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Avoid Maximum Cost Method for Solving Linear Fractional Transshipment Problem

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Abstract
This study contributes valuable insights into linear fractional transshipment problem which is a special class of mathematical programming problem. We present the mathematical model for the linear fractional transshipment problem and develop an efficient algorithm based on the 'Avoid Maximum Cost Method (AMCM)' for finding an initial basic feasible solution (IBFS) of the given model. AMCM is based on the concept of making the maximum possible allocation to either a column or a row of the transportation cost matrix in such a way that the allocation to the corresponding cell that has the highest cost will be avoided in the further steps. The methodology is composed of the following two steps: firstly, we formulated an equivalent transportation model of the problem by considering the cost-profit ratio matrix. Secondly, we apply AMCM to find an IBFS of the problem. In a nutshell, this article finds the solution to a linear fractional transshipment model by applying AMCM to the cost-profit ratio matrix. The applicability of the proposed approach is illustrated with some suitable numerical examples. The contribution ends by introducing a comparative analysis to show the efficiency of the proposed algorithm.

Keywords- Transshipment problem, Fractional programming problem, Avoid maximum cost method.

1. Introduction

In this modern-era of ever-increasing competitive market of production, we need to develop more efficient and cost-effective solution to the problem of delivering raw materials or in-process inventory or final product or related information from the source point or origin to the destination or the final consumption points. Logistic management can help us to solve this problem. For every industry, transportation cost is one of the crucial cost components of logistics costs and it contributes almost one-third to two-third of the total logistics cost (Cano et al., 2020; Mathirajan et al., 2022). To maximize the profit, we need to minimize the product cost. The product cost consists of several costs like raw material's cost, worker's cost, equipment's charges, and transportation cost. Now-a-days, it's very difficult to reduce the cost of raw materials, worker and equipment. So, to minimize the product cost, we need to minimize the transportation cost involved in the process.

Transportation problem (TP) is one of the most used linear programming models for solving several real-world problems. The French mathematician Monge (1781) was first to propose the transportation problem. In 1920s, Tolstoi (1930) first studied the transportation problem mathematically. This classical transportation problem is used when a large amount of homogeneous goods is transported from one side to another in a network. A transportation network is defined by its two characteristics, namely vertex and edges. Moreover, the vertices are of two types: (i) Source, from where the goods are transported and (ii) Sink, to where goods are transported. Edges represent the transportation cost, time, profit, etc. In a general framework, not only homogeneous goods but we can transport any kind of homogeneous things like data in a transportation network. In the literature, one of the following three solution procedure is followed to minimize the transportation cost or time of the TP: (a) developing linear programming (LP) model of the TP and solve it using standard LP solution method, (b) developing network model and solve it using network approach, and (c) special transportation algorithms which are consists of a simple heuristic technique followed by an optimality test of the solution obtained by the heuristic (Mathirajan et al., 2022). Several heuristic techniques namely, North-West Corner method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM) etc. are there in the literature for finding initial basic feasible solution (IBFS) of a transportation problem. Recently, Mathirajan et al. (2022) conducted an experimental study and performance analysis of 34 recently developed heuristic techniques.

In a transportation model, the homogeneous goods are transported from the sources only and the goods are received at destinations. Thus, each vertex in the network model of a TP acts as a shipper or source only or as a receiver or sink only. Sometimes, it is observed that the transportation cost can be decreased if the transshipment is allowed in the model. For example, let us consider a very small transportation problem with two sources (O_1, O_2) and one destination (D). The availability at the sources O_1 and O_2 be 100 units and 50 units respectively and demand at the destination be 150 units. The unit transportation cost from O_1 to D be 7 units and the transportation cost from O_2 to D be 3 units. Then the total transportation cost is $(7 \times 100 + 3 \times 50) = 850$ units. Now if the transshipment is allowed in the model and the unit transportation cost from O_1 to O_2 is 2 units, then the total transportation cost becomes 650 units $((2 \times 100)\, for\ transshipment\ of\ product\ + (3 \times 150)\, for\ transportation\ of\ product)$ which is less than the cost of transportation model. Keeping this point in mind, transshipment model is developed. Transshipment model can be viewed as a more generalized version of the transportation model. Orden (1956) was the first person who studied the transshipment problem. In a transshipment problem, transshipment is permitted with the additional feature that the shipments may go via any sequence of vertices in the network rather than being restricted to direct connections from one source to one of the sinks. Consequently, in a transshipment network model, all the vertices may act as sources as well as sinks i.e., all the vertices of the network can act as transit points. In **Figure 1**, a transshipment network in which the vertices $1, 2, 3$ are sources, vertices 7 and 8 are sinks, and vertices $4, 5, 6$ are pure transit points, is illustrated. Note that in the network, vertices $1, 2, 3, 7, 8$ are also acting as transit points. The supplies are there at vertices 1, 2, 3 and demands are there at vertices 7 and 8 whereas there are no demand or supply at vertices 4, 5, 6 (hence pure transit point). All sources and destinations in a transshipment problem can work in any direction. For a transshipment model, it is assumed that a large amount of goods to be shipped is available at each vertex and act as stockpile, which can be drawn or replenished. Also, the unit transportation cost at a vertex considered as a shipper to the same vertex considered as receiver is set equal to zero (Ignizio, 1982). "The solution to the transshipment problem is lies in the fact that withdrawals from and compensating additions to the stockpiles are equivalent to transshipment" (Khurana, 2015). Khurana (2015) studied various forms of a transshipment problem by considering different real-life scenarios.

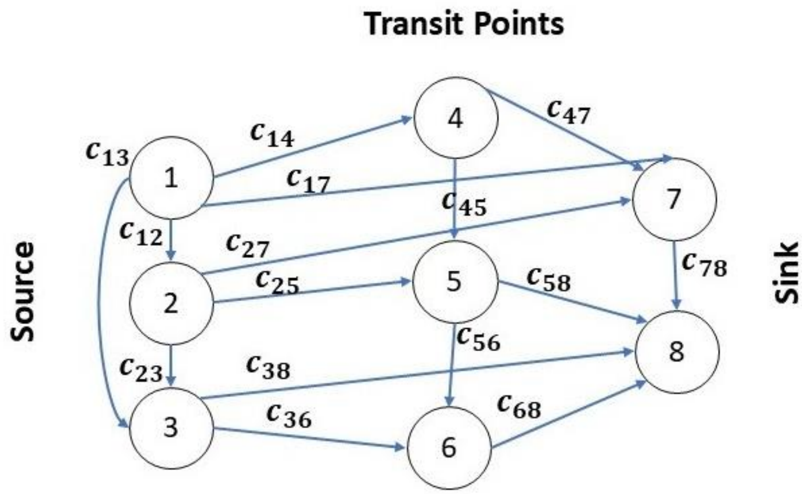

Figure 1. Transshipment network.

The classical single-objective transshipment problem is a typical linear programming problem where network flow cost is optimized. This transshipment technique is also used to determine the shortest route between two vertices in a network. King and Logan (1964) use a transshipment model to determine the flow of primary products to the final products in the market through processors. Later, Judge et al. (1965) extent this model to a multi-regional, multi-product and multi-plant problem and formulate a general linear programming model for the problem. Afterwards, several researchers developed alternative formulations of the transshipment model and solution methodologies for solving such typical optimization problem (Goldberg, 1997; Garg and Prakash, 1985; Ghosh and Mondal, 2017; Hurt and Tramel, 1965). Khurana et al. (2012, 2015) modelled three-dimensional time minimizing transshipment problem and developed algorithm for solving these models. Khurana (2013) studied multi-index transshipment problems. There are number of transshipment models with multiple-objectives also. Among the multi-objective models, bi-objective programming problems are most used model in practical. As, in our real life, when we buy something or make a prediction about something, we mainly focused on two aspects of that product, in particular, one is good and the other is bad. For example, if we go to the market to sell a product, we look at the profit and cost of that product (Schaible and Shi, 2003). So, two objective functions work together. A bi-objective transshipment problem can be tackled by using fractional transshipment model which is a single objective model.

