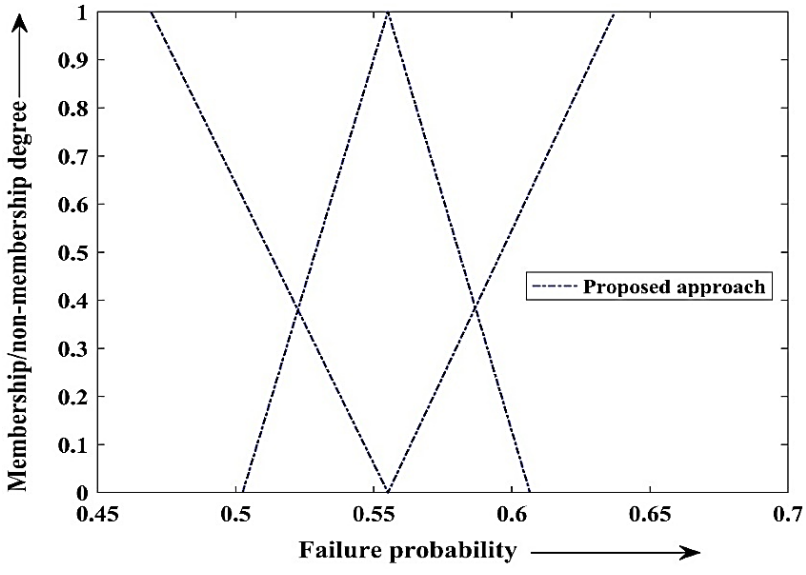

Figure 5. Failure probability of a rope on a crane.

Table 3. Failure probability of a rope on a crane.

α, β	Crisp approach	FFT approach	Proposed approach	
0.0	0.55520	[0.50257, 0.60661]	[0.50257, 0.60661]	[0.46941, 0.63734]
0.1	0.55520	[0.50635, 0.60254]	[0.50635, 0.60254]	[0.47799, 0.62955]
0.2	0.55520	[0.51014, 0.59846]	[0.51014, 0.59846]	[0.48657, 0.62176]
0.3	0.55520	[0.51393, 0.59439]	[0.51393, 0.59439]	[0.49515, 0.61397]
0.4	0.55520	[0.51771, 0.59031]	[0.51771, 0.59031]	[0.50373, 0.60618]
0.5	0.55520	[0.52150, 0.58624]	[0.52150, 0.58624]	[0.51231, 0.59839]
0.6	0.55520	[0.52529, 0.58216]	[0.52529, 0.58216]	[0.52089, 0.59060]
0.7	0.55520	[0.52908, 0.57809]	[0.52908, 0.57809]	[0.52947, 0.58281]
0.8	0.55520	[0.53286, 0.57401]	[0.53286, 0.57401]	[0.53805, 0.57502]
0.9	0.55520	[0.53665, 0.56994]	[0.53665, 0.56994]	[0.54663, 0.56723]
1.0	0.55520	[0.55520, 0.55520]	[0.55520, 0.55520]	[0.55520, 0.55520]

Table 4. Reliability of a rope on a crane.

α, β	Crisp approach	FFT approach	Proposed approach	
0.0	0.44480	[0.39339, 0.49743]	[0.39339, 0.49743]	[0.36266, 0.53059]
0.1	0.44480	[0.39848, 0.49269]	[0.39848, 0.49269]	[0.37188, 0.52197]
0.2	0.44480	[0.40357, 0.48795]	[0.40357, 0.48795]	[0.38109, 0.51334]
0.3	0.44480	[0.40866, 0.48321]	[0.40866, 0.48321]	[0.39030, 0.50471]
0.4	0.44480	[0.41375, 0.47847]	[0.41375, 0.47847]	[0.39952, 0.49609]
0.5	0.44480	[0.41884, 0.47373]	[0.41884, 0.47373]	[0.40873, 0.48746]
0.6	0.44480	[0.42393, 0.46899]	[0.42393, 0.46899]	[0.41794, 0.47884]
0.7	0.44480	[0.42902, 0.46425]	[0.42902, 0.46425]	[0.42716, 0.47021]
0.8	0.44480	[0.43411, 0.45951]	[0.43411, 0.45951]	[0.43637, 0.46159]
0.9	0.44480	[0.43920, 0.45477]	[0.43920, 0.45477]	[0.44558, 0.45296]
1.0	0.44480	[0.44480, 0.44480]	[0.44480, 0.44480]	[0.44480, 0.44480]

