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Achieving Consensus in Groups with Low Authoritarianism of Participants in Decision Making

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Abstract
The complexity of formalizing negotiation procedures leads to the search for fundamentally new approaches to building mathematical models. The theoretical study of consensus makes it possible to analyze various situations encountered by social groups participating in the group decision-making process, leaving aside specific characteristics of the groups. In the article, we built a mathematical model for a group with low authoritarianism among participants based on modeling using Markov chains. The analysis of the model showed that as the leniency of the group members increases, the time to reach consensus increases exponentially, apparently due to the negotiators' lack of desire to take responsibility for the decision. The differences between the number of negotiations in groups of participants with low authoritarianism are greater as the size of the group is smaller. This indicates a lack of coordination in such groups in the absence of a desire to take responsibility for the decision-making. It was revealed that in a dyad of two absolutely lenient participants, consensus is unattainable. For a group with low authoritarianism among participants, psychological "traps" such as false consensus or groupthink are possible, which can sometimes lead to managerial decisions with serious consequences.

Keywords- Decision making, False consensus, Groupthink, Social groups, Markov chains, Time to reach consensus.

1. Introduction

The process of reaching a consensus based on the possibility and ability of the group members to compromise has been poorly understood. Consensus decision-making is an alternative to the commonly practiced voting-based group decision-making processes. Critics of such processes believe that the latter generate competitive debate and create excessive competition in the group of experts (for example, in a social group). These dynamics can damage group members' relationships and destroy their ability to implement a common solution (Newcomb, 1959; Krichevsky and Dubovskaya, 2009). Moreover, a vote-based decision imposes rules of hierarchical relations and sets the parties against each other, since it creates winners and losers in the discussions.

The complexity of formalizing the negotiation procedure leads to the search for fundamentally new approaches for building mathematical models. The need to build such models for studying interaction in groups and the factors affecting the time to reach consensus are related to the possibility of increasing the duration of the real process. The theoretical study of consensus makes it possible to analyze various real situations, abstracting from the specific features of certain social groups in which decisions are made by consensus. It is important to note that the study of consensus in practice within the framework of social

psychology raises many questions related to ensuring the reproducibility of research results. As follows from the fundamental study of the collaboration by evidence-based psychologists, out of 100 original experimental studies in the field of social psychology, other groups managed to reproduce no more than 39 experiments (Aarts et al., 2015).

The work of DeGroot demonstrated the fundamental possibility of describing the process of reaching consensus based on Markov chains (DeGroot, 1974). Markov chains are widely used in solving applied questions in various fields of activity (Buechel et al., 2015; Das and Chakraborty, 2022; Kumar and Kumar, 2023; Niu and Zhou, 2023; Tandon et al., 2023). The model of reaching consensus is based on the assumption that participants exchange opinions and can influence the opinions of other group members during the discussion.

Recently, this model has been applied in different fields, for example, in network automation management (Chebotarev, 2010), in negotiation processes (Mazalov and Tokareva, 2012), as well as when managing social networks (Chkhartishvili et al., 2018).

Previous research on group dynamics conducted by the authors did not cover cases where the group consisted of negotiators with a low level of authoritarianism. Meanwhile, in such groups, a situation where the trust of each member in his own opinion is less than the trust in the opinion of the rest (a group of lenient negotiators) is possible. This situation may seem unusual: a participant trusts himself less than the rest of the group. However, an analysis of the literature showed that such cases are not uncommon (Aronov and Maksimova, 2023). In this case, such a group may find itself in a psychological "trap", which is called a false consensus. As a rule, a false consensus occurs when the members of the group are lenient, incompetent in the issue under consideration, or indifferent; therefore, they try to give in to each other during the negotiations. As a result of discussions in the group, a decision is made that does not fit anyone in the group. This situation was first considered by the American social psychologist John Harvey who called it the Abilene paradox (Harvey, 1988). The Abilene paradox is a paradox in which the family, against their will, went to the Abilene (Texas) only so that the members of the family (group) would not be upset; none of the family members showed their will. As a result, everyone was dissatisfied with the decision they themselves came to.

A group of lenient members can also form a consensus decision, which is called "groupthink" (Janis, 1972). The consequences of such compliance might lead to incorrect management decisions. One recalls the decision of US President John F. Kennedy and his advisers to carry out the famous invasion of Cuba in 1962, the decision of US President Lyndon Johnson and his team regarding the escalation of the Vietnam War in 1964, the decision of NASA in 1986 about the launch of the Space Shuttle Challenger, which exploded immediately after the launch, and the collapse of the Enron Corporation. An overview of some cases related to groupthink is given in (Hart, 1991).

In other words, lenient negotiators can sometimes reach a consensus that, in practice, could entail dire consequences, which makes the present study relevant.

