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Reliability Analysis of Cutting Tools for Industrial Applications: An Integrated AHP-RSM-PHM Approach

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Abstract
In manufacturing industries, reliability analysis of cutting tools is of paramount importance, as their frequent failures may result in enhanced downtime of production lines, leading to reduced throughput, enhanced process cycle times, and low profits. There are numerous factors that govern the desired operations of cutting tools, e.g., tool cutting speed, feed, depth of cut, and many others. Existing literature on cutting tools' reliability estimation emphasizes mainly three variables, as mentioned earlier while neglecting other important factors. Including a greater number of factors in the process of estimating reliability increases the number of covariates, hence rendering the data acquisition costlier and estimation models highly complex. This work initially utilizes Analytical Hierarchy Process (AHP) to assess the importance of various factors that are responsible for the cutting tool's performance, followed by the reliability estimation of the cutting tools using proportional hazards model (PHM) considering the four "critical to reliability" factors as obtained through AHP as covariates. The proposed method also helps in determining the relationship of these sub-factors with the hazard rate and reliability of the cutting tools. Experimental results are then used to verify the model's predictions through response surface methodology (RSM) and Weibull fit. Furthermore, the paper also presents a proposed technique to estimate the required number of cutting tools for one machine per day and the number of job completions that can be an essential takeaway for various industries. Thus, this research paper proposes an integrated AHP-RSM-PHM based approach for a comprehensive reliability analysis of cutting tools.

Keywords- Cutting tools, Reliability, Analytic hierarchy process, Proportional hazard model, Response surface methodology.

1. Introduction
In recent decades, machining has been a vital technique in aerospace, automobile, defence, and railways industries for producing various products through cutting tools (Cheng et al., 2017). Cutting tools are an essential element of manufacturing processes, used to shear off the workpieces into specific shapes and sizes. During shear-off, the cutting tools experience wear and tear, which reduces their reliability and useful

life (Bayraktar, 2021; Tang et al., 2023). The sudden failures and extreme wear of the cutting tools can give rise to challenges that can cause the depletion of valuable resources, such as time and costs. According to a study conducted by Sakharov et al. (1990), it was demonstrated that the cost of tooling in flexible manufacturing systems accounts for approximately 25% of the overall machining expenses. Wiklund (1998) found that cutting tools are frequently replaced when they reach 50-80% of their actual life to prevent failures and their associated consequences, rather than using the cutting tools until they reach the end of their actual lifespan. Hence, it is vital to know the actual life of the cutting tools to reduce distortion of the workpieces and tools' wastage before the cutting tools fail. The reliability and tool life of the cutting tools is highly influenced by the machining parameters, tool materials, operating temperature, and coatings used to manufacture the tool insert (Upadhyaya, 2004). Hence, it is critical to establish a framework of guidelines to determine a series of variables affecting cutting tool reliability, as deviations from prescribed operating parameters might lead to poor process and product quality. After that, to achieve dimensional accuracy and high-quality machined surfaces, it is crucial to estimate and analyze the reliability of the cutting tools.

Wager and Barash (1971) conducted over a hundred tool life tests with a high-speed steel turning tool, revealing that the tool life values exhibited a normal distribution. Klim et al. (1996) estimated the tool life and reliability function for the constant and variable feed cases but did not consider cutting speed and depth of cut (DOC). Lin (2008) derived reliability function with the help of tool wear limit, cutting speed, feed, and DOC. However, there are various other external and internal factors that may affect the mechanical system's reliability and tool life. Some researchers enlisted such factors affecting cutting tools' performance, e.g., Wang et al. (2016) proposed a selection method by calculating energy consumption influenced by cutting speed, undeformed chip thickness, and tool rake angle. Gaddafee and Chinchanikar (2020) modelled the reliability function with gamma and Weibull distributions by considering speed, feed, and DOC as factors. Niu et al. (2020) consolidated ten attributes of tool materials, including physical, mechanical, chemical, and cost parameters. Rao (2022) investigated the reliability of cutting tools in plasma-assisted turning and explored the prediction of machining characteristics by collecting data on tool wear, surface roughness, and other relevant parameters. Le (2022) performed the experimental study using three cutting tools' parameters, namely the number of pieces, cutting piece material, and tip radius, considering three cutting mode parameters, i.e., cutting speed, feed rate, and DOC variation in each experiment. Zhang et al. (2023) investigated the reliability of CNC machine tools concerning machining accuracy by considering geometric and vibration errors. Therefore, to obtain accurate reliability estimates, it is crucial to consider various factors that can influence the failure process (Bazaz et al., 2023a; Bazaz et al., 2023b). The Proportional Hazards Model (PHM) is suitable for this purpose, as it can analyze the relationship between failure time and multiple predictor variables (Sharma et al., 2022). Using this model, we can determine which factors impact the likelihood of failure and how they affect failure rates over time. This information can help us design more reliable cutting tools and minimize the risk of failure.

Some researchers have used PHM to estimate the reliability of the cutting tools. Liu and Makis (1996) utilize PHM to analyze the reliability of cutting tools at different cutting speeds, feed, and DOC by determining the distribution of failure times of the cutting tools. Equeter et al. (2016) investigated the Cox PHM for predicting the lifespan of the cutting tools by considering cutting speed as a covariate. The literature is limited in exploring more variables and considering only three variables, namely cutting speed, feed, and DOC, which affect tool wear when estimating reliability. However, there is still a scope to include additional co-variates such as cutting tool thermal conductivity, cutting edge angle, hardness, and machinability, among others (Bazaz et al., 2023a).

1.1 Problem Statement

The problems leading to under or overestimation of reliability, while conducting reliability studies, arise because of neglecting crucial factors that affect life of cutting tools. Including more number of factors for reliability analysis may increase experimental setup cost and require significant amount of data. Hence, undertaking qualitative studies before reliability estimation is essential to determine which factors impact tool lifespan most. Additionally, there is a limited application of Multi-Criteria Decision-Making (MCDM) techniques to evaluate factor weights in case of unavailability of cutting tools failure data, which can be further explored to evaluate the cutting tools' performance. Thus, there is a need to establish the effect of prominent factors on tool life through MCDM models to further develop a reliability estimation model whose performance can be validated with the help of advanced experimental design techniques. Addressing this problem will lead to more precise reliability assessments, benefiting industries reliant on predictive maintenance and system longevity. Furthermore, the absence of validated analytical models for cutting tool inventory estimation hinders efficient resource allocation and may lead to overstocking or shortages. Solving this problem will enhance manufacturing efficiency, reduce costs, and ensure optimal tool availability for production processes.

1.2 Research Gaps

The literature review reveals the following research gaps in the domain of reliability analysis of the cutting tools:

(i) The literature is limited in performing the qualitative analysis for prioritizing the factors that may affect the tool's life and reliability. Non-consideration of such factors may underestimate or overestimate cutting tools' reliability, leading to imprecise tool life prediction.
(ii) Most of the papers utilized a maximum of three factors for estimating reliability through PHM since studies related to qualitative analysis for identifying various factors affecting the tool's life are found limited in the literature, as mentioned in point 1.
(iii) The literature has primarily utilized the PHM model to estimate reliability parameters with only three factors as covariates. The inclusion of the fourth covariate in PHM after qualitative investigation, along with the validation of the estimated parameters with the use of advanced experimental design models, is found to be limited.
(iv) The literature is also found limited in estimating the inventory of cutting tools required by using the parameters obtained with the help of the analytical model that uses important covariates duly validated with the help of experimental design models.

1.3 Aim and Objectives

To address the research gaps mentioned above, this paper endeavors to propose an integrated Analytic Hierarchy Process - Response Surface Methodology - Proportional Hazard Model (AHP-RSM-PHM) based approach for a comprehensive reliability analysis of cutting tools.

The aim of the research paper is to precisely estimate and enhance the life and reliability of the cutting tools. This is achieved by initially identifying and prioritizing factors influencing tool lifespan through Analytic Hierarchy Process (AHP) followed by integration of such factors into a Proportional Hazard model for reliability estimation, validated experimentally with the help of Response Surface Methodology (RSM). Ultimately, the established reliability model is utilized to determine optimal job completions and cutting tool requirements. Towards achieving the aim of this research work, four objectives are formulated, as stated below:

(i) To identify and prioritize the factors that influence cutting tool lifespan using the Analytic Hierarchy Process (AHP).
(ii) To develop a model using PHM that can estimate the reliability of cutting tools by considering the

prioritized factors as covariates.

(iii) To develop an advanced experimental design model, such as Response Surface Methodology (RSM), that can be used to validate the parameters as obtained from the analytical reliability model as mentioned in point 2.

(iv) To determine the number of job completions and cutting tools required per machine tool in a day by application of the developed reliability model duly validated with the help of an advanced experimental design model for industrial applications.

To achieve the first objective, an extensive literature review (Weinert and Kempmann, 2004; Ezugwu et al., 2005; Musfirah et al., 2017) is carried out supported by a series of industrial visits to understand and finally arrive at seventeen factors that affect the reliability of the cutting tools. It is worthwhile mentioning here that no such precedence is found in the literature for comprehension of factors affecting the reliability of the cutting tools. After establishing the factors, an AHP model is utilized to assess the relative importance of each factor through pair-wise comparisons. This model allows decision-makers to model relative probabilities and risk-adjusted values, providing flexibility in assessing various scenarios. Unlike other MCDM methods, AHP does not impose specific time scales or geographical limitations (Hontoria and Munier, 2021). However, it carries limitations, assuming a second-degree polynomial, which might not always be accurate, presenting an approximation that may not precisely represent the actual relationship. The study prioritizes four significant "critical to reliability" factors out of the identified seventeen factors that affect the reliability of cutting tools using AHP.