Till now very little research has been done regarding fractional transshipment model. Pradhan and Biswal (2015) developed two algorithms based on VAM for solving a fractional transportation problem. Recently, Garg et al. (2021) have worked on a fractional transshipment model under fuzzy environment. But they have considered the model as a two-stage model where transshipment is done through some transit points which are not exactly the source or destinations of the network. As per our knowledge, there is no study on the linear fractional transshipment model in which all the source and sink points are considered as transit point. So, this research gap motivates us to focus on a linear transshipment problem where all the sources and the destinations of the network are transit points. The main aim of this contribution is not to cover all the ranges of linear transshipment problem but merely to develop a methodology for finding an IBFS of the problem using a new heuristic technique, namely Avoid Maximum Cost Method (AMCM). Furthermore, the performance of the heuristic technique is analyzed. Besides the theoretical aspects of linear fractional transshipment problem, the article also explores the experimental-based results.

The structure of this paper is as follows: Section 2 discusses basic notion of Linear Fractional Programming (LFP) model. Mathematical formulation of linear fractional transshipment problem is given in Section 3. Section 4 proposes an approach to solve linear fractional transshipment problem. A numerical example is described in Section 5 to illustrate the applicability of proposed method. Also, this section analyses the experimental results and after that, a comparative study has carried out to validate this result. Finally, conclusions are drawn in Section 6.

2. Linear Fractional Programming Problem

In real-life, most of the decision-making problems are having more than one objective. In literature, these problems are addressed by the Multi-Objective Programming problem. If the decision-making problem has only two objectives, then it is known as bi-objective problem. One of the efficient ways to solve these bi-objective problem is the use of fractional programming technique. This technique not only make a single-objective problem but also gives an insight into the situation of the problem. LFP problems are of great interest because of their extensive application areas such as resource allocation, transportation, production, finance, location theory, stochastic process, Markov renewal programme, information theory, applied linear algebra, large scale programming, game theory, etc. (Ozkok, 2020). To study the relative efficiency of the objectives in different fields such as education, hospital administration, court systems, air force maintenance units, bank branches, etc., fractional programming are used. Sometimes, decision makers need to take decision depending on the optimization of inventory/sales, actual cost/standard cost, output/employee, etc with respect to some constraints (Chakraborty and Gupta, 2002). The linear fractional (ratio) criteria are frequently encountered in finance as illustrated by the many situations as follows, (i) Corporate planning (min (actual cost to standard cost), max (return on investment)); (ii) Bank Balance sheet Management (min (risk-assets to capital), max (actual capital to required capital)); etc. (Steuer, 1986). The exhaustive overview of theory and application on fractional programming problem can be found in the book 'Linear-fractional programming: Theory, methods, applications and software' (Bajalinov, 2003).

Mathematically, if the objective function of the problem is the ratio of two different linear functions and the constraint set contains only the linear functions of the decision variables then the problem is called a LFP problem (Pradhan and Biswal, 2015). LFP is a special type of nonlinear programming problem. The mathematical model of LFP is developed by the Hungarian mathematician B. Matros and his associates in 1960. Afterwards, Charnes and Cooper proposed the most used method for solving the LFP problem in 1962. The method is based on the variable transformation, and the method transforms the LFP to equivalent LP (Charnes and Cooper, 1962). The updated objective function method of Bitran and Novaes (1973) is also very popular approach for solving a LFP problem. There are several analytical methods and heuristic approaches for solving the LFP problem. The solution methodologies are mainly based on two of these basic ideas, "Variable Transformation" and "Updated Objective Function".

Standard mathematical model of a linear fractional programming problem is given by:

Find $\mathrm{X} \in \mathrm{R}^{\mathrm{n}}$ *so as to*

$$\max \ Z(X) = \frac{N(X)}{D(X)} = \frac{c^T X + \alpha}{d^T X + \beta} \tag{1}$$

$$subject\ to, AX \leq b, X \geq 0 \tag{2}$$

where, $\alpha, \beta \in \mathbb{R}, c, d \in \mathbb{R}^n, b \in \mathbb{R}^m, A \in \mathbb{R}^{m\times n}$. The basic assumption of the model is that the denominator of the objective function i.e., $d^T X + \beta$ is positive over the feasible region. Note that the feasible region of the problem is given by $S = \{X \in \mathbb{R}^n | AX \leq b, X \geq 0, b \in \mathbb{R}^m, d^T X + \beta > 0\}$ which is a nonempty convex set (Ozkok, 2020). Also, the fraction objective function $Z(X)$ is continuous on the domain S.

Since, the numerator as well as the denominator of the objective function are linear functions, LFP is a pseudo convex programming problem. An LFP problem can be single objective linear fractional programming as well as multi-objective linear fractional programming. It has an important role in logistic and supply chain management for reducing cost and improving profit. In this article, we studied about the single objective linear fractional problem for the transshipment problem.

3. Mathematical Model Formulation

In this section, we present the mathematical model of a linear fractional transshipment model. Let us consider a linear fractional transshipment problem (LFTP) with m sources and n sinks. In this study, we consider a transshipment network where all the sources and sinks are transit points and there is no pure transit point. So, in this model any source or sink can ship to any other source or sink. To develop the mathematical model, it is convenient to number them successively so that the sources are numbered from station 1 to m and sinks are numbered from station $(m+1)$ to $(m+n)$. The parameters of the problem are given by:

i, j: index variable

$m =$ the number of source points

$n =$ the number of original sink points

$a_i =$ availability at station i; $\quad i = 1,2, \dots, m$ (original source points)

(Note that the availabilities at the sinks are zero. Hence $a_i = 0$; $\quad i = m+1, m+2, \dots, m+n$)

$b_j =$ demand at station j; $\quad j = m+1, m+2, \dots, m+n$ (original sink points)

(Note that the demands at the sources are zero. Hence $b_j = 0$; $\quad j = 1,2, \dots, m$)

$c_{ij} =$ unit shipping cost from station i to station j; $\quad i, j = 1,2, \dots, m+n, i \neq j$

$d_{ij} =$ unit preference due to the shipping from station i to station j; $\quad i, j = 1,2, \dots, m+n, i \neq j$

α = the total fixed cost of the shipment,

β = the total fixed benefit or profit of the shipment,

(We assume that $c_{ii} = d_{ii} = 0, \forall\, i$ i.e., the shipping cost and the profit from the shipping within the same station is zero. Note that c_{ij} need not be same as c_{ji} and d_{ij} need not be same as d_{ji}.)

For the discussion purpose, we assumed that $total\, availability = total\, demand$ i.e., $\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j$.

Let X_{ij} be the quantity shipped from station i to station j, $i, j = 1,2, \dots, m+n$. Then the mathematical model of a LFTP is given by:

$$\text{minimize } \frac{\sum_{i=1}^{m+n} \sum_{\substack{j=1 \\ j\neq i}}^{m+n} c_{ij} x_{ij} + \alpha}{\sum_{i=i}^{m+n} \sum_{\substack{j=1 \\ j\neq i}}^{m+n} d_{ij} x_{ij} + \beta} \tag{3}$$

subject to,

$$\sum_{\substack{j=1 \\ j\neq i}}^{m+n} x_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{m+n} x_{ji} = a_i, i = 1,2, \dots, m \tag{4}$$

$$\sum_{\substack{i=1 \\ i\neq j}}^{m+n} x_{ij} - \sum_{\substack{i=1 \\ i\neq j}}^{m+n} x_{ji} = b_j, j = m+1, m+2, \dots, m+n \tag{5}$$

$$x_{ij} \geq 0, i, j = 1,2, \dots, m+n; i \neq j \tag{6}$$

In the above model, $\sum_{\substack{j=1\\j\neq i}}^{m+n} x_{ij}$ represent the amount of outflow from the station i and $\sum_{\substack{j=1\\j\neq i}}^{m+n} x_{ji}$ represent the amount of inflow to the station i. Hence the constraint given by the Equation (4) is representing the net inflow-outflow at the station $i, i = 1,2, \dots, m$; which are the source points of the problem. So, constraints given by Equation (4) are the supply constraint of the problem. Similarly, constraints given by the Equation (5) are the demand constraints. Equation (6) reflects that the amount transshipped from station i to station j cannot be negative. The above model has similar structure of linear fractional transportation problem, but in this case the coefficient of $\sum x_{ji}$ are '-1'. To convert the model into a linear fractional transportation model, let us define $T_i = \sum_{\substack{j=1\\j\neq i}}^{m+n} x_{ji}, i = 1,2,\dots,m$ and $T_j = \sum_{\substack{i=1\\i\neq j}}^{m+n} x_{ji}, j = m+1, m+2, \dots, m+n$, where, T_i and T_j represents the total amount of transshipment through the i-th source point and the j-th sink point respectively. Let $T =$ total availability= total demand. Then we have two constraints $T_i \leq T, i = 1,2,\dots,m$ and $T_j \leq T, j = m+1, m+2, \dots, m+n$. Using the nonnegative slack variables x_{ii} and x_{jj} for the respective inequalities, we obtain the equality constraints as,

$$\sum_{\substack{j=1\\j\neq i}}^{m+n} x_{ji} + x_{ii} = T, i = 1,2,\dots,m \tag{7}$$

$$\sum_{\substack{i=1\\i\neq j}}^{m+n} x_{ji} + x_{jj} = T, j = m+1, m+2, \dots, m+n \tag{8}$$

Table 1. Tabular representation of transshipment problem.