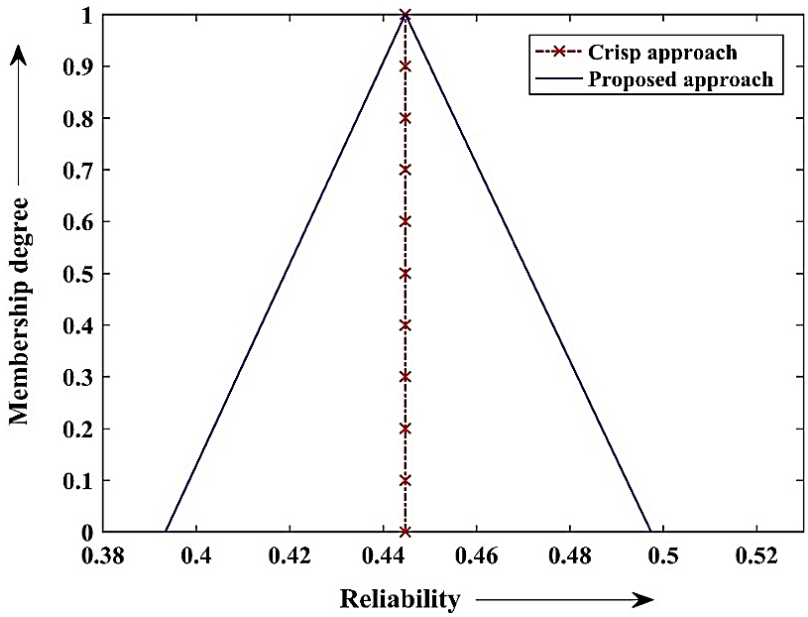

Figure 6. Reliability of a rope on a crane.

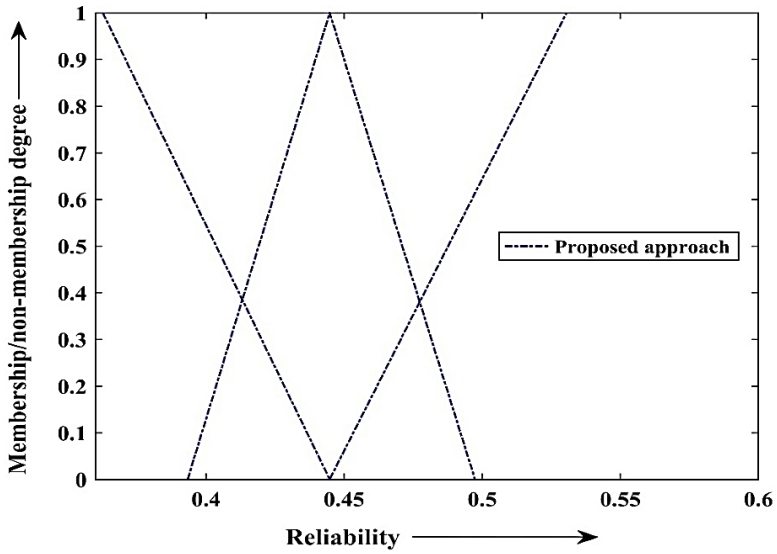

Figure 7. Reliability of a rope on a crane.

Table 5. The failure difference between f_T and f_{T_i}, $D(f_T, f_{T_i})$.

Basic events	Failure probability	$D(f_T, f_{T_i})$	Rank
A	(0.44614,0.47914,0.53179,0.58372,0.61501)	0.21533	8
B_1	(0.45524,0.48825,0.54144,0.59361,0.62457)	0.16902	7
B_2	(0.46834,0.50108,0.55341,0.60464,0.63515)	0.00851	4
D_1	(0.46893,0.50208,0.55431,0.60543,0.63588)	0.00450	2
D_2	(0.46898,0.50213,0.55475,0.60583,0.63625)	0.00319	1
D_3	(0.46888,0.50158,0.55386,0.60503,0.63551)	0.00627	3
E	(0.46781,0.50058,0.55296,0.60424,0.63478)	0.01076	5
F_1	(0.46296,0.49500,0.54612,0.59653,0.62727)	0.14325	6
F_2	(0.08518,0.09559,0.11040,0.12581,0.13652)	2.31763	9