In the second objective, prioritized factors identified by the AHP are chosen as covariates for the development of a PHM model that can estimate the reliability, hazard rate, and tool life. Then, the unknown parameters of the PHM are determined using the maximum likelihood estimation method. The PHM Model offers advantages such as not assuming a specific distribution for survival times, suitability for censored data, and the ease of interpreting hazard ratios. This makes it a robust option for model the reliability of cutting tools, particularly when dealing with complex and dynamic industrial scenarios (Harrell, 2015). The ability to handle censored data aligns well with the often-incomplete nature of reliability studies in practical applications, providing a more realistic representation of the survival experience of cutting tools in a manufacturing environment, making it a sound choice compared to the other traditional reliability estimation models. Nonetheless, its results can be biased if the proportional hazards assumption is violated, and it does not directly estimate the survival function.

Moreover, in the next step, RSM and Weibull fit are employed to validate the estimated values of the unknown parameters in the PHM. RSM is advantageous for not assuming a specific distribution, requiring fewer experiments, and aiding in the identification of variable interactions (Khuri and Mukhopadhyay, 2010). However, its reliance on a second-degree polynomial model as an approximation introduces the risk of bias if the assumed model class is incorrect.

In consideration of the industrial applications outlined in the fourth objective of this research paper, the developed methodology proposes model to determine the daily inventory of cutting tools per machine tool by applying the proposed model's median life and comparing it to experimental time-to-failure data. Hence, the research contribution of this paper lies in developing an AHP-RSM-PHM based integrated approach for the estimation of tool life by including the prioritized factors as covariates for reliability estimation.

The paper is structured into four sections. The second section outlines the methodology and algorithms used in the study. The third section presents the results and discussions based on the findings. Next, the fourth section states the findings and real industry applications. Finally, the fifth section concludes the paper by summarizing the key contributions of the research.

2. Proposed Methodology

This section outlines the proposed methodology to better understand the anticipated approach for prioritizing factors, reliability estimation, validation, and inventory estimation while considering "critical to reliability" factors. The flow chart of the proposed methodology is illustrated in **Figure 1**. The first step of this flow chart initiates a thorough qualitative study of the factors that impact cutting tools' performance and tool life with the help of an extensive literature review (Weinert and Kempmann, 2004; Ezugwu et al., 2005; Lee and Lee, 2009; Musfirah et al., 2017; Bazaz et al., 2023a) and investigative studies with the help of a series of industrial visits, which led to the identification of seventeen such factors and sub-factors. Then, four sub-factors deemed "critical to reliability" are prioritized as per the weights obtained through pairwise comparisons, considering their impact on the reliability of the cutting tools through the Analytic Hierarchy Process (AHP). Furthermore, these are selected as covariates for the Proportional Hazard Model (PHM) to estimate reliability, hazard rate, and tool life using Maximum Likelihood Estimation (MLE). The estimated parameters are then validated using Response Surface Methodology (RSM) on previously conducted experimental data. Finally, the methodology enables the estimation of reliability, median life, number of job completions, and number of cutting tools required per day of the working period for a machine tool, which can be used for industrial applications.

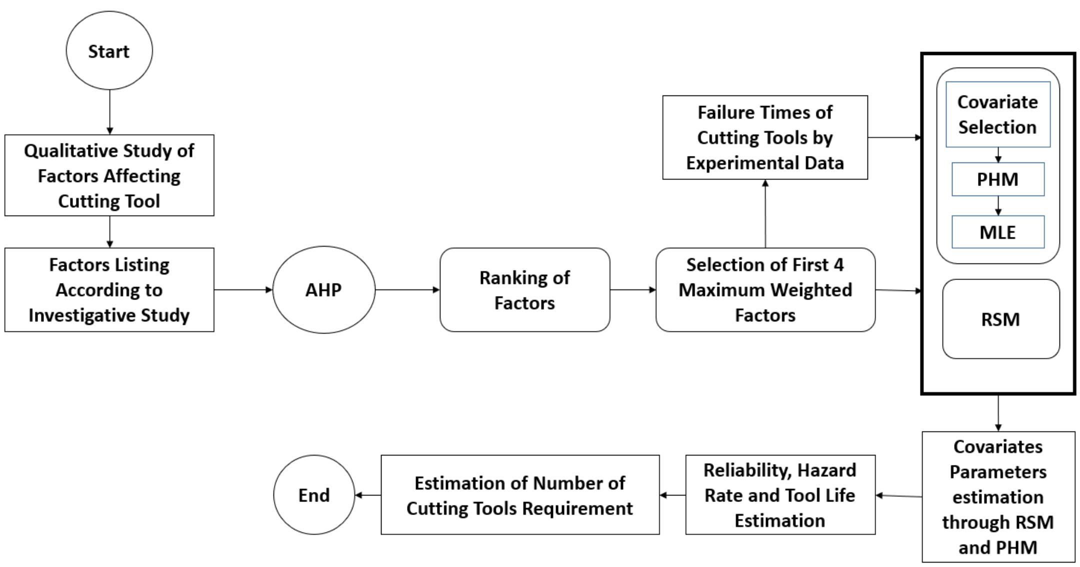

Figure 1. Flowchart of the proposed methodology.

This section is divided into six sub-sections. Section 2.1 discusses the AHP and the factors that impact the reliability of cutting tools with their hierarchical framework. The subsequent step is to create the PHM model, as illustrated in Section 2.2. The MLE method for determining the unknown parameters of the PHM model is covered in Section 2.3. In Section 2.4, the experimental data of the cutting tool has been explained. The design of the experiment using the RSM is further discussed in Section 2.5. Finally, Section 2.6 explains the methodology for determining the number of jobs or workpiece completion and the optimal number of cutting tools required for machining.

2.1 Factors Listing and Hierarchical Framework for AHP

The analytic hierarchy process is a decision-making tool developed by Saaty (Whitaker, 1987), which helps in breaking complex problems into simple criteria. The AHP relies on three fundamental principles: problem decomposition, comparative judgment, and relative importance or rankings synthesis. The associated goal, categorial factors, and sub-factors affecting the reliability of the cutting tools, as identified through extensive surveys, are organized into a hierarchical structure, as shown in **Figure 2**, with their respective acronyms. In this Figure, categorial factors are Cutting Parameters (CP), Cutting Tools Properties (CTP), Cutting Tools Geometry (CTG), Workpiece Properties (WP), and lastly External and Environmental factors (EE). These categorical factors (CF) are enlisted with their respective sub-factors (SF), which are Cutting Speed (CS), Depth of Cut (DOC), Feed Rate (FR), Hardness (HR), Toughness (TH), Thermal Conductivity (TCO), Strength (SH), Rake Angle (RA), Nose Radius (NR), Cutting Edge Angle (CEA), Workpiece Hardness (WH), Workpiece Surface finish (WS), Workpiece Machinability (WM), Lubricants (LU), Environmental Temperature (ET), Environmental Vibration (EV) and Cutting Cost (CC).

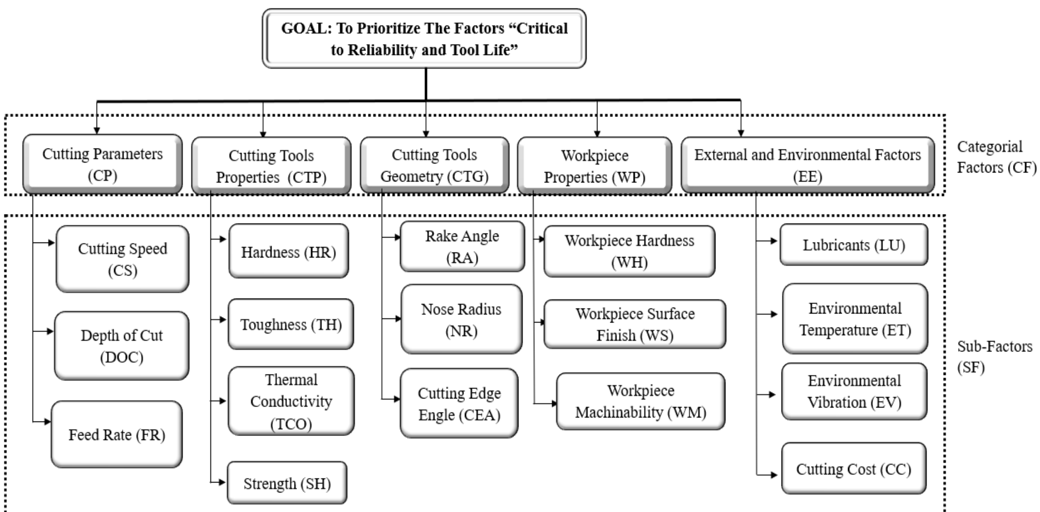

Figure 2. Hierarchy of the AHP model.

Further to this, pair-wise comparisons of all seventeen factors and sub-factors are performed. The pairwise comparison involves assigning values to express the relative importance or preference between two elements or factors, as shown in **Table 1** for categorical factors. Typically, a numerical scale, such as the Saaty scale (1 to 9), is employed, where a higher value indicates a stronger preference or greater importance for the factor on the left compared to the one on the right. Pair-wise comparisons for other sub-factors are executed in similar manners. The consistency ratio (CR) values have been found well within limits i.e., less than 0.1.

Table 1. Comparison matrix for CF.