The aim of the study is the development and analysis of a consensus model for a group of members with low authoritarianism[1] (including a group of lenient negotiators) based on statistical modeling of regular Markov chains as well as identification of the specific features of this model in relation to those previously

[1]A group of lenient experts implies that the trust of each member in his own opinion is less than the trust in the opinion of the rest. For a group of negotiators with low authoritarianism, the requirement for a level of trust in everyone else that is less than in themselves is not needed.

considered.

It is relevant for practice to study the dynamics of a social group where participants have low authoritarianism and often give way to each other in the process of finding a solution.

2. Systematic Literature Review

2.1 Game Theory and Social Networks

Negotiation models ultimately lead the researcher to the tools of game theory since negotiators pursue their own (often antagonistic) goals (Mazalov and Tokareva, 2012; Aronov et al., 2018; Das and Chakraborty, 2022; Niu and Zhou, 2023). However, the model considered in the study is closest to social networks, where participants, as a rule, are united by common interests and tasks, and relationships of trust of various levels are built. This important problem of trust at various levels in social group negotiations is considered, for example, in the works of (Hua and Jing, 2023; De Vreede et al., 2013), by constructing an index of closeness (disagreement) of initial preferences of group members.

The main advantages of these models are that this type of model helps to avoid the situation of forming a group of "ignoramuses". Moreover, the work (Del Moral et al., 2018) shows that at each stage, the moderator of the negotiation process knows the degree of agreement, which allows him to determine whether a state of consensus in the group has been reached or not. However, it is not specified how this index can be measured in practice. The work (Vreede et al., 2013) examines the influence of mental models on the effectiveness of negotiation results in small groups.

The disadvantages of these models are that they usually explore the influence of only one factor on the result in the group, although there are many others.

Conclusion. It was revealed that teams with a higher shared mental model index score will have a higher level of consensus.

Note that in most sociological studies, the negotiation process is well studied only in small groups (Hare, 1973; Baron and Kerr, 2003; Del Moral et al., 2018; Myers and Twenge, 2021).

2.2 Socio-psychological Research

Issues of ensuring consensus have been considered in various socio-psychological studies related to group dynamics (Lewin, 1973; Rogov, 2007; Podoprigora and Tytar, 2014). Among the factors that positively influence group effectiveness, the researchers distinguish high competence in social interaction, the ability to defend one's point of view and listen to the opinions of all group members, personal motivation, and goodwill. Personality characteristics that negatively affect teamwork include lack of flexibility, desire to dominate, inability to reconsider one's point of view, and aggressiveness (Mitchell, 2017).

The main advantages of these studies are that they focus on the technology of negotiations, the formation of rules and mechanisms governing the resolution of specific conflicts, as well as the systemic activities of governmental institutions and socio-political organizations in order to ensure public harmony, etc.

Conclusion: Understanding the significance of such approaches is developed only in a few disciplinary fields - institutional-sociological, socio-psychological, jurisprudential, and in the theory of international relations" (Grishina, 2008).

2.3 Delphi Method

Another very popular method for achieving consensus, where special attention is paid to the initial characteristics of the group, is the Delphi method. The Delphi method was developed in the 1960s to achieve consensus among a group of qualified experts by the following authors: Olaf Helmer, Norman Dalkey and Nicholas Rescher (Helmer, 1977). The Delphi procedure has four essential features: anonymity, iteration, controlled feedback of the participants' judgments, and statistical aggregation of group members' responses (Rowe and Wright, 2001). This method has been widely used for business forecasting, in many health-related fields, including clinical medicine, public health, this method helps to help develop professional guidelines (Taylor, 2019; Zabolotskikh et al., 2021; Gluckman et al., 2022; Lazarus et al., 2022; Niederberger and Renn, 2023).

The main advantages of this model: Delphi is based on the principle that forecasts (or decisions) from a structured group of individuals are more accurate than those from unstructured groups (Rowe and Wright, 2001).

The disadvantages of this model: There are obvious deficits in the practice and rigor of consensus measurement for Delphi research.

(i) The response of experts to the task will depend upon the extent of their knowledge about the task to be forecast. This may affect their confidence they have in their own opinions and how much weight they give to feedback from anonymous participants (Rowe and Wright, 2001).

(ii) There may be a disregard for other points of view related to the issue.

(iii) With larger groups come greater administrative cost in terms of times and finances (Rowe and Wright, 2001; Von Der Gracht, 2012).

(iv) Moderator powers are too high.

(v) The method takes a long time. Therefore, it is not suitable for operational analysis.

(vi) As the group grows, the conformity of experts increases; experts often strive to become part of the majority. Note that the majority opinion is not necessarily correct. Therefore, the method often discards creative solutions (minority opinion) that may be most effective. Many authors note that groups for this method should be from 5 to 20 people. Groups that exceed this size stop improving decision accuracy as new members are added (Settle and Armstrong, 1979; Rowe and Wright, 2001).

(vii) Special attention has to be paid to the formulation of the Delphi theses and the definition and selection of the experts in order to avoid methodological weaknesses that severely threaten the validity and reliability of the results; mistakes in statistical tests or their premises have even been made (Von Der Gracht, 2012; Mauksch et al., 2020).