6. Conclusions

Reliability analysis is vital in determining a product's overall performance and enhancing its competitiveness in the market, underscoring its significance in system reliability design and evaluation. However, the process often encounters the challenge of navigating expert information, which can be complex, ambiguous, incomplete, and inconsistent. Effectively addressing these issues is essential to achieve precise assessments of system reliability. Therefore, the traditional probability reliability approach is no longer appropriate for incomplete and inconsistent information about the failure probability of a basic event. To address the abovementioned issue, intuitionistic fuzzy sets, and fault tree analysis are incorporated into this study. The failure and reliability intervals were then calculated for the top events. The comparative results of the proposed approach and traditional probabilistic reliability and FFT approaches are shown in **Tables 3** and **4**, as well as **Figures 4**, **5**, **6** and **7**. A comparison of existing approaches (the traditional probability reliability and FFT approaches) shows that the proposed approach is more general and appropriate for evaluating the fuzzy reliability of complex engineering systems other than top event, where failure probabilities are represented by an intuitionistic fuzzy set. Moreover, the component with the highest criticality, denoted as D is F_2, indicating that it is the most critical bottom event in the system based on this analysis. This finding implies that F_2 significantly contributes to the overall failure of the system and

requires immediate attention to prevent unreliability. When a component reaches its "limit of failure", any small degradation in its performance could jeopardize the entire system's integrity. To mitigate the criticality of F_2, the following steps can be taken:

(i) By adding additional components that perform the same function as F_2, backups can be created to take over in case of failure. This will reduce reliance on a single critical component and lower the overall risk. Implementing a preventive maintenance schedule, supported by condition monitoring systems, would allow early detection and repair or replacement of F_2 before a failure occurs.
(ii) Integrating fault-tolerant design techniques into the system, such as adding compensatory components or control algorithms, can help mitigate the impact of F_2's failure. Additionally, optimizing system operations to reduce the load on F_2 could prevent it from reaching its limit of failure.
(iii) Enhancing quality control measures during manufacturing can minimize the risk of F_2 failure. Rigorous testing under various operational conditions will ensure that the component can withstand the expected stresses and help identify weaknesses in its design or production.
(iv) Educating crane operators on best practices, ensuring proper rope handling, and addressing each identified failure mode will help minimize risks and enhance operational safety. This systematic process aids in continuously improving safety protocols.

Therefore, implementing these strategies, the system's overall reliability can be enhanced, reducing the criticality of F_2 and preventing it from pushing the system to the limit of failure.

6.1 Limitations of the Study

(a) Defining the membership, non-membership, and hesitation degrees precisely can be challenging, especially in the absence of sufficient expert knowledge or historical failure data.
(b) The IFFT approach involves more complex calculations than traditional fault tree analysis due to the need to handle intuitionistic fuzzy sets and their associated operations.

6.2 Future Scope of the Study

(a) Future research could focus on integrating the IFFT approach with other advanced reliability assessment techniques, such as Bayesian networks**,** Markov chains, and Monte Carlo simulations, to enhance its capability for dynamic system analysis and to handle time-dependent failure data more effectively.
(b) Incorporating real-time data analytics with the IFFT approach could enable proactive fault diagnosis and prognosis, allowing for predictive maintenance and reducing unexpected system failures.

Conflict of Interest
The authors declare that there is no conflict of interest regarding the publication of this research.

AI Disclosure
The author(s) affirm that no part of this article has been written or generated with the assistance of generative AI.

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References

Abdelgawad, M., & Fayek, A.R. (2011). Fuzzy reliability analyzer: quantitative assessment of risk events in the construction industry using fuzzy fault-tree analysis. *Journal of Construction Engineering and Management, 137*(4), 294-302. https://doi.org/10.1061/(asce)co.1943-7862.0000285.

Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*(1), 87-96. https://doi.org/10.1016/s0165-0114(86)80034-3.

Chachra, A., Kumar, A., & Ram, M. (2023). Intuitionistic fuzzy approach to reliability assessment of multi-state systems. *Mathematics and Computers in Simulation, 212,* 489-503. https://doi.org/10.1016/j.matcom.2023.05.014.