	CP	CTP	CTG	WP	EE
CP	1	2	4	3	5
CTP	0.5	1	2	1	3
CTG	0.25	0.5	1	0.33	0.5
WP	0.33	1	3	1	3
EE	0.2	0.33	2	0.33	1

The next step of the AHP approach is to normalize the matrix by dividing each element by the sum of its column. The eigenvectors found from the pair-wise matrix are normalized so that its elements sum to 1. These normalized values represent the relative importance of the elements in the given level. Finally, this process is repeated for each level in the hierarchy for categorial factors and sub-factors. At last, the local weights for each element are combined across all levels to obtain the global weights. The weights of categorial factors and sub-factors so obtained are placed in **Table 2**. In this Table, the second column denotes the weights assigned to categorical factors, while the fourth column indicates the local weights assigned to sub-factors. On the other hand, the last column reflects the overall global weights assigned across all factors.

Table 2. Local and global weights of factors and sub-factors.

CF	Weight	SF	Local weight	Global Weight
CP	0.4298	CS	0.54995	0.23641
		DOC	0.24021	0.10326
		FR	0.20984	0.09020
CTP	0.2021	HR	0.27777	0.05616
		SH	0.36586	0.07397
		TCO	0.23256	0.04702
		TH	0.12381	0.02503
CTG	0.0868	RA	0.29696	0.02579
		CEA	0.16342	0.01419
		NR	0.53961	0.04686
WP	0.2051	WH	0.4126	0.0847
		WS	0.25992	0.0533
		WM	0.32748	0.0672
EE	0.0759	LU	0.27718	0.02105
		ET	0.09543	0.00725
		EV	0.16009	0.01216
		CC	0.4673	0.03549

Figure 3 illustrates the sub-factors arrangement in descending order of significance, determined by their corresponding global weights as outlined in **Table 2**. The results indicate that the cutting speed, DOC, feed rate, and workpiece hardness are the most crucial sub-factors that are "critical to reliability" for the cutting tool. The effects of the four most influencing factors are appended in **Table 3** after an investigative study. In addition to the enumeration of such factors, the evidence and testimony of their impact on the reliability of cutting tools are illustrated in the third column of **Table 3**. The experimental study further testifies these effects, and these four sub-factors are shortlisted to be used as covariates in the PHM model, as discussed in the subsequent section.

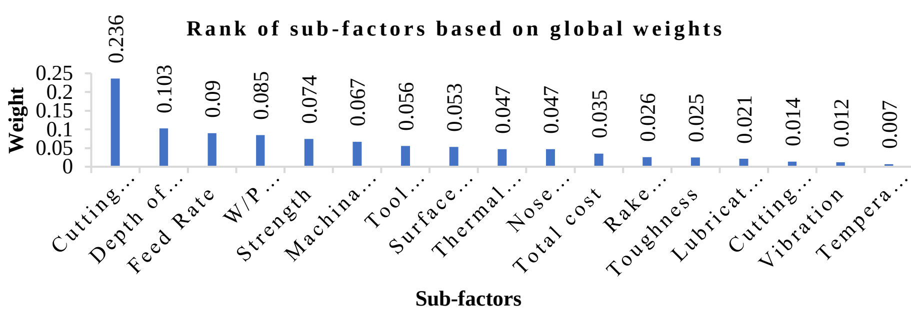

Figure 3. Ranking of sub-factors.

Table 3. Factors and sub-factors list.

CF	SF	Effects
CP	CS	Higher CS generates higher temperatures, accelerating the tool's wear and tear.
	DOC	Higher DOC leads to higher cutting forces and temperatures, speeding up the tools' deterioration.
	FR	High FR can lead to increased cutting temperatures, speeding up tool wear, while inadequate feed rates can cause tool failure by generating heat as the tool rubs against the workpiece.
WP	WH	Harder materials require more effort and energy to cut. Due to this stress and increased wear, the cutting tool may dull or break faster.

2.2 Cox Proportional Hazards Model

The proportional hazards model proposed by Cox (Therneau and Grambsch, 2000) has been utilized to model the tools' reliability and hazard functions. According to Cox, reliability depends not solely on time but also on external factors that may influence failure rates, as illustrated in Equation (1).

$$\lambda(t, X) = \lambda_o(t). g(X, A) \tag{1}$$

The baseline hazard rate is a time-dependent function, denoted as $\lambda_o(t)$. The function $g(X, A)$ is a positively valued entity that remains constant over time and includes the influence of multiple covariates. The covariates are represented as a row vector $X = (x_1, x_2, x_3 \ldots x_m)$, while the regression parameters are represented as a column vector $A = a_1, a_2, a_3, \ldots a_m$, where, "$m$" denotes the number of variables associated with stress.

In this study, Weibull distribution is used as a base function for estimating the hazard rate. The Weibull distribution is commonly employed in the reliability study of diverse systems because of its ability to adapt to different shapes of distribution (Ndlovu and Ayomoh, 2023). To confirm that the experimental data follows Weibull distribution, we have conducted a Goodness-of-Fit test in section 3.1. This distribution is characterized by certain parameters, which are used to determine the shape and characteristics of the hazard rate function. Equation (2) provides a hazard rate function for Weibull distribution.

$$\lambda_o(t) = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{(\beta-1)} \tag{2}$$

where, β represents the shape parameter, and θ the characteristic life of the Weibull distribution. The relationship between the dependent variable (tool life) and the independent variables of the process parameters (cutting speed v, feed f, DOC d, workpiece hardness h) is depicted by modified Taylor's tool life Equation (3).

$$TL = Cv^{\rho_1} f^{\rho_2} d^{\rho_3} h^{\rho_4} \tag{3}$$

The PHM used in this study includes Taylor's tool life equation to predict the likelihood of the cutting tools' failures based on the cutting parameters being used. Equation (4) provides the proposed formula to estimate the failure rate of the cutting tool.

$$g(X, A) = v^{\rho_1} f^{\rho_2} d^{\rho_3} h^{\rho_4} \tag{4}$$

where, X is a row vector consisting of the covariates $X =$ vector of (v, f, d, h) and A is a column vector consisting of the unknown parameters ρ_1, ρ_2, ρ_3 and ρ_4. Therefore, from Equations (1), (2), and (4), the PHM hazard rate function is reformed as illustrated in Equation (5).

$$\lambda(t) = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{(\beta-1)} v^{\rho_1} f^{\rho_2} d^{\rho_3} h^{\rho_4} \tag{5}$$

The hazard rate can be estimated by dividing the probability density function $f(t)$ by the survival or reliability function $R(t)$ as shown in Equation (6). The reliability function is placed at Equation (7).

$$\lambda(t) = \frac{f(t)}{R(t)} \tag{6}$$

$$R(t,X) = exp(-\int_0^t \lambda(u)du) \tag{7}$$

Further, Equation (8) can be derived by integrating Equation (7) over the time interval from *0* to *t*, which is the function of covariates and time.

$$R(t,X) = exp\left[\left(-(\frac{t}{\theta})^{(\beta)}\right) v^{\rho_1} f^{\rho_2} d^{\rho_3} h^{\rho_4}\right] \tag{8}$$

The $f(t)$ Equation (9) for cutting tools can be derived with the help of Equations (5), (6), and (8).

$$f(t,X) = \frac{\beta}{\theta}\left[\frac{t}{\theta}\right]^{(\beta-1)} v^{\rho_1} f^{\rho_2} d^{\rho_3} h^{\rho_4}\, exp\left[\left(-(\frac{t}{\theta})^{(\beta)}\right) v^{\rho_1} f^{\rho_2} d^{\rho_3} h^{\rho_4}\right] \tag{9}$$

Further, Equation (9) is utilized for parameter estimation with the help of the MLE technique, as explained in the next section.

2.3 Maximum Likelihood Estimation (MLE)

Maximum likelihood estimation is a statistical method that maximizes the likelihood function to estimate probability distribution parameters (Myung, 2003). In this paper, the MLE method aims to determine the unknown parameter values of a proposed model (Equation 9) and maximize the likelihood of observing the given data. The derived $f(t)$ (Equation 9) is characterized by six unknowns, namely β, θ, ρ_1, ρ_2, ρ_3 and ρ_4. A likelihood function is formalized to determine the unknown parameters, as demonstrated through Equation (10).

$$L(\theta,\beta,\rho_1,\rho_2,\rho_3,\rho_4) = \prod_{i=1}^{n}\frac{\beta}{\theta}\left[\frac{t_i}{\theta}\right]^{(\beta-1)} v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4} exp\left[\left(-(\frac{t_i}{\theta})^{(\beta)}\right) v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4}\right] \tag{10}$$

The log-likelihood function is derived by taking the natural logarithm of Equation (10), resulting in Equation (11).

$$ln\,(L) = ln\prod_{i=1}^{n}\frac{\beta}{\theta}\left[\frac{t_i}{\theta}\right]^{(\beta-1)} v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4} exp\left[\left(-(\frac{t_i}{\theta})^{(\beta)}\right) v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4}\right] \tag{11}$$

The final log-likelihood Equation (12) for the cutting tools is obtained by reworking Equation (11).

$$ln(L) = nln\beta - n\beta ln\theta + (\beta-1)\sum ln\, t_i + \rho_1 ln\sum v_i + \rho_2 ln\sum f_i + \rho_3 ln\sum d_i + \rho_4 ln\sum d_i - \frac{1}{\theta^\beta}\left[\sum t_i^\beta v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4}\right] \tag{12}$$

In order to obtain the maximum value, log-likelihood Equation (12) is differentiated partially with respect to each unknown parameter, equated to zero, resulting in Equations (13-18). These six equations are simultaneously solved in MATLAB with the "*fsolve*" function in an optimization toolbox to determine the unknown parameters' values.

$$n\theta^\beta - \beta\theta^\beta n\, ln\theta + \beta\theta^\beta\sum ln\, t_i - \beta[\sum t_i^\beta ln\frac{t_i}{\theta} v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4}] = 0 \tag{13}$$

$$\theta = \left[\frac{\sum t_i^\beta v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4}}{n}\right]^{1/\beta} \tag{14}$$

$$ln\sum v_i - \frac{1}{\theta^\beta}\left[\sum t_i^\beta v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4} lnv_i\right] = 0 \tag{15}$$

$$ln \sum f_i - \frac{1}{\theta^\beta}\left[\sum t_i^\beta v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4} ln f_i\right] = 0 \quad (16)$$

$$ln \sum d_i - \frac{1}{\theta^\beta}\left[\sum t_i^\beta v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4} ln d_i\right] = 0 \quad (17)$$

$$ln \sum h_i - \frac{1}{\theta^\beta}\left[\sum t_i^\beta v_i^{\rho_1} f_i^{\rho_2} d_i^{\rho_3} h_i^{\rho_4} ln h_i\right] = 0 \quad (18)$$

The obtained values are further discussed in the results and discussions in section 3.1. In order to ensure the accuracy of the proposed model, unknown parameters are validated using RSM and Weibull fits by utilizing the experimental data, as explained in section 3.1.