From \ To	Sources $(1 \;\; 2 \;\; \cdots \;\; i \;\; \cdots \;\; m)$	Sink $(1 \;\; 2 \;\; \cdots \;\; j \;\; \cdots \;\; n)$	Supply
Sources $(1, 2, \vdots, i, \vdots, m)$	Source-to-Source Submatrix (P) (c_{ij}, d_{ij})	Source-to-Sink Submatrix (Q) (c_{ij}, d_{ij})	$(a_1+T, a_2+T, \vdots, a_i+T, \vdots, a_m+T)$
Sinks $(1, 2, \vdots, j, \vdots, n)$	Sink-to-Source Submatrix (R) (not necessarily equals to Q^T) (c_{ij}, d_{ij})	Sink-to- Sink Submatrix (S) (c_{ij}, d_{ij})	$(T, T, \vdots, T, \vdots, T)$
Demand	$(T \;\; T \;\; \cdots \;\; T \;\; \cdots \;\; T)$	$(b_1+T \;\; \cdots \;\; b_n+T)$	

Here, x_{ii} represent the difference between the total availability and the actual amount of transshipment through the i-th source point. x_{jj} represent the difference between the total availability and the actual amount of transshipment through the j-th sink point.

Combining Equations (3) to (8), we obtain the reformulated model as,

$$\text{minimize } \frac{\sum_{i=1}^{m+n} \sum_{\substack{j=1\\j\neq i}}^{m+n} c_{ij}x_{ij} + \alpha}{\sum_{i=i}^{m+n} \sum_{\substack{j=1\\j\neq i}}^{m+n} d_{ij}x_{ij} + \beta} \tag{9}$$

subject to,

$$\sum_{j=1}^{m+n} x_{ij} = a_i + T, i = 1,2,\dots,m \tag{10}$$

$$\sum_{j=1}^{m+n} x_{ij} = T, i = m+1, m+2, \dots, m+n \tag{11}$$

$$\sum_{i=1}^{m+n} x_{ij} = T, j = 1,2,\ldots,m \tag{12}$$

$$\sum_{i=1}^{m+n} x_{ij} = b_j + T,\ j = m+1, m+2, \ldots, m+n \tag{13}$$

$$x_{ij} \geq 0, i,j = 1,2,\ldots,m+n; i \neq j \tag{14}$$

$$c_{ii} = 0, i = 1,2,\ldots,m+n \tag{15}$$

$$d_{ii} = 0, \quad i = 1,2,\ldots,m+n \tag{16}$$

$$\alpha, \beta \in \mathbb{R} \text{ and } \alpha \geq 0, \beta > 0 \tag{17}$$

Rim condition for the problem is given by:

$$\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j \tag{18}$$

The mathematical model given by Equations (9) to (18) represent a standard balanced linear fractional transportation problem.

To solve this model using transportation algorithm, we need to construct the matrix form of the problem. The matrix form of the linear fractional transshipment problem is given in **Table 1**. In the matrix form of the problem, first m rows are corresponding to the supply points and next n rows are corresponding to the transit points (in this case sinks are acting as transit points). Similarly, first m columns are corresponding to the transit points (in this case supply points are acting as transit points) and the next n columns are corresponding to the sink points. Each supply points and each demand points are having their original supplies and demands respectively, whereas the transit points don't have any supply or demand. Note that the transshipped amount through the transit points are equals to the maximum amounts of buffer stock i.e., $T = \max\{\sum_{i=1}^{n} a_i, \sum_{j=1}^{m} b_j\}$.

Then each transshipment points will have supplies equals to its original supplies + T and demands equals to its original demand + T. This ensures that if any transshipment point is a net supplier, then it will have a net outflow equal to the point's original supply, and, similarly, if a transshipment point is a net demander, then it will have a net inflow equal to the point's original demand. Although, we don't know how much will be shipped through each transshipment point, but by the construction of the model we can be assured that the total transhipped amount will not exceed T. In the matrix form, each cell of the matrix contains two values, one is the unit shipping cost c_{ij} and other one is the unit profit from the shipping d_{ij}. The matrix has four blocks. The first block (P in the **Table 1**) is consists of the costs and preferences due to the transshipment from the sources to sources. Note that the diagonal elements of the block i.e., the unit shipping costs and preferences for the transshipment from source i to itself are zero. It is true for the fourth block S also, where the elements represent the shipping costs and preferences for the transshipment among the sink points. The second and the third block of the matrix are corresponding to the transshipment from source points to the sink points and sink points to source points respectively.

In the next section, we present the algorithm developed for solving the linear fractional transshipment model. Note that the cost-preference matrix of the problem has $(m+n)$ rows and $(m+n)$ columns hence the optimal non-degenerated solution of the problem contains $2(m+n)-1$ number of basic variables. However, among these basic variables $m+n$ variables appearing in the diagonal cells as they represent the remaining buffer stock and if they are omitted, then we end up with exactly $(m+n-1)$ basic variables of our interest.

4. Proposed Methodology to Solve Linear Fractional Transshipment Problem

The mathematical model established in last section can be treated as a linear fractional transportation problem. So, we can use well-known transportation algorithms like NWCM, VAM, LCM, Modified VAM Methods with some modification for solving the linear fractional transshipment problem. But use of these methods is not straight forward. Before applying these heuristic techniques, we need to create a ratio matrix. The numerator of the ratio in the ratio matrix is the coefficient of the decision variable in the numerator objective function (e.g., cost function) and the denominator of the ratio in the ratio matrix is the coefficient of the decision variable in the denominator objective function (e.g., profit function). The ratio of the two objective function gives us one objective function (e.g., minimum $z = \min\left\{\frac{cost\ function}{profit\ function}\right\}$). Then we can apply the transportation heuristics to find an IBFS. To find the optimal solution, we can use Modified distribution (MODI) or stepping stone method taking IBFS as the starting solution. In our study, we propose a new method to find the IBFS for the fractional transshipment problem. The name of the new method is AMCM.

4.1 Avoid Maximum Cost Method

Mutlu et al. (2022) used the avoid maximum cost method for finding the IBFS for the classical transportation problem. We adopt this method for solving fractional transshipment problem. At the beginning of the methodology, we construct the ratio matrix for the transshipment problem. To construct the ratio matrix, we should have $d_{ij} \neq 0, \forall\, i, j$, but as per the construction of the model $d_{ii} = 0, \forall\, i$. To avoid this situation, at the initial stage, we assume that $d_{ii} = 1, \forall\, i$. On the ratio matrix we apply the AMCM which is based on making the allocation to either a row or a column in such a way that the allocation to the cell that has the highest ratio (cost/profit) will be avoided in the further steps. After getting an IBFS, we proceed for the optimality test where we take $d_{ii} = 0, \forall\, i$ again. The proposed algorithm is depicted below:

***Algorithm*: Avoid Maximum Cost Method for LFTP**

***Input*:** The number of source points m; number of sink points n; availability at the i-th source point $a_i, i = 1,2, \dots m$, $(a_i = 0, i = m+1, m+2, \dots, m+n)$; demand at the j-th sink point $b_j, j = m+1, m+2, \dots, m+n$, $\left(b_j = 0, j = 1,2, \dots, m\right)$; unit cost and unit profit matrices $c_{ij}, d_{ij}, i, j = 1,2, \dots, m+n$, ($c_{ii} = 0$, and $d_{ii} = 1, i = 1,2, \dots, m+n$).