Conclusion: Thus, the Delphi method requires the fulfillment of conditions for the formation of a group: it must consist of professional experts to solve problems not related to the development of innovative solutions. Thus, the Delphi method does not build trust at various levels. Social groups may be more diverse. For example, coalitions may arise within a social group. Coalition formation is a dynamic process in any social group (Myers and Twenge, 2021). In coalitions, regardless of the vector of initial opinions, consensus is impossible. For example, in standardization, this has led to the formation of a new type of standardization

documents – incomplete consensus documents. The effectiveness of these documents was manifested during the COVID-19 pandemic, when many national standardization bodies in order to accelerate the process began to develop standardization documents on personal protective equipment (for example, face masks) based on incomplete consensus standards instead of common consensus standards (Aronov et al., 2021).

All described consensus methods and articles do not explore issues related to the number of negotiations before reaching consensus under different initial conditions, for example:
a) The influence of group size on the number of approvals,
b) How the process will be reduced when the initial characteristics of the group change.
The model considered in this study allows us to answer these questions.

2.4 Model Based on Regular Markov Chains

In the works of Aronov et al. (2018) and Zazhigalkin et al. (2019), a theoretical consensus model based on regular Markov chains was built (Aronov et al., 2018; Zazhigalkin et al., 2019). The analysis of this model was devoted to the issues of the time to reach consensus and the factors influencing this time. It is shown that the time until consensus is reached and the quality of the decision made under the conditions of a fixed number of group members and group structure is associated with two main characteristics: the initial opinions of experts and their authoritarianism.

The main advantages of this model are the ability to identify and study the key characteristics of the group and form practical recommendations to prevent delaying the process of making an agreed decision.

The disadvantages of this model: despite these advantages, the model does not reveal the problem of correlating consensus and truth, when the deliberate formation of a group of "ignoramuses" leads to a consensus that is far from the truth (Myers and Twenge, 2021). Therefore, the problem of approaching the truth of a collective decision and analysis of the experience of using consensus procedures deserves special attention but is beyond the scope of the current study.

Conclusion. As a result, we can conclude that researchers are trying to investigate the influence of the initial socio-psychological characteristics of the group and the proximity of the initial preferences of its members on the outcome of the negotiations.

The present study examines the authoritarianism of group members as one of the integral characteristics that can be measured using preliminary tests (Adorno, 2001; Krichevsky and Dubovskaya, 2009). In these tests, an indicator is formed, measured as a percentage, which can easily be converted into the interval (0, 1), which is used in this study.

3. Theoretical Model of Consensus in a Group with Low Authoritarianism of Participants and Description of Modeling

Let us describe a model of consensus in the process of reaching agreement based on regular Markov chains (Aronov and Maksimova, 2023). Let a group of n negotiators discuss some issue. Each of the participants adheres to its initial opinion, which is described by the vector,

$$S(0) = (S_{01}, S_{02,\ldots,} S_{0n})$$

where, $S_{0i}$is the opinion of the i-th member, $i = 1, \ldots, n$. Members of the group exchange views on the vector $S(0)$.

Let's consider how the members opinions changes as a result of rounds of negotiations. Let $0 < p_{ij} < 1$ (i = 1, …, n; j = 1, …, n) define the probability that the i-th member of the group trusts the opinion of the j-th expert. At the same time, the i-th member also trusts himself with a certain probability $0 < p_{ii} < 1$, which is interpreted as the level of authoritarianism of the i-th member of the group: the higher the value of p_{ii}, the higher his authoritarianism[2]. The resulting confidence matrix $\mathbf{P} = (p_{ij})$ is stochastic (the condition $\forall i \in \overline{1,n}\left[\sum_{j=1}^{n} p_{ij} = 1\right]$ is satisfied).

At *the first step* of the negotiations, we get the vector,

$$S^T(1) = \mathbf{P}\cdot S^T(0) = (S_{11}, S_{12,\dots,} S_{1n})^T.$$

Thus, when a group member revises his opinion, his new opinion will be a linear combination of the opinions of the remaining group members. For example, the first component of the new vector of opinions after the first step is equal to,

$$S_{11} = p_{11} \cdot S_{01} + p_{12} \cdot S_{02} + \cdots + p_{1n} \cdot S_{0n}.$$

At *the second step* of the negotiations, we get the vector,

$$S^T(2) = \mathbf{P}\cdot S^T(1).$$

Consider, for example, the first component of the new vector of opinions after the second step is equal to

$$S_{21} = p_{11} \cdot S_{11} + p_{12} \cdot S_{12} + \cdots + p_{1n} \cdot S_{1n}$$

$$= p_{11} \cdot (p_{11} \cdot S_{01} + p_{12} \cdot S_{02} + \cdots + p_{1n} \cdot S_{0n}) + \cdots + p_{1n} \cdot (p_{n1} \cdot S_{01} + p_{n2} \cdot S_{02} + \cdots + p_{nn} \cdot S_{0n})$$

$$= (p_{11} \cdot p_{11} + p_{12} \cdot p_{21} + \cdots p_{1n} \cdot p_{n1}) \cdot S_{01} + \cdots + (p_{11} \cdot p_{1n} + p_{12} \cdot p_{2n} + \cdots p_{1n} \cdot p_{nn}) \cdot S_{0n}.$$