Chachra, A., Kumar, A., & Ram, M. (2024). A Markovian approach to reliability estimation of series-parallel system with Fermatean fuzzy sets. *Computers & Industrial Engineering, 190*, 110081. https://doi.org/10.1016/j.cie.2024.110081.

National Research Council, Division on Engineering, Physical Sciences, Board on Mathematical Sciences, Their Applications, Committee on Mathematical Foundations of Verification, & Uncertainty Quantification. (2012). *Assessing the reliability of complex models: mathematical and statistical foundations of verification, validation, and uncertainty quantification*. National Academies Press, Washinton.

Domingues, M.S.Q., Baptista, A.L.F., & Diogo, M.T. (2017). Engineering complex systems applied to risk management in the mining industry. *International Journal of Mining Science and Technology, 27*(4), 611-616. https://doi.org/10.1016/j.ijmst.2017.05.007.

Garg, H. (2016). A novel approach for analyzing the reliability of series-parallel system using credibility theory and different types of intuitionistic fuzzy numbers. *Journal of the Brazilian Society of Mechanical Sciences and Engineering, 38*(3), 1021-1035. https://doi.org/10.1007/s40430-014-0284-2.

Hennig, P., Osborne, M.A., & Girolami, M. (2015). Probabilistic numerics and uncertainty in computations. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 471*(2179), 20150142. https://doi.org/10.1098/rspa.2015.0142.

Hong, Y.Y., & Lee, L.H. (2009). Reliability assessment of generation and transmission systems using fault-tree analysis. *Energy Conversion and Management*, *50*(11), 2810-2817.

Huang, H.Z., Tong, X., & Zuo, M.J. (2004). Posbist fault tree analysis of coherent systems. *Reliability Engineering & System Safety*, *84*(2), 141-148. https://doi.org/10.1016/j.ress.2003.11.002.

Khajuria, R. (2023). Intuitionistic fuzzy fault tree analysis of PCBA using novel arithmetic operations. *Journal of Quality in Maintenance Engineering*, *29*(4), 822-841. https://doi.org/10.1108/jqme-10-2022-0062.

Kumar, M. (2014). Applying weakest t-norm based approximate intuitionistic fuzzy arithmetic operations on different types of intuitionistic fuzzy numbers to evaluate reliability of PCBA fault. *Applied Soft Computing, 23*, 387-406. https://doi.org/10.1016/j.asoc.2014.06.036.

Kumar, M., & Singh, S.B. (2022). System reliability analysis based on different types of Pythagorean fuzzy failure rates of components. *Nonlinear Studies*, *29*(3), 779-808.

Kumar, M., & Singh, S.B. (2023). Reliability analysis of parted rope injuries during ship mooring operation under neutrosophic environments. *Applied Ocean Research*, *141*, 103798. https://doi.org/10.1016/j.apor.2023.103798.

Kumar, M., & Singh, S.B. (2024). Analysis of system reliability based on weakest t-norm arithmetic operations using Pythagorean fuzzy numbers. *International Journal of System Assurance Engineering and Management*, *15*(4), 1467-1482. https://doi.org/10.1007/s13198-023-01906-3.

Kumar, M., Pant, H., & Singh, S.B. (2024). Fuzzy reliability and availability of system under a calendar-based inspection involving multiple failures and its application to wind turbine system. *Journal of Systems Science and Systems Engineering*, *33*(2), 187-206. https://doi.org/10.1007/s11518-023-5580-x.

Kumar, M., Singh, S.B., & Kumar, D. (2023). System reliability analysis based on Pythagorean fuzzy set. *International Journal of Mathematics in Operational Research, 24*(2), 253-285. https://doi.org/10.1504/ijmor.2023.129440.

Mhalla, A., Dutilleul, S.C., Craye, E., & Benrejeb, M. (2014). Estimation of failure probability of milk manufacturing unit by fuzzy fault tree analysis. *Journal of Intelligent & Fuzzy Systems*, *26*(2), 741-750. https://doi.org/10.3233/ifs-130764.

Mo, J., & Beckett, R. (2018). *Engineering and operations of system of systems*. CRC Press. Boca Raton. ISBN: 9781315206684. https://doi.org/10.1201/9781315206684.