***Output*:** An initial basic feasible solution of the LFTP.

Step 0: Calculate $\sum_{i=1}^{m} a_i$ and $\sum_{j=m+1}^{m+n} b_j$; Set $T = \max\left\{\sum_{i=1}^{m} a_i, \sum_{j=m+1}^{m+n} b_j\right\}$

Step 1: Construct the supply vector and the demand vector for the transshipment as $\boldsymbol{supply} = \boldsymbol{a} + T$ and $\boldsymbol{demand} = \boldsymbol{b} + T$, respectively

Step 2: Test if $\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j$, then go to next **Step;** otherwise
if $\sum_{i=1}^{m} a_i > \sum_{j=m+1}^{m+n} b_j$
append a dummy sink or column with $c_{i(m+n+1)} = 0$ and $d_{i(m+n+1)} = 1, \forall i$, and append a scalar $\sum_{i=1}^{m} a_i - \sum_{j=m+1}^{m+n} b_j$ to the vector $\boldsymbol{demand}$
elseif $\sum_{i=1}^{m} a_i < \sum_{j=m+1}^{m+n} b_j$
append a dummy source or row with $c_{(m+n+1)j} = 0$ and $d_{(m+n+1)j} = 1, \forall j$, and append a scalar $\sum_{j=m+1}^{m+n} b_j - \sum_{i=1}^{m} a_i$ to the vector $\boldsymbol{supply}$
end

Step 3: Calculate the ratio matrix $\boldsymbol{r} = \boldsymbol{c}./\boldsymbol{d}$

Step 4: Find the maximum ratio cell and consider the row and column corresponding to this cell. If there is more than one alternative, consider all the rows and columns corresponding to these alternatives.

Step 5: Find the cell with the lowest ratio within these rows and columns selected in **Step 4**. If there is more than one candidate for the minimum ratio, select the cell whose row or column contains the highest ratio.

Step 6: Make the maximum allocation to the selected cell from **Step 5** by considering the corresponding legitimate row and column capacity. After the allocation, remove the corresponding row or column whose legitimate capacity becomes zero, if both the row and column capacity become zero, then remove the column only.

Step 7: If there is only one row or column left, go to **Step 8**, otherwise go to **Step 4**.

Step 8: Make the necessary assignments for the last-left row or column.

The **MATLAB** code of the algorithm is given in the **Appendix.**

5. Numerical Illustration and Result Discussion

To illustrate the model and methodology presented in the last two sections, we consider a fractional transshipment problem with three sources (A, B, C) and four destinations (I, II, III, IV). We extended the example of a fractional transportation problem taken by Sirvi et al. (2011) to a linear fractional transshipment problem which is a balanced problem. For the extension purpose, we generate the remaining data (cost and profit data for source to source and sink to sink transshipment) randomly. Randomly generated data for c_{ij} and d_{ij} is in the ranges [4,13] and [3,17] respectively. The mathematical model of the problem is given by:

Find the amount of transshipment x_{ij} from station i to station j so as to,

$$min: z = \frac{\sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij}}{\sum_{i=1}^{m+n} \sum_{j=1}^{m+n} d_{ij} x_{ij}} \tag{19}$$

subject to

$$\sum_{j=1}^{7} x_{ij} = a_i + 48, \ i = 1,2,3 \tag{20}$$

$$\sum_{j=1}^{7} x_{ij} = 48, \ i = 4,5,6,7 \tag{21}$$

$$\sum_{i=1}^{7} x_{ij} = 48, \ j = 1,2,3 \tag{22}$$

$$\sum_{i=1}^{7} x_{ij} = b_j + 48, j = 4,5,6,7 \tag{23}$$

$$x_{ij} \geq 0, \ i, j = 1,2,3,4,5,6,7 \tag{24}$$

The values of the unit transshipment cost (c_{ij}) and preference (d_{ij}) are given in the matrix form of the problem (**Table 2**). Note that, to start the solution procedure we consider $d_{ii} = 1, i = 1,2, \ldots, 7$.

The transshipment cost and preference per unit of transshipment are given by the above table where top left corner's values of each cell represent the transshipment cost per unit and bottom right corner's values of each cell represent the profit cost per unit of transshipment. Our target is to minimize the total transshipment cost per unit of profit. The IBFS obtained by using the proposed algorithm (AMCM) is given in **Table 3**. For this example, we compare the IBFSs obtained by different methodologies (NWCM, LCM, least ratio method, VAM, Modified VAM (Sirvi et al., 2011), Modified VAM (Pradhan and Biswal, 2015)) and we present the comparison result in the **Table 4**. From the comparison table, we observe that the proposed methodology performed better than the other considered methodologies from the literature although the computation time is a bit higher than few of the methods like NW method, LCM, least ratio method. We use LINGO 11.0 (Schrage, 2008) to find optimal solution of the mathematical model of the problem given by Equations (19) to (24). The optimal solution of the problem is given in the **Table 5**. The optimal value of the problem is 0.383721. In the optimal solution, we observe that 48 units are transshipped from source A to source C again 36 units are transshipped from source C to source A. Similar kind of things happened in the transshipment between the sinks I and IV. Although, this solution is an optimal solution of the problem but practically it is not a smart solution as it seems to be very odd to transport 48 unit from station A to station C and then transport back 36 units from station C to Station A. This happens due to higher profit value in the corresponding cells. To make this solution as a smart solution we make some adjustment in the allocation. The new solution is given in the **Table 6**. For the new solution, objective function's value is 0.484772, where the total transshipment cost is 573 units and the corresponding profit is 1182.

Table 2. Linear transshipment model example data.

	A	B	C	I	II	III	IV	$Supply$
A	0 0	11 13	6 16	8 9	10 12	12 7	9 6	12 + 48
B	11 13	0 0	9 17	6 11	4 9	8 17	11 6	19 + 48
C	6 16	9 17	0 0	9 5	13 4	11 3	7 9	17 + 48
I	8 9	6 11	9 5	0 0	9 15	5 6	5 16	48
II	10 12	4 9	13 4	9 15	0 0	6 8	12 5	48
III	12 7	8 17	11 3	5 6	6 8	0 0	6 6	48
IV	9 6	11 6	7 9	5 16	12 5	6 6	0 0	48
$Demand$	48	48	48	3 + 48	22 + 48	18 + 48	5 + 48	

Table 3. IBFS by AMCM.

	A	B	C	I	II	III	IV
A	31			7	22		
B		48		1		18	
C	17		48				
I				43			5
II					48		
III						48	
IV							48

Table 4. Comparison of IBFS by different methods.

Method:	Value of the numerator function $(N(X))$	Value of the denominator function $(D(X))$	Objective function's value of the IBFS	Computation time (in 's')
NWCM	1416	1812	0.781457	0.004965
LCM	369	357	1.033613	0.008917
Minimum Ratio	369	357	1.033613	0.009537
VAM	369	357	1.033613	0.024836
Modified VAM (Sirvi et al., 2011)	369	357	1.033613	0.036518
Modified VAM (Pradhan and Biswal, 2015)	510	705	0.723404	0.025175
AMCM	553	996	0.555221	0.021516

Table 5. Optimal solution of the problem.

	A	B	C	I	II	III	IV
A	12		48				
B		24		3	22	18	
C	36	24					5
I							48
II					48		
III						48	
IV				48			

Table 6. Practical solution of the problem.

	A	B	C	I	II	III	IV
A	48		12				
B		24		3	22	18	
C		24	36				5
I				48			
II					48		
III						48	
IV							48

5.1 Experimental Results

To analyze the performance of the proposed methodology, we conduct experiments in two phases: i) on 40 small size fractional transshipment problems with different number of sources and sinks and ii) on 64 large size fractional transshipment problems with different number of sources and sinks. For both the phases, we find the solution of these problems by using well-known methods from the literature (North-West corner method, least cost method (LCM), Minimum Ratio method (MRM), VAM, Modified VAM (Sirvi et al., 2011), Modified VAM (Pradhan and Biswal, 2015)) as well as the new method (AMCM). For the first phase, details of the problems and the optimal solutions are given in **Table 7**. Some of the test problems are extension of examples related to the transportation problems taken from the literature. For these problems, the data related to c_{ij} is generated from the range with the lower bound as the minimum value and upper bound as the maximum value of the transportation costs. The preference data d_{ij} is generated randomly from interval $[10, 50]$. Rest of the problems are generated randomly. For these problems, average unit transportation cost is 500 units, average unit profit is 200 units, and average supply and average demand is 120 units. The problems taken from the literature are all balanced whereas the randomly generated problems are all unbalanced. The methods used for the comparison are all coded using MATLAB (www.mathworks.com), and the experiments were run on a Windows-based PC with a 3.20 GHz Intel 11 generation i5 quad core processor and 8 GB RAM. The optimal solutions of the problems are obtained by using LINGO 11.0 software.