Carrying out such calculations for each component, the resulting vector at the second stage can be represented as,

$$S^T(2) = \mathbf{P}\cdot S^T(1) = \mathbf{P}\cdot\mathbf{P}\cdot S^T(0) = \mathbf{P}^2\cdot S^T(0) = (S_{21}, S_{22,\dots,} S_{2n})^T.$$

And after the k-th *step* we get the vector

$$S^T(k) = (S_{k1}, S_{k2,\dots,} S_{kn})^T = \mathbf{P}\cdot S^T(k-1) = \mathbf{P}^k\cdot S^T(0).$$

We get a Markov chain. Note that despite the fact that after each step the opinion of group members changes, the formula for $S^T(k)$ contains the degrees of the confidence matrix **P** and the initial opinion vector $S(0)$.

The process ends at the m-th step, when all rows of the matrix $\mathbf{P}^m$ become the same (with a given accuracy), i.e. group cohesion has been achieved. The confidence matrix **P** after m iterations reaches the final matrix **F**, in which all corresponding row elements are equal. Thus, in subsequent discussions, the matrix **P** will not change, therefore, the opinion vector,

$$S^T(m) = \mathbf{P}^m\cdot S^T(0) = \left(S_{m1}, S_{m2,\dots,} S_{mn}\right)^T,$$

will also not change. Thus, after m iterations, a consensus is reached. This approach provides a mathematical definition of consensus. From the theory of convergence of the initial matrix **P** to the final matrix **F** (a

[2]Authoritarianism [from lat. Autoritas – influence, power] – a socio-psychological characteristic of a person reflecting his/her desire to maximally subordinate partners in interaction and communication to his/her influence.

necessary and sufficient condition for reaching consensus) for any vector $S(0)$ is the regularity[3] of the matrix **P**. For the regularity of the matrix, it is sufficient that the sums over the rows of the matrix **P** are equal to 1 and for any probabilities p_{ij} the strict inequality $0 < p_{ij} < 1$ was satisfied (Theorem 4.1.2; Kemeny and Snell, 1960). If the confidence matrix **P** is regular, then no matter what the initial opinions of the group members was, consensus is achievable, although it may take a significant number of agreements.

The key to the theoretical study of the phenomenon of consensus in a group, as noted in the introduction, is the analysis of the change in the initial opinion of each member of the group. Let us build a consensus model in large ($n > 5$) and small groups ($n \leq 5$) of participants with low authoritarianism.

Prerequisites for modeling

(i) Let $(S_{01}, S_{02,\ldots,} S_{0n})$ be the vector of initial opinions, and $(S_{m1}, S_{m2,\ldots,} S_{mn})$ be the vector of opinions after *m* steps.

(ii)The model considered earlier by the authors described groups of $n = 5, 10, 20, 50$ people with authoritarianism for which the following condition was fulfilled:

$$\forall i = \overline{1,n}\ \ \exists j = \overline{1,n};\ \left[p_{ii} > p_{ij}\right],$$

$$j \neq i$$

i.e. there is at least one member whose level of confidence in his opinion exceeds the level of confidence of any other member of the group. This was provided by the lower boundary of authoritarianism modeling $p_{ii} = 0.2$ for all groups. For a group of negotiators with low authoritarianism, we will assume that,

$$\forall\ i = \overline{1,n}\left[p_{ii} < \frac{1}{n}\right].$$

At the same time, for a group of lenient[4] negotiators, an additional condition is required: each of them trusts his own opinion less than the opinion of any of the others:

for each $i = \overline{1,n}\ \ \forall j \neq i\ \left[p_{ij} \geq p_{ii}\right]$ (1)

It is easy to see that this condition is equivalent to the following:

$$\forall\ i = \overline{1,n}\left[p_{ii} < \frac{1}{n}\right].$$

For a group of authoritative members, the condition must be met,

$\forall i = \overline{1,n}\ \ \exists j \neq i\ \left[p_{ij} < p_{ii}\right]$ (2)

***Modeling step* 1:** selection of the number of group members. Let us consider small groups with $n = \overline{2,5}$, members and large groups with $n = 10, 20, 50$ members.

Next, let us describe the modeling process for a group of lenient participants. For a group of authoritative participants, the modeling was carried out similarly, but condition (2) was satisfied.

***Modeling step* 2:** choice of levels of authoritarianism p_{ii} $(i = \overline{1,n})$. Based on assessment (1), for groups of

[3]Matrices, where the sums of the elements of all rows are equal to one, are referred to as *stochastic*. If for some *n* all elements of the matrix $\mathbf{P}^n$ are not equal to zero, then such a transition matrix is called *regular*.