2.4 Experimental Setup and Data

The unknown parameters of the likelihood Equations (13-18) are estimated using experimental data, which are collected with the help of an experiment conducted by Qehaja et al. (2017). For more details on experiment setup, please refer Qehaja et al. (2017). The tool is considered to have failed under two conditions: (1) When the maximum width of end clearance wear or nose wear approached 0.2 mm threshold, and (2) When a catastrophic failure took place. Four factors are selected from the AHP model to develop a tool life prediction model with three levels of each factor. **Table 4** outlines the selected parameters for the experiment, including their respective units, limits, and their three levels as obtained from the experiment. The three levels are coded in such a way that the low level corresponds to -1, the middle level 0, and the high level corresponds to $+1$ by transformation of the equations, as shown in the third column of **Table 4**. For example, for factor v (m/min) at serial number 1, the calculation is as follows:

For high level: - $\frac{ln\, v - ln\, 135}{ln\, 180 - ln\, 135} \rightarrow \frac{ln\, 180 - ln\, 135}{ln\, 180 - ln\, 135} \rightarrow 1$

For Middle level: - $\frac{ln\, v - ln\, 135}{ln\, 180 - ln\, 135} \rightarrow \frac{ln\, 135 - ln\, 135}{ln\, 180 - ln\, 135} \rightarrow 0$

For low level: - $\frac{ln\, v - ln\, 135}{ln\, 180 - ln\, 135} \rightarrow \frac{ln\, 100 - ln\, 135}{ln\, 180 - ln\, 135} \rightarrow -1.04 \approx -1$

The calculation for remaining factors with all three code levels has been performed in a similar manner. Further to this, the experiment design is conducted at three code levels using RSM, which involves a total of 24 runs, as discussed in the subsequent section.

Table 4. Code level of cutting tools factors (Qehaja et al., 2017).

Cutting actors and their levels					
S. No.	**Factors**	**Code level**	**High level**	**Middle level**	**Low level**
			+1	0	-1
1.	v (m/min)	$X_1(\frac{ln\, v - ln\, 135}{ln\, 180 - ln\, 135})$	180	135	100
2.	f (mm/rev)	$X_2(\frac{ln\, f - ln\, 0.214}{ln\, 0.285 - ln\, 0.214})$	0.285	0.214	0.178
3.	d (mm)	$X_3(\frac{ln\, d - ln\, 0.85}{ln\, 1.5 - ln\, 0.85})$	1.5	0.85	0.5
4.	h (HRC)	$X_4(\frac{ln\, h - ln\, 45}{ln\, 55 - ln\, 45})$	55	45	35

2.5 Response Surface Methodology (RSM)

Box and Wilson (1951) developed RSM as a means to enhance production in chemical process engineering. Response surface methodology consists of a group of mathematical and statistical techniques used in the development of an adequate functional relationship between a response of interest, Y, and a number of associated control (or input) variables denoted by x_i and x_j. The RSM regression equation is obtained using second-order polynomial equations, represented by Equation (19).

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i{}^2 + \sum_{i=1}^{k-1} \sum_{j>1}^{k} \beta_{ij} x_i x_j. \tag{19}$$

Equation (19) includes a response variable, denoted by Y, as well as a constant coefficient (β_0), linear coefficient (β_i), quadratic coefficient (β_{ii}), and interaction coefficient (β_{ij}). Furthermore, the independent factors are denoted by coded values (x_i and x_j). In our case, there are four factors: speed, feed, DOC, and w/p hardness. The reason to use a second-order polynomial in response surface methodology with four factors is to account for curvature in the response surface, which may indicate a minimum or maximum of the response variable. A second-order model can also include interaction terms between the factors, which is important for understanding the effects of the factors on the response.

The study used cutting speed, feed, DOC, and workpiece hardness as *v*, *f*, *d*, and *h*, respectively for experiment design and reliability estimation. To achieve this, a circumscribed central composite design (CCD) is used as RSM design for the selected factors and levels. The aim of the RSM is to assess the unknown parameters and their interactions in order to determine the optimal response while conducting the least number of trials possible. This design includes 16 factorial points and 8 centre points. The experimental error and reproducibility of data are examined by analyzing the centre points. To minimize experimental costs, the axial points are avoided in this experimental design. Hence, the experimental design proposed by the RSM involved a total of 24 runs, as outlined in **Table 5**.

Table 5. Experiment factors combination and response (Qehaja et al., 2017).

Test No.	Coded factors					Performance Measures	
	X_0	X_1	X_2	X_3	X_4	Tool wear (mm)	Tool failure time (TTF) (min)
1.	+1	-1	-1	-1	-1	0.201	95
2.	+1	-1	-1	-1	1	0.198	50
3.	+1	-1	-1	1	-1	0.207	48
4.	+1	-1	-1	1	1	0.192	85
5.	+1	-1	1	-1	-1	0.199	60
6.	+1	-1	1	-1	1	0.210	50
7.	+1	-1	1	1	-1	0.194	70
8.	+1	-1	1	1	1	0.208	46
9.	+1	1	-1	-1	-1	0.200	35
10.	+1	1	-1	-1	1	0.203	55
11.	+1	1	-1	1	-1	0.191	37
12.	+1	1	-1	1	1	0.211	61
13.	+1	1	1	-1	-1	0.206	35
14.	+1	1	1	-1	1	0.201	48
15.	+1	1	1	1	-1	0.197	42
16.	+1	1	1	1	1	0.196	9
17.	+1	0	0	0	0	0.209	80
18.	+1	0	0	0	0	0.198	68
19.	+1	0	0	0	0	0.199	75
20.	+1	0	0	0	0	0.192	52
21.	+1	0	0	0	0	0.200	74
22.	+1	0	0	0	0	0.198	40
23.	+1	0	0	0	0	0.191	55
24.	+1	0	0	0	0	0.198	45

The unknown parameters obtained from the RSM regression model are utilized for the validation of the proposed model (sections 2.2 and 2.3), and the results are shown in section 3.1. Further to this, the next section is dedicated to proposing the methods and equations for the estimation of the number of job completions and cutting tools required for a machine tool in a day.

2.6 Methodology for Determining Cutting Tools Inventory for Industrial Applications

This paper proposes methodology in the form of Equations (20-25) for determining the number of jobs or workpieces that can be completed and the optimal number of cutting tools required for machining within an eight-hour working period in a day. Where, *D*, *TL*, and L_{piece} denote the diameter of a workpiece, cutting tool life, and length of a workpiece, respectively.

***Step* 1:** Calculate the length of machining per cutting tool:

$$L_{total} = \left(\frac{f*v*1000}{\pi*D}\right) * TL \tag{20}$$

***Step* 2:** Calculate the number of workpieces cut per cutting tool:

$$N_{total} = \frac{L_{total}}{L_{piece}} \tag{21}$$

***Step* 3:** Calculate the effective time to process the workpiece per cutting tool:

$$T_{total} = TL + (N_{total} - 1) * 0.5 \tag{22}$$

***Step* 4:** Calculate the effective working time per workpiece to complete:

$$T_{piece} = \frac{T_{total}}{N_{total}}. \tag{23}$$

***Step* 5:** Calculate the number of workpieces produce in 8 hrs. shift:

$$N_{8\,hr} = \frac{N_{total}*8*60}{T_{total}} \tag{24}$$

***Step* 6:** Calculate the number of times cutting tools change in 8 hrs. shift:

$$CT_{required} = \frac{N_{8\,hr}}{N_{total}} \tag{25}$$

This paper then demonstrates the results of the proposed model with the help of experimental data to estimate the optimal number of tools required for machining the workpiece duly illustrated in the results and discussion in section 3.3.

3. Results & Discussion

This section provides a comprehensive overview of the outcomes obtained from the proposed methodology. This section is divided into four subsections as follows:
Section 3.1. Validation of unknown parameters of the proposed model using RSM estimated unknowns.
Section 3.2. Analysis of cutting tools' reliability, hazard curve, and median life.
Section 3.3. Determination of workpiece completion and cutting tools' requirements for a day.

3.1 Validation of Unknown Parameters of the Proposed Model using RSM Estimated Unknowns

Equation (27) has been developed in the form of a modified Taylor's tool life equation. This Equation has been derived by substituting code-level transformation equations (**Table 4**) in the RSM Equation (26).

$$Y_{est} = 4.085 - 0.264 * x_1 - 0.164 * x_2 - 0.088 * x_3 - 0.059 * x_4 \tag{26}$$

$$TTF = 6653.9613 * v^{-0.918} * f^{-0.5723} * d^{-0.15520} * h^{-0.295} \tag{27}$$

The RSM is utilized to determine the optimal settings of the four factors to determine optimal tool life. The tool life is estimated with the help of Minitab software (version 19.1 and 64-bit) by considering the four independent variables in RSM (Equation 27). The response surface plots generated to obtain the optimal tool life are placed in **Figures 4**, **5**, and **6**. In **Figure 4**, the response surface plot shows the dynamic relationship between velocity or cutting speed with (a) w/p hardness and (b) feed on tool life. As cutting speed increases, tool life decreases convexly, with an optimal range identified between low and moderate cutting speeds. Additionally, an increase in w/p hardness positively impacts tool life at higher speed, and if feed increases, tool life decreases.