First, we compare the performance of the considered methods based on the relative percentage deviations of the IBFS from the optimal solutions. The value of the objective function for the IBFS obtained by using different heuristics are presented in **Table 8**. Although it is not expected that an initial basic feasible solution will produce an optimal solution, however finding IBFS close to the optimal solution will reduce the computation time and number of iterations to be performed in the second stage (Sirvi et al., 2011) to find the optimal solution of the problem. The relative percentage deviations of the test problems obtained from each method are given in **Table 9**. The average relative percentage of deviation for different heuristics are given in the last row of the table. We observe that the proposed method is the second-best performer after modified VAM (Pradhan and Biswal, 2015) for the problems considered in the experiment.

Table 7. Details of the test problems and their optimal solutions.

Problem	Data source	Number of customers	Number of suppliers	Problem size	Optimal value
Pro1	Russell (1969)	5	5	10 × 10	0.2949171
Pro2	Shafaat and Goyal (1988)	4	6	10 × 10	0.06582403
Pro3	Kirca and Satir (1990)	3	4	7 × 7	0.392514395
Pro4	Mutlu et al. (2022)	4	4	8 × 8	0.03547586
Pro5	Ahmed et al. (2016)	3	4	7 × 7	0.04197531
Pro6		3	4	7 × 7	0.4435068
Pro7		4	4	8 × 8	0.06637168
Pro8	Tze-San (1986)	3	3	6 × 6	0.04749119
Pro9	Karagul and Sahin (2020)	5	5	10 × 10	0.3805142
Pro10	Soydan et al. (2023)	5	5	10 × 10	0.9559556
Pro11	Random	3	3	6 × 7	1.732678
Pro12	Random	3	6	10 × 9	1.987512927
Pro13	Random	4	3	7 × 8	1.142019
Pro14	Random	3	4	7 × 8	2.14745
Pro15	Random	3	3	7 × 6	2.143285
Pro16	Random	3	6	10 × 9	1.614801847
Pro17	Random	3	3	6 × 7	2.104338
Pro18	Random	3	4	7 × 8	1.610843
Pro19	Random	3	4	7 × 8	1.674791859
Pro20	Random	4	3	7 × 8	1.994896
Pro21	Random	4	4	8 × 9	1.990392666
Pro22	Random	3	6	9 × 10	1.862482052
Pro23	Random	3	5	8 × 9	2.074266612
Pro24	Random	3	4	7 × 8	1.945059223
Pro25	Random	3	5	8 × 9	1.951150313
Pro26	Random	4	6	10 × 11	1.963354123
Pro27	Random	4	3	7 × 8	1.963338
Pro28	Random	4	5	10 × 9	1.960253504
Pro29	Random	4	3	7 × 8	2.00219
Pro30	Random	4	4	9 × 8	1.927028
Pro31	Random	4	5	9 × 10	1.856282594
Pro32	Random	4	4	8 × 9	1.716677483
Pro33	Random	3	5	8 × 9	1.416936317
Pro34	Random	4	6	11 × 10	1.941616271
Pro35	Random	3	4	7 × 8	1.78139715
Pro36	Random	4	3	7 × 8	1.665374
Pro37	Random	3	5	8 × 9	1.699302891
Pro38	Random	4	3	7 × 8	1.699356
Pro39	Random	4	6	10 × 11	1.676518572
Pro40	Random	4	6	11 × 10	1.654524568

Table 8. Objective function's values for IBFS obtained by different methods.

Problem	NWCM	LCM	MRM	VAM	Modified VAM (Sirvi et al., 2011)	Modified VAM (Pradhan and Biswal, 2015)	AMCM
Pro1	1.73193	0.761356	0.813482	0.760786802	0.802743	0.713508613	0.754058
Pro2	0.199359	0.12043	0.137868	0.136363636	0.121996	0.122201493	0.125757
Pro3	1.404553	0.413408	0.414894	0.413407821	0.512821	0.414893617	0.414894
Pro4	0.188389	0.072937	0.061605	0.067823344	0.067823	0.054716611	0.066805
Pro5	0.178276	0.065095	0.06051	0.073529412	0.06051	0.057677319	0.062281
Pro6	1.108632	0.549969	0.549969	0.508266993	0.547489	0.547489413	0.538813
Pro7	0.190989	0.122881	0.122581	0.113939394	0.141026	0.079051383	0.118684
Pro8	0.096861	0.065126	0.058623	0.094293194	0.058553	0.058170962	0.06171

Table 8 continued…

Pro9	2.025761	0.946083	0.926934	0.534238823	0.757537	0.532079112	0.618839
Pro10	2.343587	1.434149	1.589281	1.786211067	1.507919	1.439893617	1.371383
Pro11	2.433579	2.3711	2.362249	1.906764344	1.906764	1.815778646	1.732678
Pro12	2.274084	2.099846	2.043278	2.11594962	2.13285	2.028442116	2.048091
Pro13	3.020539	1.625345	1.625345	1.520376792	1.520377	1.520376792	1.179492
Pro14	2.487469	2.445064	2.4097	2.436775654	2.334663	2.286111509	2.237636
Pro15	2.87081	2.218373	2.218373	2.218372857	2.2177	2.213141534	2.241344
Pro16	2.449132	2.509277	1.994649	2.496506959	2.100095	1.989674436	1.704133
Pro17	2.591478	2.694491	2.526115	2.669865931	2.46939	2.473170222	2.221503
Pro18	2.531464	1.945741	1.887905	1.944131659	1.893512	1.935764989	1.70152
Pro19	2.271565	2.784023	2.651662	2.38568761	2.385688	1.950847527	1.771352
Pro20	2.323161	2.19027	2.089707	2.267834113	2.141447	2.178850076	2.111966
Pro21	2.28746	2.20892	2.129022	2.33659732	2.197521	2.283373541	2.108779
Pro22	2.585845	2.268646	2.244842	2.024593674	2.164261	1.874418479	1.999951
Pro23	2.614645	2.199292	2.187248	2.256012814	2.213307	2.158783165	2.228277
Pro24	2.415764	2.205139	2.14292	2.063044744	2.063045	2.187707957	2.09173
Pro25	2.355293	2.31876	2.303997	2.13751212	2.391666	2.003612869	2.099606
Pro26	2.30542	2.331396	2.232537	2.338139788	2.205581	2.09479983	2.118457
Pro27	2.50299	2.360691	2.45725	2.057503543	2.154105	1.973006524	2.126543
Pro28	2.488989	2.342803	2.128711	2.242017859	2.165886	2.107867331	2.124065
Pro29	2.479793	2.346556	2.295504	2.296084104	2.296084	2.302071113	2.171247
Pro30	2.464094	2.430009	2.126783	2.454358261	2.231914	2.122961224	2.0899
Pro31	2.389029	2.389595	2.326611	2.132717814	1.96633	1.966330007	2.018517
Pro32	2.166096	2.427395	2.260495	2.131079414	2.393028	2.013971128	1.883733
Pro33	3.076709	2.048467	1.687322	1.741115643	1.759159	1.773758338	1.556882
Pro34	2.655775	2.555957	2.164503	2.414274224	2.181326	2.189409669	2.133522
Pro35	3.731177	2.138335	1.881284	1.937546156	1.912156	1.908151691	1.982972
Pro36	2.290419	1.992561	1.881602	1.937608036	1.848371	1.823565669	1.856142
Pro37	1.974873	2.051047	1.880886	2.08559872	1.911406	1.849004835	1.897381
Pro38	2.372701	2.195649	2.195649	2.715215311	2.698051	2.048311261	1.897589
Pro39	2.498863	1.94437	1.846753	1.796875935	1.878746	1.834683746	1.884151
Pro40	2.401774	2.624907	1.961537	2.596905161	2.292885	1.949714975	1.870855

Table 9. Relative percentage deviation (RPD) from the optimal solutions.