[4]Leniency is a socio-psychological characteristic of a person, opposite to authoritarianism.

different sizes, a different upper boundary for members is provided (for example, for $n = 5$ it is 0.2, and for $n = 20$ we obtain an upper boundary of 0.05). Thus, different initial conditions (boundaries of authoritarianism) were for different sizes of groups. Authoritarianism was varied in the range of ± 0.005 from the average level. For example, for $n = 5$, the simulation included the following levels:
$p_{ii} = 0 - 0.01; 0.02 - 0.03; 0.04 - 0.05; 0.07 - 0.08; 0.09 - 0.1; 0.14 - 0.15; 0.19 - 0.2.$

For $n = 20$ the simulation included the following levels:
$p_{ii} = 0 - 0.01; 0.02 - 0.03; 0.04 - 0.05.$

***Modeling step* 3:** simulation of confidence probabilities p_{ij} $(i \neq j;\ i, j = \overline{1,n})$ from the uniform distribution law when condition (1) is met. The implementation of this step was carried out in three stages:

(i) Modeling the elements of the auxiliary matrix Δ with the size *n* x *n* by uniform distribution law on the interval [0, 1], obtaining the initial elements of the matrix Δ_{ij} $(i, j = \overline{1,n})$;

(ii) Formation of the matrix Δ^0 with the size *n* x *n* by adjusting the elements of the matrix Δ: elements of each row,

$$\Delta_{ij}^0 = \frac{\Delta_{ij}}{\sum_{j=1, i\neq j}^{n}(\Delta_{ij})} \cdot (1 - np_{ii}),\ \Delta_{ij}^0 \geq 0.$$

(iii) Modeling of the confidence matrix **P**: for the elements of each row $p_{ij} = p_{ii} + \Delta_{ij}^0$ $(i \neq j)$. The auxiliary matrix Δ is necessary to ensure the lenient condition: for each $i = \overline{1,n}$ $\forall j \neq i$ $\left[p_{ij} \geq p_{ii}\right]$.

Let us prove that the resulting matrix **P** will be stochastic:

$$\sum_{j=1}^{n} p_{ij} = p_{ii} + \sum_{j=1, i\neq j}^{n} \left(p_{ii} + \Delta_{ij}^0\right) = p_{ii} + \sum_{j=1, i\neq j}^{n} \left(p_{ii} + \frac{\Delta_{ij}}{\sum_{j=1, i\neq j}^{n}(\Delta_{ij})} \cdot (1 - np_{ii})\right) =$$

$$= p_{ii} + (n-1)p_{ii} + \frac{\sum_{j=1, i\neq j}^{n}(\Delta_{ij})(1-np_{ii})}{\sum_{j=1, i\neq j}^{n}(\Delta_{ij})} = np_{ii} + 1 - np_{ii} = 1.$$

Thus, after implementing steps 1-3, a simulated confidence matrix **P** is obtained, which is then used to calculate the required degree *m*.

In order the average time to reach consensus with changes in other parameters, 100 simulations were performed in *Excel* for each fixed level of factors. As shown in (Efron and Tibshirani, 1991), such a number is sufficient to obtain consistent conclusions about the average value.

For a group of lenient members, 58 series of 100 simulations were conducted.

For a comparative analysis of the results with the results for a group of authoritative members, another series of simulations was carried out (99 simulations were performed). For such simulations, condition (2) was required, which was ensured by simple normalization of p_{ij} $(i \neq j)$ in each row:

$$\text{since } \forall\ i = \overline{1,n}\ \left[p_{ii} \geq \frac{1}{n}\right], \text{ then } \exists j \neq i\ \left[p_{ij} < p_{ii}\right].$$

Note that for a group of two members ($n = 2$) with authoritarianisms $p_{11} = p_{22} = 0$, a decomposable matrix[5] is obtained

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

In the theory of Markov chains, it is shown, that the corresponding transition matrix does not converge to the final matrix (Kemeny and Snell, 1960).

4. Analysis of Simulation Results and their Interpretation

Let us estimate the convergence time of the opinion matrix **P** to the final matrix **F** = $\mathbf{P}^m$. This time is given by the number *m* of iterations (negotiations within the group) until a consensus is formed. Mathematically, *m* is defined as the power of the matrix **P** such that in the final matrix **F** the elements inside each column *j* satisfy the condition $|p_{ij} - p_{kj}| < \delta$ for all i, k $(j, i, k = \overline{1, n})$. The value of *m* was calculated from the condition δ = 0.001 (this number of decimal places provides an acceptable variability in authoritarianism, based on the levels selected at the 2nd step of modeling). In practice, the value of *m* determines the time it takes to reach consensus.

Figures 1-3 show simulation results for a group of negotiators with low authoritarianism. To interpret the simulation results, we introduce average authoritarianism (*p*) of a group as the arithmetic mean of the authoritarianism of its members.