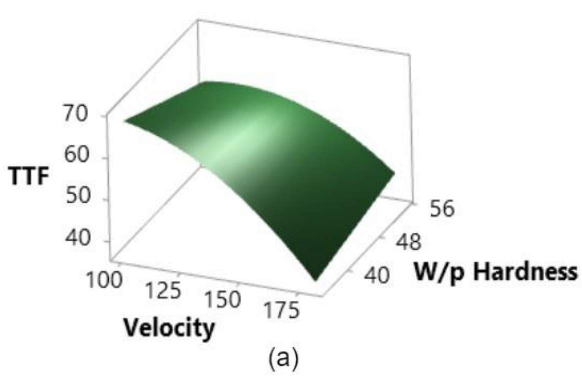

Figure 4. (a) Surface plot of time to failure vs w/p hardness, velocity. (b) Surface plot of time to failure vs feed, velocity.

The surface plots of **Figure 5** illustrate the dynamic relationship between (a) velocity vs DOC vs tool life and (b) feed vs DOC tool life. As velocity increases, tool life decreases convexly, with an optimal range identified between low and moderate cutting speeds. Additionally, with an increase in DOC, tool life decreases at higher speed and feed, and if feed increases, tool life decreases.

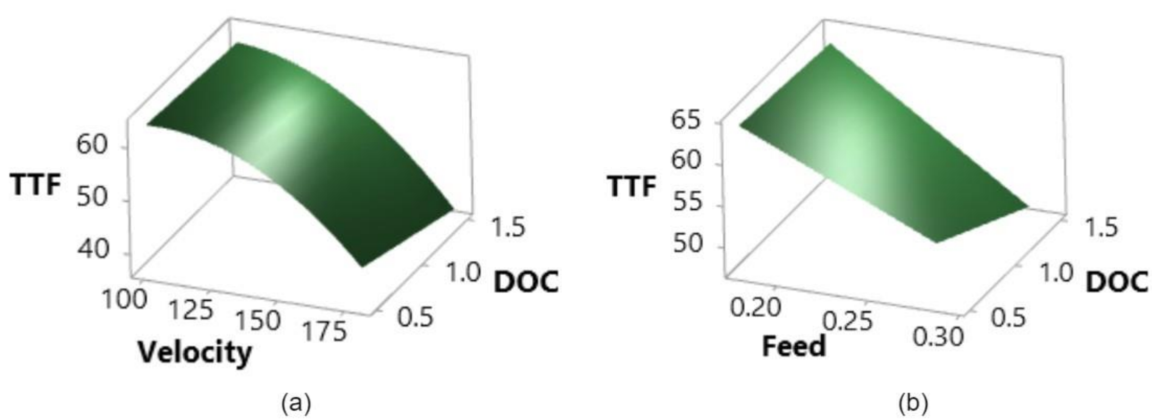

Figure 5. (a) Surface plot of time to failure vs DOC vs velocity. (b) Surface plot of time to failure vs DOC vs feed.

The 3D response surface plot **Figure 6** illustrates the dynamic relationship between (a) DOC vs w/p hardness vs tool life and (b) feed vs w/p hardness vs tool life. As DOC increases, tool life decreases and an optimal range of DOC is identified between low and moderate levels of DOC where tool life in maximum.

Additionally, with an increase in w/p hardness, tool life slightly increases at higher DOC and lower feed, but decreases at higher feed. Similarly, if feed increases, tool life decreases at higher w/p hardness.

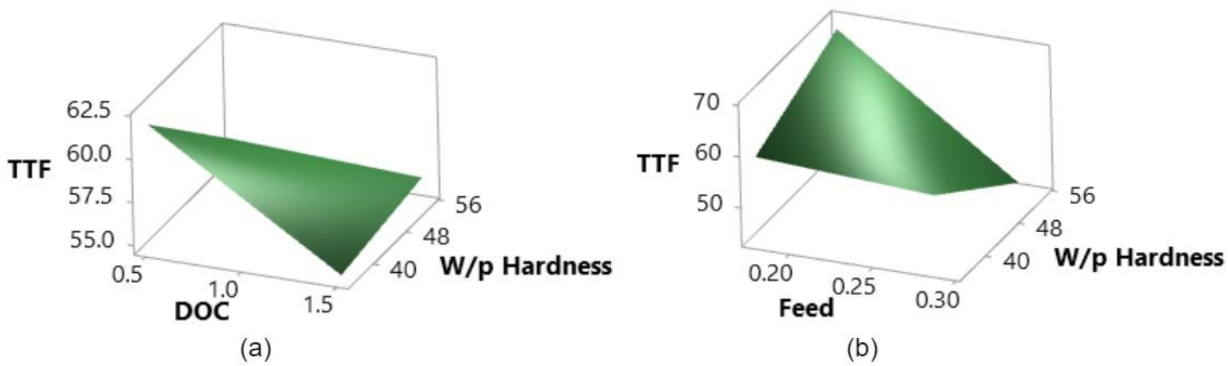

Figure 6. (a) Surface plot of time to failure vs w/p hardness, DOC. (b) Surface plot of time to failure vs w/p hardness, feed.

The surface plot aids in identifying the optimal parameter combinations for maximizing tool life, which is crucial for effective machining processes. The optimum tool life obtained is 72.550 mins, and the optimal setting values of the factors are presented in **Table 6**.

Table 6. Optimal parameters and tool life obtained from RSM model.

v (m/min)	*f* (mm/rev)	*d* (mm)	*h* (HRC)	Tool life (min)
110.5051	0.17820	1.5	55	72.550

The Weibull fit test is then performed to examine the fitness of the Weibull distribution of twenty-four failure times for the cutting tools (**Table 5**). The results of the test, depicted in **Figure 7**, indicate a good fit for the Weibull distribution.

In addition to the aforementioned method, Mann's Goodness-of-Fit test (Mann, 2006) for the Weibull failure distribution is performed. The parameters obtained from this test are presented in **Table 7**. Since $M < F_{Critical}$, at $\alpha = 0.05$ the data follows a Weibull distribution.

Table 7. Mann's test parameters obtained.

Test Statistics *(M)*	Degree of Freedom 1	Degree of Freedom 2	Significance level α	$F_{Critical}$ from *F*-distribution
1.3123	23	24	0.05	1.9932

The four most important sub-factors, as identified in **Figure 3**, are selected as covariates for the proposed model, as shown in Equation (9). Next, the unknown parameters are estimated using the maximum likelihood estimation method (Equations (13-18)) and validated using the RSM model and Weibull fit test, as presented in **Table 8**. The obtained results demonstrate that the beta (β) and theta (θ) values determined using the proposed model exhibit insignificant difference when compared with the Weibull fit. Additionally, when comparing the remaining unknown parameters with the RSM, the difference is again insignificant, except for parameter ρ_2. The limitation of applying the Weibull fit directly to the time-to-failure lies in its failure to account for the effects of factors. This is evident in **Table 8**, where it is observed that the characteristic life is overestimated. Additionally, while Response Surface Methodology (RSM) is employed, it does not offer insights into the shape and characteristics life. But the analysis of unknown parameters through the proposed PHM model considers the effects of covariates as well and reveals that the influence of feed is the most significant, followed by speed, then depth of cut (DOC), and lastly, workpiece hardness.

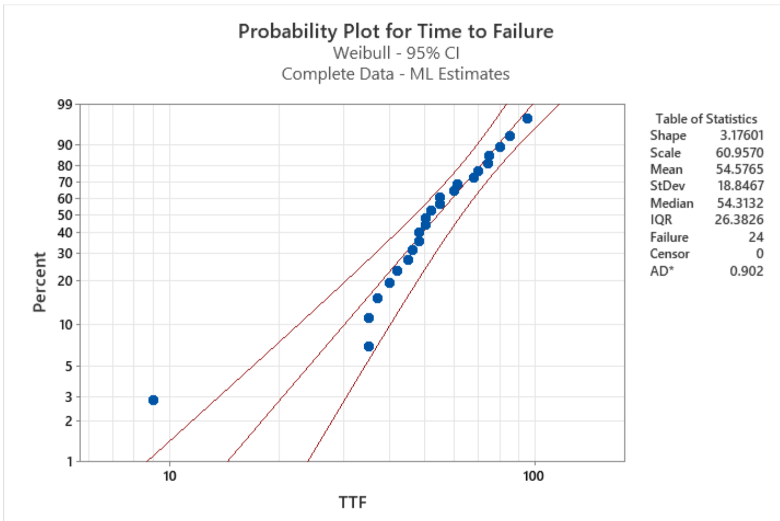

Figure 7. Weibull fit plot.

Table 8. Validation of proposed model unknowns with RSM and Weibull Fit.