Problem	NWCM	LCM	MRM	VAM	Modified VAM (Sirvi et al., 2011)	Modified VAM (Pradhan and Biswal, 2015)	AMCM
Pro1	487.26	158.1593	175.834	157.9663	172.1929	141.9353	155.6847
Pro2	202.8663	82.95766	109.4488	107.1639	85.33703	85.64876	91.04962
Pro3	257.8349	5.322971	5.701503	5.322971	30.65012	5.701503	5.701503
Pro4	431.0347	105.5952	73.65212	91.18168	91.18168	54.23618	88.31242
Pro5	324.717	55.08021	44.15511	75.173	44.15511	37.40773	48.37461
Pro6	149.9696	24.00454	24.00454	14.60185	23.44555	23.44555	21.48918
Pro7	187.7571	85.14125	84.68818	71.66869	112.4786	19.10409	78.81708
Pro8	103.955	37.13322	23.43954	98.54881	23.29246	22.4879	29.94024
Pro9	432.3745	148.6327	143.6004	40.39918	99.08242	39.8316	62.6323
Pro10	145.1565	50.02257	66.25048	86.85084	57.73939	50.62348	43.45679
Pro11	40.45188	36.84599	36.33513	10.04724	10.04724	4.796081	1.54E-05
Pro12	14.41856	5.651954	2.805793	6.462182	7.312494	2.059317	3.04792
Pro13	164.4911	42.32209	42.32209	33.1306	33.1306	33.1306	3.281302
Pro14	15.83363	13.85895	12.21218	13.47299	8.717937	6.457031	4.199668
Pro15	33.94441	3.5034	3.5034	3.5034	3.472026	3.259321	4.575156
Pro16	51.66764	55.39224	23.52283	54.60144	30.05281	23.21477	5.532013
Pro17	23.14932	28.04459	20.04321	26.87439	17.34761	17.52723	5.567769
Pro18	57.15152	20.79024	17.1998	20.69033	17.54789	20.17093	5.629153
Pro19	35.63268	66.231	58.32784	42.44681	42.44681	16.48298	5.765472
Pro20	16.45523	9.793683	4.752657	13.68182	7.346286	9.221236	5.868462
Pro21	14.92505	10.97911	6.964905	17.39379	10.40643	14.71975	5.947911

Table 9 continued…

Pro22	38.83867	21.80768	20.52959	8.704064	16.20307	0.640888	7.380959
Pro23	26.05153	6.027439	5.446817	8.76195	6.703093	4.074527	7.4248
Pro24	24.20002	13.37131	10.17246	6.065909	6.065909	12.47513	7.540701
Pro25	20.71306	18.84065	18.08405	9.551381	22.57722	2.688801	7.608616
Pro26	17.42254	18.74558	13.71036	19.08905	12.33741	6.694957	7.899873
Pro27	27.48644	20.23865	25.15674	4.796196	9.716438	0.492453	8.312631
Pro28	26.97283	19.51531	8.593634	14.37387	10.49011	7.530344	8.356633
Pro29	23.85404	17.19945	14.64965	14.67863	14.67863	14.97766	8.443629
Pro30	27.87015	26.1014	10.36597	27.36495	15.82159	10.16764	8.451982
Pro31	28.69962	28.73011	25.33709	14.89187	5.928376	5.928376	8.739771
Pro32	26.17953	41.40077	31.67846	24.13977	39.39884	17.31797	9.731352
Pro33	117.1382	44.57013	19.08244	22.87889	24.15228	25.18264	9.876636
Pro34	36.78164	31.64071	11.47947	24.34353	12.3459	12.76222	9.883793
Pro35	109.4523	20.03698	5.607232	8.765536	7.340242	7.115457	11.31557
Pro36	37.53181	19.64647	12.98378	16.34672	10.98837	9.498867	11.45494
Pro37	16.21663	20.69929	10.68575	22.73261	12.48176	8.809609	11.65642
Pro38	39.62354	29.20476	29.20476	59.77908	58.76903	20.53456	11.6652
Pro39	49.05074	15.97665	10.15402	7.179006	12.06235	9.434144	12.38472
Pro40	45.164	58.65025	18.55592	56.95779	38.58274	17.8414	13.07507
Average RPD	98.25734	37.94666	32.00607	34.06458	31.60067	20.64073	21.40192

Table 10. Descriptive statistics of the percentage deviation (PD).

Method	Minimum RPD	Q_1	Median (Q_2)	Q_3	Maximum RPD
NWCM	14.41856	25.58865	38.18524	124.1427	487.26
MCM	3.5034	18.35905	25.05297	45.93324	158.1593
MRM	2.805793	10.31759	18.81918	32.84263	175.834
VAM	3.5034	9.923277	19.88969	45.48547	157.9663
Modified VAM (Sirvi et al., 2011)	3.472026	10.31663	16.77534	38.78676	172.1929
Modified VAM (Pradhan and Biswal, 2015)	0.492453	6.635475	13.74099	22.66962	141.9353
AMCM	1.54×10^{-05}	5.842714	8.447805	12.55731	155.6847

To check the consistency of the proposed method for LFTP, we complete the descriptive analysis of the relative percentage deviation of IBFS from the optimal solution. The summary statistics is given in the **Table 10**. From **Table 10**, we can conclude that the proposed method produces a very consistent result. In fact, for the considered problems, the proposed methodology produces the most consistent result.

To analyse the time complexity of the proposed algorithm, we do an analysis of the computational time taken for producing the IBFS for different problems. The details of the computation time taken by different methodologies for finding IBFS are presented in **Table 11**. Last row of the table indicates the average time taken for finding IBFS of a problem. Here, we observe that the proposed method takes less average computation time than the computation time taken by the methods based on VAM for finding an IBFS of a problem.

In the second phase of the experiment, we consider the problems with large number of sources and sinks. To do so, we set up an experimental design similar to the design defined in Mathirajan et al. (2022). The summary of this experimental design is given in **Table 12**. The experiments are conducted on the above-mentioned system configurations using the same software. We developed a MATLAB code for randomly generating problem instances using the given experimental design. For each combination of the problem factors: problem size (m x n) where m: number of sources and n: number of sinks, cost structure – range (CR), profit structure – range (DR), and degree of imbalance (K), we consider 10 problem instances which are randomly generated for performance evaluation of the heuristic methods pertaining to a fractional transshipment problem. In total 640 fractional transshipment problem instances are randomly generated following the experimental design and note that each of the randomly generated problems are unbalanced.

Table 11. Computation time (in 's') for finding IBFS using different heuristics.

Problem	NW rule	MCM	MRM	VAM	Modified VAM (Sivri et al., 2011)	Modified VAM (Pradhan and Biswal, 2015)	AMCM
Pro1	0.004644	0.007998	0.00789	0.008294	0.02662	0.034304	0.022887
Pro2	0.003928	0.00757	0.007007	0.006918	0.022119	0.027657	0.019725
Pro3	0.003693	0.007374	0.006972	0.006897	0.020403	0.028393	0.019138
Pro4	0.003599	0.007131	0.007051	0.007035	0.022596	0.027745	0.020044
Pro5	0.005806	0.011306	0.00764	0.00787	0.023795	0.042166	0.032827
Pro6	0.005428	0.008381	0.009766	0.010251	0.023703	0.027909	0.022118
Pro7	0.005333	0.011876	0.01315	0.009001	0.033666	0.034297	0.022337
Pro8	0.003916	0.008167	0.010596	0.010547	0.031095	0.027286	0.020515
Pro9	0.004135	0.007318	0.007901	0.007149	0.033776	0.050026	0.033147
Pro10	0.005269	0.010332	0.011722	0.012199	0.04046	0.049348	0.024093
Pro11	0.003708	0.006678	0.006458	0.006811	0.019307	0.020766	0.022281
Pro12	0.000008	0.000059	0.000085	0.000058	0.001337	0.002579	0.000824
Pro13	0.000007	0.000083	0.000083	0.000073	0.002071	0.003063	0.001376
Pro14	0.000007	0.000077	0.000065	0.000069	0.001427	0.002349	0.001226
Pro15	0.000007	0.000083	0.000073	0.000075	0.001857	0.003084	0.00265
Pro16	0.000007	0.000083	0.000069	0.000075	0.003022	0.00255	0.00142
Pro17	0.000008	0.000064	0.000058	0.000061	0.001069	0.001556	0.000824
Pro18	0.000402	0.00175	0.00231	0.001749	0.007877	0.017025	0.008762
Pro19	0.000012	0.000093	0.00009	0.000089	0.001308	0.001891	0.000981
Pro20	0.000007	0.000074	0.000057	0.000061	0.000862	0.001466	0.000691
Pro21	0.000008	0.000061	0.000052	0.000055	0.000932	0.001519	0.000774
Pro22	0.000008	0.00007	0.000064	0.000064	0.003455	0.002371	0.001067
Pro23	0.000009	0.000061	0.000053	0.000056	0.00101	0.001555	0.00081
Pro24	0.000164	0.000217	0.000209	0.000213	0.003734	0.002058	0.001106
Pro25	0.000008	0.000136	0.000055	0.000057	0.001014	0.001497	0.000763
Pro26	0.000007	0.000066	0.000061	0.000072	0.001154	0.002815	0.000986
Pro27	0.000008	0.00006	0.00006	0.000067	0.001076	0.001375	0.00081
Pro28	0.000009	0.000068	0.000059	0.000084	0.001858	0.001777	0.001128
Pro29	0.000008	0.00007	0.000056	0.00007	0.000932	0.001453	0.001603
Pro30	0.00001	0.000145	0.000053	0.000143	0.00095	0.002451	0.002073
Pro31	0.000008	0.000065	0.000074	0.000061	0.001173	0.001816	0.000951
Pro32	0.000008	0.000074	0.000065	0.000068	0.002554	0.002384	0.001997
Pro33	0.000052	0.000069	0.000062	0.000063	0.001196	0.002628	0.001083
Pro34	0.000007	0.000071	0.00006	0.000064	0.001202	0.001883	0.00095
Pro35	0.000007	0.000068	0.000062	0.000064	0.001155	0.001828	0.000959
Pro36	0.000007	0.000073	0.000062	0.000067	0.002499	0.002168	0.001187
Pro37	0.000007	0.000049	0.000048	0.000048	0.001257	0.004016	0.000679
Pro38	0.000008	0.000074	0.000066	0.000067	0.001905	0.007047	0.001161
Pro39	0.000007	0.00008	0.000073	0.000076	0.002163	0.004163	0.001884
Pro40	0.000006	0.000075	0.000068	0.00007	0.001389	0.003627	0.001222
Average computation time	0.001257	0.002453725	0.002420275	0.00877445	0.011397275	0.007526475	0.0070559