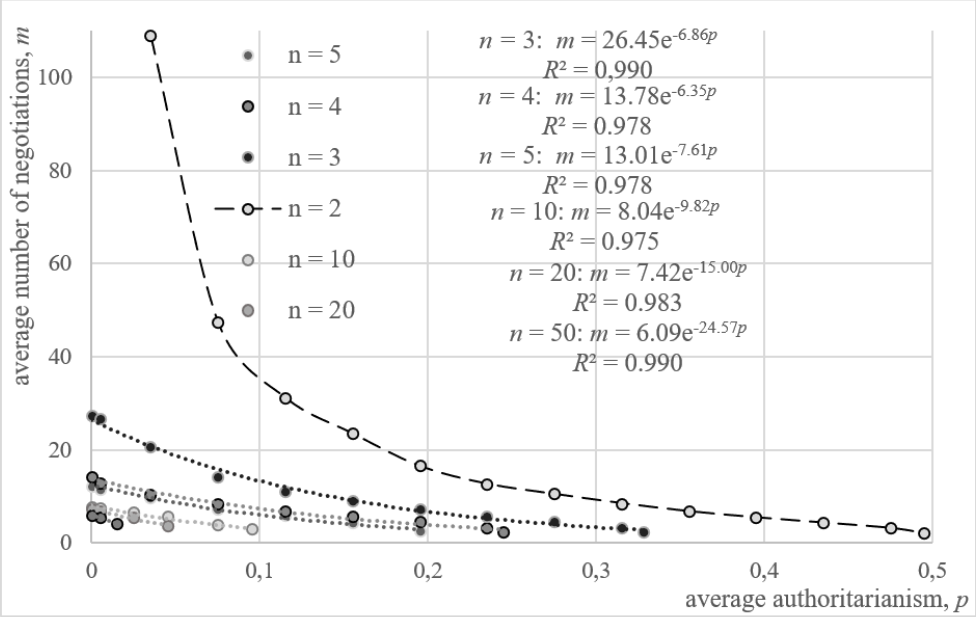
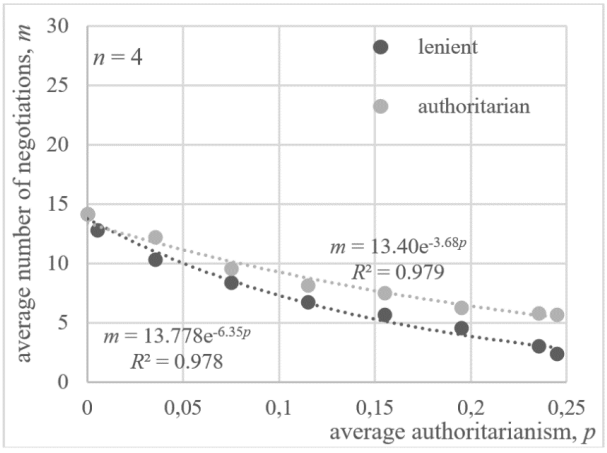
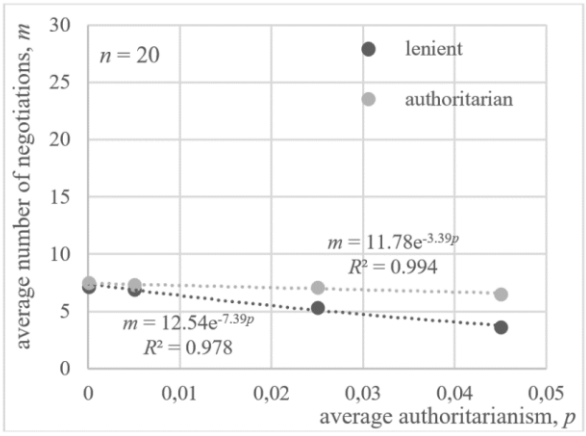


Figure 1. Dependence of the number of negotiations (*m*) on the average authoritarianism (*p*) in the group of lenient participants $n = \overline{2, 5}; 10, 20, 50$.

[5]Matrix A is said to be *decomposable* if it can be reduced to the form $\tilde{A} = \begin{pmatrix} B & 0 \\ C & D \end{pmatrix}$, by a permutation of rows, where B and D are square matrices.



(a)



(b)

Figure 2. Dependence of the number of negotiations (m) on the average authoritarianism (p) in the group of lenient 395 and authoritarian participants with the number of members (a) n = 4, (b) n = 20.

(i) The dependences of the average number of negotiations in a group on the average authoritarianism at fixed n are quite well described by exponential curves (Figures 1-2 show the forms of the dependence and the corresponding coefficient of determination R^2, which exceeds the value of 0.975 for all cases).

(ii) With a decrease in the authoritarianism of group members, the number of negotiations grows (see Figures 1-2). At the same time, the average number of negotiations for the group of authoritative members is slightly higher than for the group of lenient members (see Figure 2). Consensus, as a way to solve a problem, according to N. Taleb makes it necessary for a group member to "stick his neck" forming symmetrical responsibility in the group (Taleb, 2018), which is not acceptable for lenient negotiators! Therefore, a decrease in the authoritarianism of members in such a group indicates the shifting of decision making to others, each expert does not want to take responsibility for the decision, as a result of this situation an increase in the number of negotiations is observed.