	β	θ	ρ_1	ρ_2	ρ_3	ρ_4
Weibull fit	3.17601	60.9570				
Cox-PHM model (proposed)	3.5261	56.5014	0.8432	3.0384	0.1749	0.091
RSM			0.918	0.5723	0.15520	0.295

3.2 Analysis of Cutting Tools' Reliability, Hazard Curve, and Median Life

The hazard and reliability plots derived from the proposed model are illustrated in **Figures 8** and **9**, respectively. These plots clearly demonstrate the significant impact of variations in factors on both the hazard and reliability curves. Furthermore, the observation reveals that in the 16th case of the cutting tool, denoted as *H16*, deteriorates significantly faster compared to other cases, exhibiting an extreme wear-out condition since the cutting tool is operated at the highest level of all four factors. Similarly, the *H15* cutting tool, representing the 15th case, shows a relatively less but noticeable degradation compared to the 16th case. This tool operates with three factors set at higher levels, while one factor, specifically the w/p hardness, is set at the lowest level. The findings suggest that the impact of w/p hardness on the hazard rate is less significant compared to other parameters, as it exhibits similar effects at both high and low levels. It is also proved from the values of the unknown parameters in **Table 8**. However, estimated accuracy could be further improved if more data is made available to estimate the parameters.

It can be further observed from the reliability curve in **Figure 9** that the reliability of the 16th case of cutting tool denoted as R16 in **Figure 9** has drastically decreased as compared to the other cases due to extreme wear out. Similarly, the R15 cutting tool degrades the same as R16 but is still noticeable due to three factors being set higher, while w/p hardness is set lowest. The findings suggest that w/p hardness has a lesser effect than other factors in showing the effect on the reliability curve.

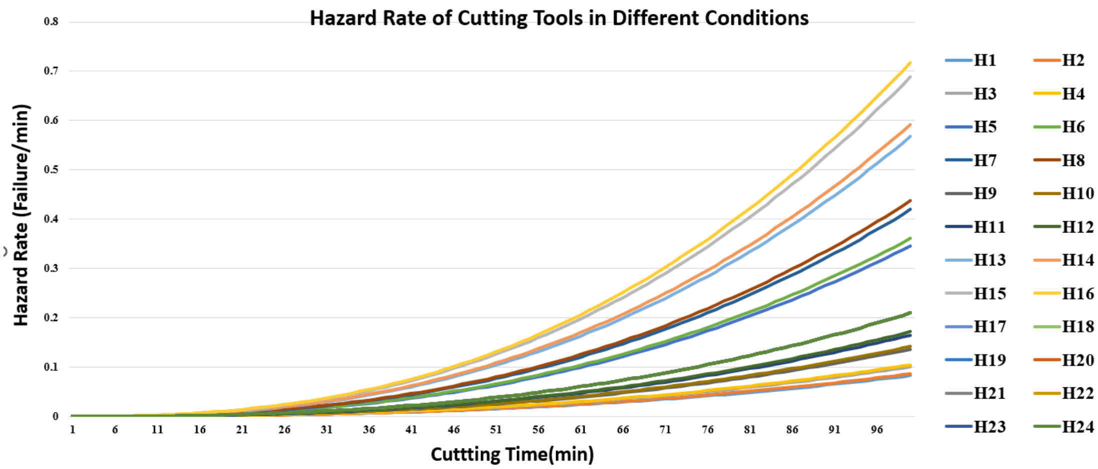

Figure 8. The hazard rate of cutting tools in different conditions.

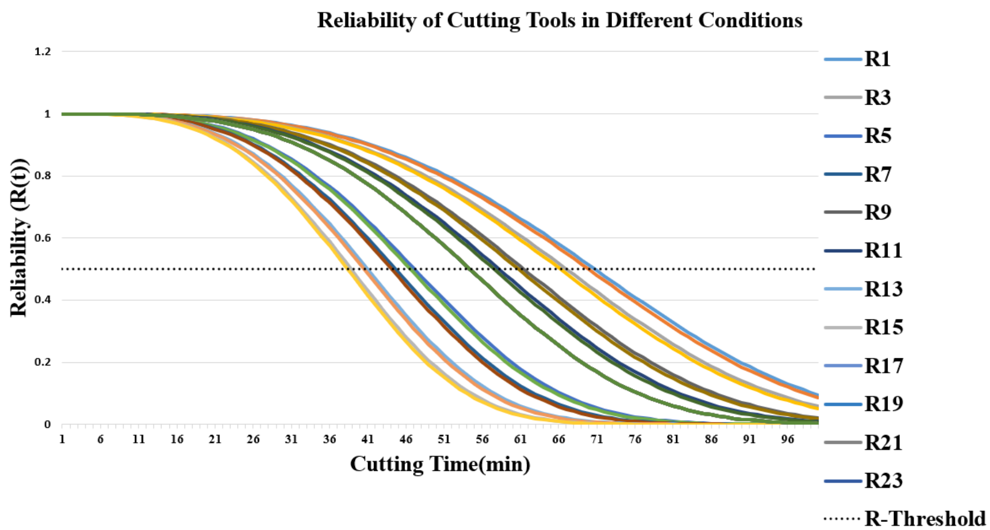

Figure 9. Reliability of cutting tools in different conditions.

In this case, the median life is considered to be a more significant outcome for cutting tools reliability analysis than the mean life of the cutting tool, as the median is not susceptible to outliers (Liu et al., 2020). The median life of a cutting tool is the point at which its reliability reaches 0.5, beyond which the likelihood of tool failure increases. The proposed model is used to predict the median lifespan of twenty-four cutting tools, as shown in **Figure 10**. For example, the coordinates (1,72) represent that 1st case of the cutting tool has a median lifespan of 72 mins. In this Figure, the points that are closer to the central point indicate a relatively lower median lifespan in relation to their respective cases. According to **Figure 10**, the highest median life value observed is 72 mins, which aligns with the optimal life obtained through RSM-based analysis, as presented in **Table 6**. Hence, the proposed model emerges as highly suitable for conducting a reliability analysis of cutting tools with varying factors.

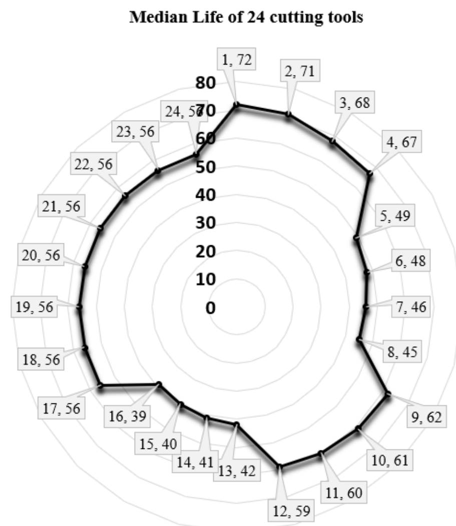

Figure 10. Median life of 24 cutting tools.

3.3 Determination of Workpiece Completion and Cutting Tool Requirements for a Day

It is essential for industries to determine the number of jobs that can be completed and the number of cutting tools required for a particular machine tool within a specific period, such as a day or a month, and there is a need to accommodate the factors that affect the lifespan of cutting tools. To address this issue, we have proposed a methodology to evaluate the number of cutting tools required per day. This assessment is based on twenty-four cases of cutting tools, as indicated in the experimental data (**Table 5**), and is determined by the median lifespan derived from the model given in section 2.2. This proposed methodology has already been explained in section 2.6.

It should be noted that these calculations are based on some assumptions, as mentioned below:

- A job consists of the machining of a workpiece with maximum length (L_{piece}) of 300 mm.
- The life of a cutting tool ends at its median life.

For the sake of better comprehension of the readers, the calculation for the results at case No. 1 is as appended below:

***Step* 1:** Calculation of length of machining per cutting tool:

$$L_{total} = \left(\frac{f*v*1000}{\pi*D}\right) * TL \rightarrow \left(\frac{0.178*100*1000}{\pi*80}\right) * 72 \rightarrow 5101.911\, mm.$$

***Step* 2:** Calculation of number of workpieces cut per cutting tool, where L_{piece} is 300 mm:

$$N_{total} = \frac{L_{total}}{L_{piece}} \rightarrow \frac{5101.911}{300} \rightarrow 17.00637.$$

***Step* 3:** Calculation of effective time to process workpiece per cutting tool:

$$T_{total} = \{TL + (N_{total} - 1) * 0.5\} \rightarrow \{72 + (18 - 1) * 0.5\} \rightarrow 80.5\, min.$$

***Step* 4:** Calculation of effective working time per workpiece to complete:

$$T_{piece} = \frac{T_{total}}{N_{total}} \rightarrow \frac{80.5}{18} \rightarrow 4.4722\, min.$$

***Step* 5:** Calculation of the number of workpieces produces in 8 hrs. shift:

$$N_{8\,hr} = \frac{N_{total}*8*60}{T_{total}} \rightarrow \frac{17.00637*8*60}{80.5} \rightarrow 101.4044.$$

***Step* 6:** Calculation of the number of times the cutting tool changes in 8 hrs. shift:

$$CT_{required} = \frac{N_{8\,hr}}{N_{total}} \rightarrow \frac{101.4044}{17.00637} \rightarrow 5.666667.$$

Similarly, calculations for other cases have been done, and the results are placed in **Table 9**. The results of this proposed methodology provide significant information for machining industries because it will help to enhance their operational efficiency and maximize their resource utilization.

Table 9. Number of jobs completed and cutting tools requirement in a day.