All the considered methodologies of the study are performed for each of the 640 problem instances. Then to measure the efficiency of the heuristics, we calculated average relative percentage error (ARPD) with respect to the optimal solution. The ARPD score over 320 instances for every cost structure range (CR), profit structure range (DR), and degree of imbalance (K) are presented graphically in **Figures 2(a)** to **2(f).** The ARPD scores over 640 instances (that is irrespective of the cost structure or any other factor) for different heuristics are calculated and presented graphically in **Figure 3**. The average computational time to obtain IBFS of the problems for different heuristics are presented in **Figure 4**.

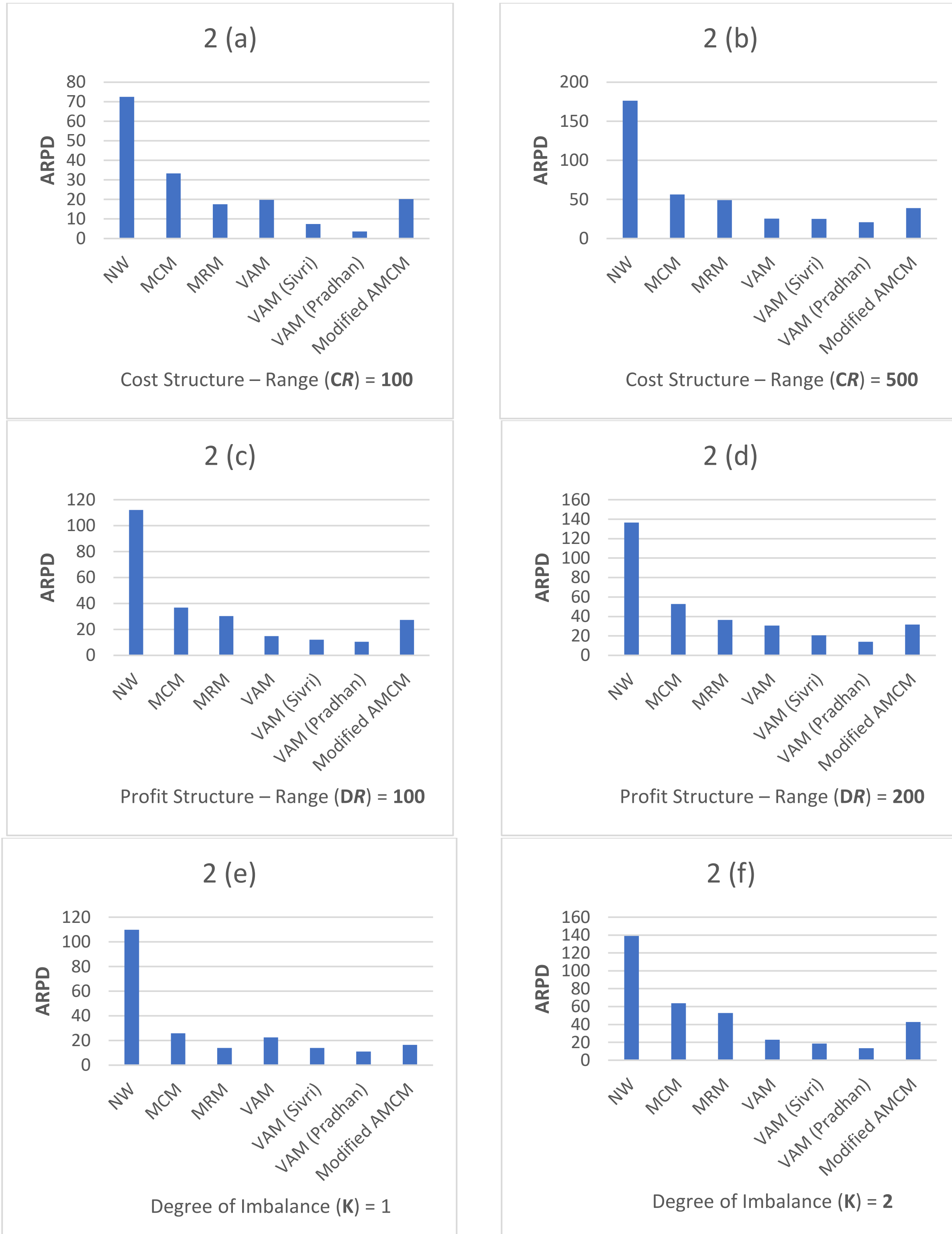

Figure 2. Average performance of the heuristics with respect to the optimal solution.

Table 12. Summary of the experimental design.

Problem factor	No. of levels	Values
Problem size ($m \times n$)	8	{10 × 10; 10 × 20; 10 × 40; 10 × 60} {20 × 10; 20 × 20; 20 × 40; 20 × 60}
Cost structure – range ($\boldsymbol{CR}$)	2	{100, 500}
Profit structure – range ($\boldsymbol{DR}$)	2	{100; 200}
Degree of imbalance (**K**)	2	{1, 2}
Number of problem configurations		(8 x 2 x 2 x 2) = 64
Number of instances per configuration		10
Number of problem instances		640
Cost Structure ($\boldsymbol{C_{ij}}$): Uniform Distribution: $U(\boldsymbol{C_{ij}}: Mean\ Cost\ -\ \boldsymbol{CR}/2, Mean\ Cost\ +\ \boldsymbol{CR}/2)$ Profit Structure ($\boldsymbol{D_{ij}}$): Uniform Distribution: $U(\boldsymbol{D_{ij}}: Mean\ Profit\ -\ \boldsymbol{DR}/2, Mean\ Profit\ +\ \boldsymbol{DR}/2)$ where Mean Cost = 500 and Mean Profit = Supply ($\boldsymbol{a_i}$): Uniform Distribution: $U(\boldsymbol{a_i}: 0.75\ \times\ Mean\ Supply, 1.25\ \times\ Mean\ Supply)$, where Mean Supply = [($\boldsymbol{K} \times \boldsymbol{n} \times$ Mean Demand)/$\boldsymbol{m}$] and Mean Demand = 100 Demand (b_j): Uniform Distribution: $U(\boldsymbol{b_j}: 75, 125)$		

From the graphs, it is clear that the proposed heuristic is a very consistent method to find initial basic feasible solution of a fractional transshipment problem. In the first phase i.e., for the smaller problem instances, the modified AMCM was the second-best performer but in this phase, we observe that for the larger problem instances the method's performance is just behind the variations of VAM. To get a better view the comparison, we present a graphical representation of the performances of the heuristics with respect to the ARPD and computational time (in millisecond) in **Figure 5**. From the graph it is observed that, with respect to the ARPD and computational time trade-off, the proposed modified AMCM is a very good heuristic along with the VAM (Pradhan and Biswaal, 2015) to find an initial basic feasible solution of a fractional transshipment problem.