(iii) An increase in the size of a group of lenient negotiators leads to a decrease in the number of negotiations when approaching absolute leniency ($p \to 0$, see Figure 1). This important fact allows us to consider the

consensus of lenient members as a special case of consensus. The fact is that in a group of "ordinary" experts, an increase in their number entails an increase in the time until consensus is reached, which is expected (Aronov et al., 2018). Since the initial conditions (authoritarian boundaries) were different for different numbers of groups, for comparison we introduce a group leniency through $p{\cdot}n$ (p is the average authoritarianism of the group members, n is the number of group members). We obtain: with a decrease in group leniency, the models diverge, and with an increase (up to the border-line case $p{\cdot}n = 1$), the models converge as shown below in Figure 3. This suggests that, generally speaking, the processes taking place in groups with a different number of members with very low authoritarianism are different.

(iv) Figure 4 shows the simulation results for a group of negotiators with varying authoritarianism from 0 to 1, which cover cases of groups with low and high authoritarianism. The resulting curves demonstrate the change in the dynamics of the number of negotiations in the transition from low to high authoritarianism. Moreover, the shape of the curve changes when passing through points $p = 1/n$: from exponential for low authoritarianism ($p < 1/n$) to fractional rational for high authoritarianism ($p > 1/n$), and with a vertical asymptote at the point $p = 1$ when a group of absolutely authoritarian participants is formed, for which consensus is unattainable. Thus, in the group of authoritarian negotiators, an increase in their number causes an increase in the time until consensus is reached, which is expected (Zazhigalkin et al., 2019).

(v) When approaching the upper threshold average value of authoritarianism $p = 1/n$, the average number of negotiations for all groups grows slowly (see Figure 5). The relation is well described by a slowly increasing logarithmic dependence, which for a group of 50 experts gives the value $m < 5$ ($R^2 = 0.986$, see Figure 2).

(vi) For a group of two experts, the number of negotiations grows hyperbolically as the authoritarianism approaches zero. With an average authoritarianism $p = 0.025$, the average number of negotiations reaches 153. The simulated situation with two experts demonstrates significant differences in the transition from $n = 2$ to $n = 5$ members in relation to the transition, for example, from $n = 10$ to $n = 20$ members (see Figures 1, 3).

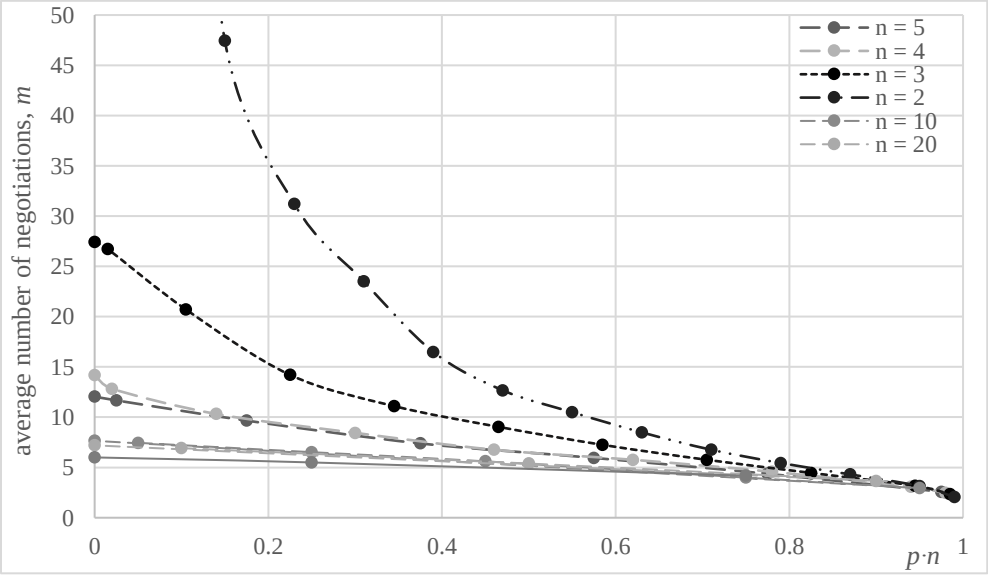


Figure 3. Dependence of the number of negotiations (m) on the authoritarianism of the group ($p{\cdot}n$) in the group of lenient participants $n = \overline{2,5}; 10, 20, 50$.