Case No.	L_{piece}(mm)	L_{total}(mm)	N_{total}	$T_{total}(min)$	T_{piece}(min)	$N_{8\,hr}$	$CT_{required}$
1.	300	5101.911	17.006	80.5	4.472	101.404	5.667
2.	300	5031.051	16.770	79.0	4.647	101.895	6.000
3.	300	4818.471	16.061	76.0	4.470	101.441	6.000
4.	300	4747.611	15.825	74.5	4.656	101.962	6.375
5.	300	5559.315	18.531	58.0	3.052	153.360	8.105
6.	300	5445.860	18.153	57.0	3.000	152.866	8.052
7.	300	5218.949	17.396	54.5	3.027	153.217	8.555
8.	300	5105.494	17.018	53.5	2.972	152.688	8.555
9.	300	7907.962	26.359	75.0	2.777	168.703	6.259
10.	300	7780.414	25.934	73.5	2.826	169.369	6.538
11.	300	7652.866	25.509	72.5	2.788	168.891	6.500
12.	300	7525.318	25.084	71.5	2.750	168.399	6.500
13.	300	8577.229	28.590	56.0	1.931	245.064	8.483
14.	300	8373.010	27.910	54.5	1.946	245.813	8.786
15.	300	8168.790	27.229	53.5	1.911	244.300	8.750
16.	300	7964.570	26.548	52.0	1.926	245.064	9.111
17.	300	6440.446	21.468	66.5	3.023	154.958	7.045
18.	300	6440.446	21.468	66.5	3.023	154.958	7.045
19.	300	6440.446	21.468	66.5	3.023	154.958	7.045
20.	300	6440.446	21.468	66.5	3.023	154.958	7.045
21.	300	6440.446	21.468	66.5	3.023	154.958	7.045
22.	300	6440.446	21.468	66.5	3.023	154.958	7.045
23.	300	6440.446	21.468	66.5	3.023	154.958	7.045
24.	300	6440.446	21.468	66.5	3.023	154.958	7.045

We also present the estimation of cutting tools required per day with the help of experimental TTF to understand a more precise inventory holding by the industry, and also conduct a comparative study between the cutting tools inventory value obtained from the proposed model considering median life and the experimental TTF data. For the sake of better comprehension of the readers, the calculation for the results at case No. 1 using the experimental TTF data is as appended below:

***Step* 1:** Calculation of length of machining per cutting tool:

$$L_{total} = \left(\frac{f*v*1000}{\pi*D}\right) * TL \rightarrow \left(\frac{0.178*100*1000}{\pi*80}\right) * 95 \rightarrow 6731.687\, mm.$$

***Step* 2:** Calculation of number of workpieces cut per cutting tool, where L_{piece} is 300 mm:

$$N_{total} = \frac{L_{total}}{L_{piece}} \rightarrow \frac{6731.687}{300} \rightarrow 22.438.$$

***Step* 3:** Calculation of effective time to process workpiece per cutting tool:

$$T_{total} = \{TL + (N_{total} - 1) * 0.5\} \rightarrow \{95 + (22 - 1) * 0.5\} \rightarrow 105.5\, min.$$

***Step* 4:** Calculation of effective working time per workpiece to complete:

$$T_{piece} = \frac{T_{total}}{N_{total}} \rightarrow \frac{105.5}{22} \rightarrow 4.795\ min.$$

***Step* 5:** Calculation of the number of workpieces produces in 8 hrs. shift:

$$N_{8\ hr} = \frac{N_{total} * 8 * 60}{T_{total}} \rightarrow \frac{22.438 * 8 * 60}{105.5} \rightarrow 102.092.$$

***Step* 6:** Calculation of the number of times the cutting tool changes in 8 hrs. shift:

$$CT_{required} = \frac{N_{8\ hr}}{N_{total}} \rightarrow \frac{102.092}{22.438} \rightarrow 4.681.$$

Similarly, calculations for other cases have been performed, and the final results of the required number of cutting tools are shown in **Table 10**. A comparative study on the cutting tools inventory required for one machine tool in a day computed using the proposed model's median life and experimental TTF data is shown in **Table 10** and **Figure 11** for better comprehension of the readers.

Table 10. Comparison table for $CT_{required}$ for one machine tool per day using proposed model's median life and experimental TTF.

Case No.	$CT_{required}$ (Using proposed model's median life)	$CT_{required}$ (Using experimental TTF)	Case No.	$CT_{required}$ (Using proposed model's median life)	$CT_{required}$ (Using experimental TTF)
1.	5.667	4.682	13	8.483	10.250
2.	6.000	8.584	14	8.786	7.454
3.	6.000	9.364	15	8.750	8.483
4.	6.375	5.100	16	9.111	42.667
5.	8.105	6.694	17	7.045	5.000
6.	8.053	8.105	18	7.045	6.000
7.	8.556	5.961	19	7.045	5.379
8.	8.500	9.117	20	7.045	7.800
9.	6.259	11.400	21	7.045	5.571
10.	6.538	7.435	22	7.045	10.467
11.	6.500	10.625	23	7.045	7.428
12.	6.500	6.538	24	7.045	9.235

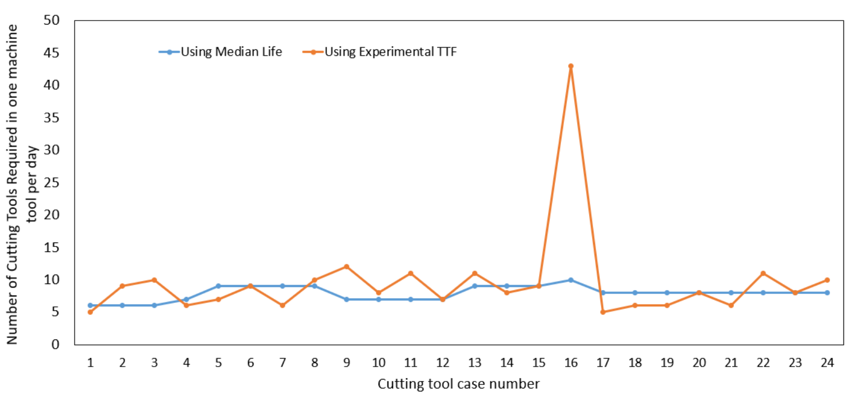

Figure 11. Represent the variation of $CT_{required}$ for one machine tool per day using proposed model's median life and experimental TTF.

It can be observed from the graph in **Figure 11** that, in the case of 16th, there is a noticeable variation in the number of cutting tools required as estimated via the median life and experimental TTF. This variation is due to the value of Experimental TTF of 09 mins and the median life of 39 mins.

4. Novelty of Findings and Real Industrial Applications

The conclusion drawn from the results and discussion underscores the novelty of the paper's findings, as outlined below:

- An AHP model has been used to identify and prioritize the factors affecting cutting tools' life and reliability. This model identifies speed, feed, depth of cut, and workpiece hardness as the primary determinants of significance, extending the analysis beyond the existing literature that primarily concentrates on three factors for the evaluation of reliability.
- This study investigates the impact of four factors on reliability and hazard curves with 24 cases of cutting parameters, filling a gap in the existing literature where these aspects have not been previously addressed.
- A comparison of the characteristic life estimates derived from Weibull fitting, which does not consider influencing factors, and those obtained from the proportional hazard model (PHM) reveals that the omission of these factors results in an overestimation of the characteristic life. Furthermore, both the Response Surface Methodology (RSM) and the proposed PHM model provide the same estimate of a tool life of 72 minutes when the factors' effects are considered, thus validating the PHM model. Such a comparative study of PHM with the RSM model is found to be limited in the literature.
- The findings from the analysis of unknown parameters, hazard curve, and reliability curve indicate that the impact of feed is highest, while the influence of w/p hardness is lowest.
- The hazard and reliability curves indicate that a cutting tool operating at its maximum level across all factors exhibits a higher rate of increasing hazard and decreasing reliability.
- This paper determines the required inventory of cutting tools per machine tool per day with the help of PHM results. This approach has received limited attention in the literature.

This reliability analysis of cutting tools finds practical applications in industries for optimizing maintenance strategies, enhancing quality control, reducing production costs, improving overall equipment efficiency, and facilitating predictive maintenance. Reliable cutting tools contribute to stable and predictable machining processes, providing valuable data for decision-making in production planning, resource allocation, and tool investment in the areas of automotive manufacturing, aerospace industry, mining, and heavy equipment manufacturing.

5. Conclusions & Future Scopes

In today's industrial environment, it is imperative to assess the reliability of cutting tools as it can affect the surface finish and dimensional accuracy of machined parts. Variations in cutting tool reliability due to factors affecting tool reliability and its life can have an adverse impact on machining operations and the economy. To address these challenges, this paper proposes a qualitative study to identify various factors impacting tools' reliability and their life before conducting any experiments. Four "critical to reliability" factors are prioritized from seventeen identified factors through pair-wise comparisons and obtaining their weights through the AHP method. Then, the tool life data for the four factors, namely speed, feed, DOC, and material hardness, at three operating levels are obtained from the experiment conducted by Qehaja et al. (2017). This paper utilized this data to develop PHM for reliability and hazard rate estimation for 24 cases of cutting tools and verified the effectiveness of the proposed model through the use of RSM and Weibull fit. Additionally, the study presented a methodology for industries to estimate the number of job completions and cutting tools required per machine per day. The tangible outcomes of this research work are (1) Identification and prioritization of factors and sub-factors affecting the life and reliability of cutting tools with the help of the AHP model; (2) Reliability parameter estimates with the help of the proposed PHM-based model considering four covariates as mentioned earlier; (3) Validation of the proposed PHM based model by comparing it with the Weibull fit and RSM; (4) Required inventory of cutting tools per machine tool in a day.

The hazard rate and reliability plots clearly demonstrate the significant impact of variations in factors on both the hazard and reliability of the cutting tools during their operation. It is also observed from the reliability and hazard curve that the 16th case of cutting tool has drastically changed as compared to the other cases due to extreme wear out. This is because the tool has been operated at the highest level of all the selected four factors. Thus, the proposed model emerges as highly suitable for conducting a reliability analysis of cutting tools with varying factors. This paper yields positive outcomes by examining four sub-factors and analyzing their respective impacts on the cutting tools' reliability, hazard rate, tool life, and necessary inventory. The paper also presents the median life of the 24 cutting tools. The highest median life value observed is 72 mins, which aligns with the optimal life obtained through RSM-based analysis.
The findings from the analysis of unknown parameters, hazard curve, and reliability curve indicate that the impact of feed is highest, while the influence of w/p hardness is lowest. The results of the proposed methodology for evaluating cutting tool requirements provide significant information for machining industries to enhance their operational efficiency and maximize their resource utilization.