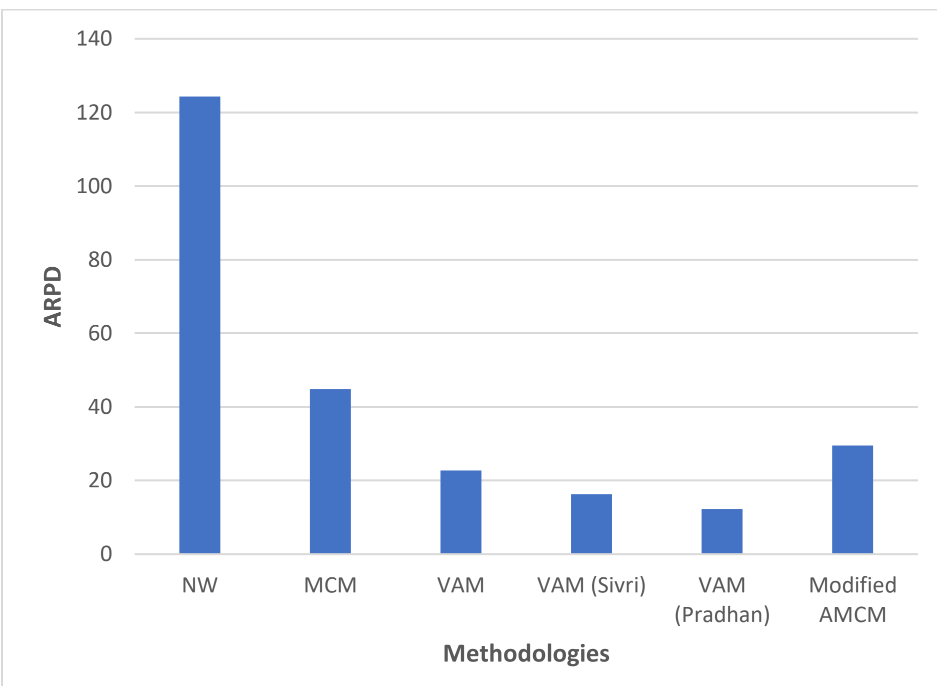

Figure 3. Average relative percentage deviation for different heuristics.

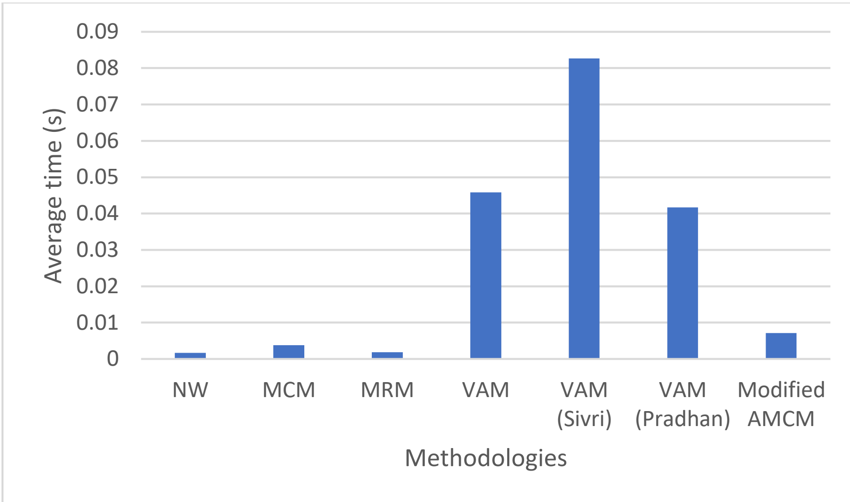

Figure 4. Average computational Time for different heuristics.

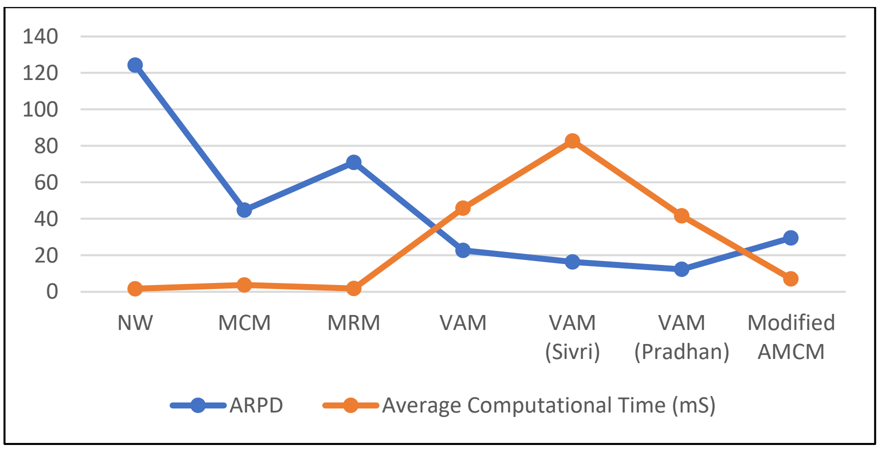

Figure 5. Performance evaluation of the heuristics.

6. Conclusion

In this study, we have investigated a special class of LFTP and its mathematical model. Moreover, we have studied about the matrix formulation of LFTP. It has been observed that LFTP is optimized by taking ratio of two linear functions subject to some linear constraints. This fact led us to investigate a single objective linear fractional problem for the transshipment problem. For this purpose, we have proposed an algorithm based on 'AMCM' for finding an IBFS of the LFTP. To demonstrate the process of proposed method, we have considered a fractional transshipment problem with two sources and three destinations. The obtained IBFS is compared with the solution obtained by other existing methods which establishes the validity and feasibility of the proposed method.

Regarding the future line of research, we will develop an efficient algorithm for optimizing the LFTP with multi-objective linear fractional functions. It will be interesting to apply this method to various real-life decision-making situations.

Conflict of Interest
The authors confirm that there is no conflict of interest to declare for this publication.

AI Disclosure
The author(s) declare that no assistance is taken from generative AI to write this article

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Appendix

```
function [X, v, N, D] = amcm(c,d,A,B)
X = zeros(size(c));
tot_dem = sum(B);
r = c./d; % Ratio matrix
temp = r;
tempc = c;
tempd = d – eye(size(d));
iteration = 0;
while(tot_dem>0)
    iteration = iteration+1;
    KE=max(max(r));
    [KR, KC]=find(r==KE);
    [row_ind, col_ind]=selection1(r,KR,KC);
    alloc=min(A(row_ind),B(col_ind));
    A(row_ind)=A(row_ind)-alloc;
    B(col_ind)=B(col_ind)-alloc;
    if (B(col_ind)==0 && A(row_ind)==0)
        r(row_ind,:)=NaN;
        r(:,col_ind)=NaN;
    elseif(A(row_ind)==0)
        r(row_ind,:)=NaN;
    elseif(B(col_ind)==0)
        r(:,col_ind)=NaN;
    end
    X(row_ind,col_ind)=alloc;
    tot_dem=tot_dem-alloc;
end
N = sum(sum(tempc.*X));
D = sum(sum(tempd.*X));
v = N/D;
end

function [KRA, KCA] = selection1(a,KR,KC)
%Find minimum of the all the selected rows and column
miniDetail = [];
secondMax = -1;
for i = 1:length(KR)
```

```
    s1 = min(a(KR(i),:));
    s2 = min(a(:,KC(i)));
    s(i) = min(s1,s2);
    col_ind = find(a(KR(i),:)== s(i));
    row_ind = find(a(:,KC(i))== s(i));
    for j = 1:length(col_ind)
        miniDetail = [miniDetail; s(i) KR(i) col_ind(j)];
        colMax = max(a(:,col_ind(j)));
        if(colMax>secondMax)
            KRA = KR(i);
            KCA = col_ind(j);
        end
    end
    for j = 1:length(row_ind)
        miniDetail = [miniDetail; s(i) row_ind(j) KC(i)];
        rowMax = max(a(row_ind(j),:));
        if(isempty(col_ind)~=1)
            if(rowMax>colMax)
                KRA = row_ind(j);
                KCA = KC(i);
            end
        else
            if(rowMax>secondMax)
                KRA = row_ind(j);
                KCA = KC(i);
            end
        end
    end
end
miniDetail;
KRA;
KCA;
end
```

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