Niroomand, S. (2018). A multi-objective based direct solution approach for linear programming with intuitionistic fuzzy parameters. *Journal of Intelligent & Fuzzy Systems*, *35*(2), 1923-1934. https://doi.org/10.3233/jifs-171504.

Parvathi, R., & Malathi, C. (2012). Intuitionistic fuzzy linear programming problems. *World Applied Sciences Journal*, *17*(12), 1802-1807.

Purba, J.H. (2014). A fuzzy-based reliability approach to evaluate basic events of fault tree analysis for nuclear power plant probabilistic safety assessment. *Annals of Nuclear Energy*, *70*, 21-29. https://doi.org/10.1016/j.anucene.2014.02.022.

Purba, J.H., Tjahyani, D.S., Ekariansyah, A.S., & Tjahjono, H. (2015). Fuzzy probability based fault tree analysis to propagate and quantify epistemic uncertainty. *Annals of Nuclear Energy*, *85*, 1189-1199. https://doi.org/10.1016/j.anucene.2015.08.002.

Rajakarunakaran, S., Kumar, A.M., & Prabhu, V.A. (2015). Applications of fuzzy faulty tree analysis and expert elicitation for evaluation of risks in LPG refuelling station. *Journal of Loss Prevention in the Process Industries*, *33*, 109-123. https://doi.org/10.1016/j.jlp.2014.11.016.

Sahoo, D., Tripathy, A.K., & Pati, J.K. (2022). Study on multi-objective linear fractional programming problem involving pentagonal intuitionistic fuzzy number. *Results in Control and Optimization*, *6*, 100091. https://doi.org/10.1016/j.rico.2021.100091.

Sankararaman, S., & Mahadevan, S. (2015). Integration of model verification, validation, and calibration for uncertainty quantification in engineering systems. *Reliability Engineering & System Safety, 138*, 194-209. https://doi.org/10.1016/j.ress.2015.01.023.

Shu, M.H., Cheng, C.H., & Chang, J.R. (2006). Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit board assembly. *Microelectronics Reliability, 46*(12), 2139-2148. https://doi.org/10.1016/j.microrel.2006.01.007.

Torra, V., & Narukawa, Y. (2009). On hesitant fuzzy sets and decision. In *2009 IEEE International Conference on Fuzzy Systems* (pp. 1378-1382). IEEE. Jeju, Korea. https://doi.org/10.1109/fuzzy.2009.5276884.

Wang, J.Q., Nie, R., Zhang, H.Y., & Chen, X.H. (2013). New operators on triangular intuitionistic fuzzy numbers and their applications in system fault analysis. *Information Sciences, 251*, 79-95. https://doi.org/10.1016/j.ins.2013.06.033.

Yager, R.R. (2013). Pythagorean fuzzy subsets. In *2013 joint IFSA world congress and NAFIPS annual meeting* (pp. 57-61). IEEE. Edmonton, AB, Canada. https://doi.org/10.1109/ifsa-nafips.2013.6608375.

Yazdi, M., & Zarei, E. (2018). Uncertainty handling in the safety risk analysis: an integrated approach based on fuzzy fault tree analysis. *Journal of Failure Analysis and Prevention, 18*(2), 392-404. https://doi.org/10.1007/s11668-018-0421-9.

Yazdi, M., Nikfar, F., & Nasrabadi, M. (2017). Failure probability analysis by employing fuzzy fault tree analysis. *International Journal of System Assurance Engineering and Management, 8*(2 suppl.), 1177-1193. https://doi.org/10.1007/s13198-017-0583-y.

Yin, H., Liu, C., Wu, W., Song, K., Liu, D., & Dan, Y. (2020). Safety assessment of natural gas storage tank using similarity aggregation method based fuzzy fault tree analysis (SAM-FFTA) approach. *Journal of Loss Prevention in the Process Industries, 66*, 104159. https://doi.org/10.1016/j.jlp.2020.104159.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338-353. https://doi.org/10.1016/s0019-9958(65)90241-x.

Zhu, B., Xu, Z., & Xia, M. (2012). Dual hesitant fuzzy sets. *Journal of Applied Mathematics*, *2012*(1), 1-13. https://doi.org/10.1155/2012/879629.

Zio, E. (2016). Some challenges and opportunities in reliability engineering. *IEEE Transactions on Reliability, 65*(4), 1769-1782. https://doi.org/10.1109/tr.2016.2591504.

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