Some potential research gaps of the proposed model are as follows: (1) Co-variate measurements' precision impact model accuracy. (2) Offline application limits real-time prediction feasibility. (3) More the number of data points better the PHM predictions. (4) The model is limited in considering censored data. The future scope and studies that could improve the results by adding more co-variates, improving covariate measurement precision, real-time application, and integration with neural networks.

Conflict of Interest
The authors confirm that there is no conflict of interest to declare for this publication.

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References

Box, G.E.P., & Wilson, K.B. (1951). On the experimental attainment of optimum conditions. *Journal of the Royal Statistical Society: Series B (Methodological)*, *13*(1), 1-38. https://doi.org/10.1111/j.2517-6161.1951.tb00067.x.

Bayraktar, S. (2021). Dry cutting: A sustainable machining technology. In *Sustainable Manufacturing* (pp. 231-257). Elsevier. https://doi.org/10.1016/B978-0-12-818115-7.00004-3.

Bazaz, S.M., Ratava, J., Lohtander, M., & Varis, J. (2023a). An investigation of factors influencing tool life in the metal cutting turning process by dimensional analysis. *Machines*, *11*(3), 393.

Bazaz, S.M., Ratava, J., Lohtander, M., & Varis, J. (2023b). Graph-based analysis of tool life parameters of the turning process in small-lot production. In *International Conference on Flexible Automation and Intelligent Manufacturing* (pp. 141-148). Springer International Publishing. https://doi.org/10.1007/978-3-031-17629-6_16.

Cheng, K., Niu, Z.C., Wang, R.C., Rakowski, R., & Bateman, R. (2017). Smart cutting tools and smart machining: development approaches, and their implementation and application perspectives. *Chinese Journal of Mechanical Engineering*, *30*, 1162-1176. https://doi.org/10.1007/s10033-017-0183-4.

Equeter, L., Letot, C., Serra, R., & Dehombreux, P. (2016). Estimate of cutting tool lifespan through Cox proportional hazards model. *IFAC-PapersOnLine*, *49*(28), 238-243. https://doi.org/10.1016/j.ifacol.2016.11.041.

Ezugwu, E.O., Bonney, J., Fadare, D.A., & Sales, W.F. (2005). Machining of nickel-base, Inconel 718, alloy with ceramic tools under finishing conditions with various coolant supply pressures. *Journal of Materials Processing Technology*, *162*, 609-614. https://doi.org/10.1016/j.jmatprotec.2005.02.144.

Gaddafee, M., & Chinchanikar, S. (2020). An experimental investigation of cutting tool reliability and its prediction using Weibull and Gamma models: A comparative assessment. *Materials Today: Proceedings*, *24*, 1478-1487. https://doi.org/10.1016/j.matpr.2020.04.467.

Harrell, F.E. (2015). Cox proportional hazards regression model. In: *Regression Modeling Strategies: With Applications to Linear Models, Logistic and Ordinal Regression, and Survival Analysis* (pp. 475-519). Springer, Cham. https://doi.org/10.1007/978-3-319-19425-7_20.

Hontoria, E., & Munier, N. (2021). Uses and limitations of the AHP method a non-mathematical and rational analysis. https://doi.org/10.1007/978-3-030-60392-2.

Khuri, A.I., & Mukhopadhyay, S. (2010). Response surface methodology. *Wiley Interdisciplinary Reviews: Computational Statistics*, *2*(2), 128-149. https://doi.org/https://doi.org/10.1002/wics.73.

Klim, Z., Ennajimi, E., Balazinski, M., & Fortin, C. (1996). Cutting tool reliability analysis for variable feed milling of 17-4PH stainless steel. *Wear*, *195*(1-2), 206-213. https://doi.org/https://doi.org/10.1016/0043-1648(95)06863-5.

Le, H.K. (2022). Multi-criteria decision making in the milling process using the PARIS method. *Engineering, Technology & Applied Science Research*, *12*(5), 9208-9216.

Lee, S.W., & Lee, H.K. (2009). Rule-based cutting condition recommendation system for intelligent machine tools. *Journal of Mechanical Science and Technology*, *23*(4), 1202-1210. https://doi.org/10.1007/s12206-009-0306-7.

Lin, W.S. (2008). The reliability analysis of cutting tools in the HSM processes. *Archives of Materials Science and Engineering*, *30*(2), 97-100.

Liu, E., An, W., Xu, Z., & Zhang, H. (2020). Experimental study of cutting-parameter and tool life reliability optimization in Inconel 625 machining based on wear map approach. *Journal of Manufacturing Processes*, *53*, 34-42. https://doi.org/10.1016/j.jmapro.2020.02.006.

Liu, H., & Makis, V. (1996). Cutting-tool reliability assessment in variable machining conditions. *IEEE Transactions on Reliability*, *45*(4), 573-581. https://doi.org/10.1109/24.556580.

Mann, N.R. (2006). Weibull distribution, mann–fertig test statistic for. In: *Encyclopedia of Statistical Sciences*. https://doi.org/10.1002/0471667196.ess1527.pub2.

Musfirah, A.H., Ghani, J.A., & Haron, C.H.C. (2017). Tool wear and surface integrity of inconel 718 in dry and cryogenic coolant at high cutting speed. *Wear*, *376*, 125-133. https://doi.org/10.1016/j.wear.2017.01.031.

Myung, I.J. (2003). Tutorial on maximum likelihood estimation. *Journal of Mathematical Psychology*, *47*(1), 90-100. https://doi.org/10.1016/S0022-2496(02)00028-7.

Ndlovu, B.N., & Ayomoh, M. (2023). Reliability analysis of the functional capabilities of an autonomous vehicle. *International Journal of Mathematical, Engineering and Management Sciences*, *8*(5), 943-965. https://doi.org/10.33889/IJMEMS.2023.8.5.054.

Niu, J., Huang, C., Li, C., Zou, B., Xu, L., Wang, J., & Liu, Z. (2020). A comprehensive method for selecting cutting tool materials. *The International Journal of Advanced Manufacturing Technology*, *110*, 229-240. https://doi.org/10.1007/s00170-020-05534-0.

Qehaja, N., Abdullahu, F., & Zhujani, F. (2017). Predictive mathematical modeling of tool life based on cutting parameters and workpiece material hardness using regression analysis. *International Journal of Mechanical Engineering and Technology*, *8*(8), 1229-1237.

Rao, T.B. (2022). Reliability analysis of the cutting tool in plasma-assisted turning and prediction of machining characteristics. *Australian Journal of Mechanical Engineering*, *20*(4), 1020-1034. https://doi.org/10.1080/14484846.2020.1769458.

Sakharov, G.N., Ilinykh, V., & Konyukhov, V.Y. (1990). Improvement of fastening elements in an assembled cutting tool. *Soviet Engineering Research*, *10*(11), 102.

Sharma, G., Sahu, P.K., & Rai, R.N. (2022). Imperfect maintenance and proportional hazard models: A literature survey from 1965 to 2020. *Life Cycle Reliability and Safety Engineering*, *11*(1), 87-103. https://doi.org/10.1007/s41872-021-00181-0.

Tang, S.H., Zhang, X.P., Xie, W.Q., Liu, Q.S., Wu, J., & Chen, P. (2023). A new evaluation method to quantify the wear failure of irregular cutting tool during shield TBM tunneling in abrasive sandy ground. *Engineering Failure Analysis*, *146*, 107011. https://doi.org/10.1016/j.engfailanal.2022.107011.

Therneau, T.M., & Grambsch, P.M. (2000). The cox model. In Therneau, T.M., & Grambsch, P. M. (eds), *Modeling Survival Data: Extending the Cox Model* (pp. 39-77). Springer, New York. https://doi.org/10.1007/978-1-4757-3294-8_3.

Upadhyaya, G.S. (2004). Trent EM, Wright PK: Metal cutting," Butterworth-Heinemann", Boston, 2000. *Science of Sintering*, *36*(1), 54.

Wager, J.G., & Barash, M.M. (1971). Study of the distribution of the life of HSS tools. *Journal of Engineering for Industry*, *93*(4), 1044-1050. https://doi.org/10.1115/1.3428041.

Wang, B., Liu, Z., Song, Q., Wan, Y., & Shi, Z. (2016). Proper selection of cutting parameters and cutting tool angle to lower the specific cutting energy during high speed machining of 7050-T7451 aluminum alloy. *Journal of Cleaner Production*, *129*, 292-304. https://doi.org/10.1016/j.jclepro.2016.04.071.

Weinert, K., & Kempmann, C. (2004). Cutting temperatures and their effects on the machining behaviour in drilling reinforced plastic composites. *Advanced Engineering Materials*, *6*(8), 684-689. https://doi.org/10.1002/adem.200400025.

Whitaker, R. (1987). The analytic hierarchy process – what it is and how it is used. *Mathematical Modelling*, *9*, 161-176. https://doi.org/10.1016/0270-0255(87)90473-8.

Wiklund, H. (1998). Bayesian and regression approaches to on-line prediction of residual tool life. *Quality and Reliability Engineering International*, *14*(5), 303-309.

Zhang, Z., Yang, Y., Li, G., Qi, Y., Yue, C., Hu, Y., & Li, Y. (2023). Machining accuracy reliability evaluation of CNC machine tools based on the milling stability optimization. *The International Journal of Advanced Manufacturing Technology*, *124*(11), 4057-4074. https://doi.org/10.1007/s00170-022-08832-x.

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