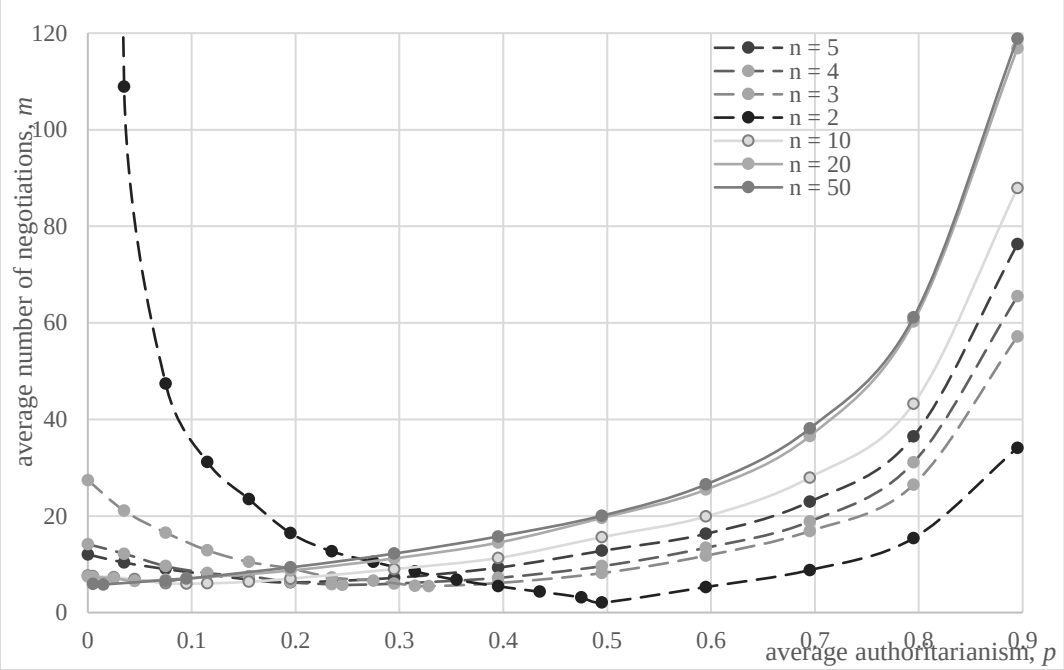


Figure 4. Dependence of the number of agreements (m) on the average authoritarianism (p) in the group of authoritarian participants $n = \overline{2,5}; 10, 20, 50$.

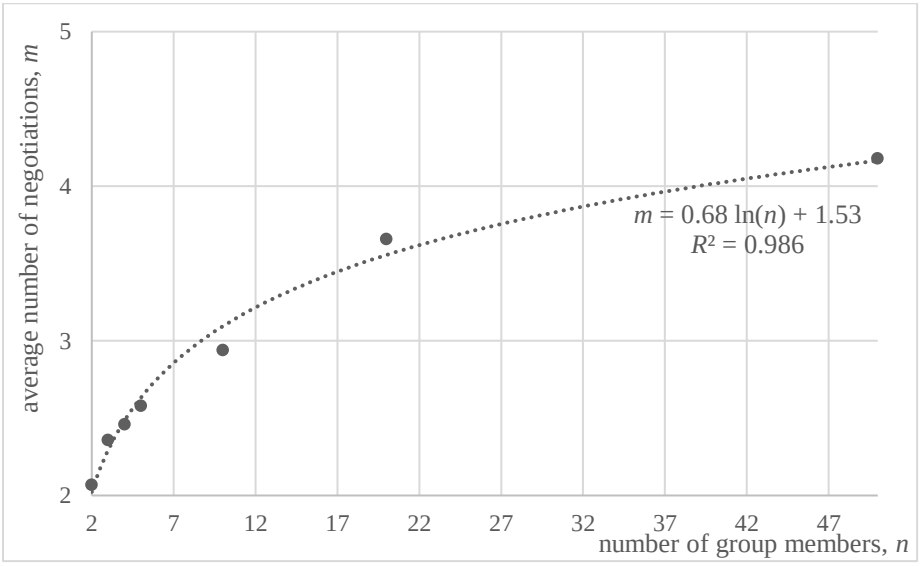


Figure 5. Dependence of the number of negotiations (m) on the number of lenient experts with an average authoritarianism $p = 1/n$.

The findings (i)-(iii) correspond to the results of studies in social psychology: disagreements can be expected to a greater extent in a group with 3-4 people, where a conflict may arise when making a decision, than in a group of 5 or more negotiators (shown in Figures 1 and 3). The increase in the number of people

in such groups from three to four has a greater effect than thein crease, for example, from 20 to 21 negotiators (Kerr, 1989). As the size of the group increases, the interactions between its members become less frequent and more formal, which leads to a weakening of the sense of belonging, and this is an obstacle to the formation of group cohesion. As the well-known American sociologist David Myers pointed out, as the size of the group grows, each additional conformist member adds less and less "strength" to the group (Myers and Twenge, 2021). Thus, with a decrease in the number of negotiators with low authoritarianism, the time difference between small groups becomes more noticeable than between large ones, in contrast to groups with high authoritarian members (on the basis Figure 4). When leniency tends to the equilibrium value $p = 1/n$ (the boundary separating the low and high authoritarianism of the negotiators), the group begins to behave as a single whole, the contribution of each new participant with such authoritarianism practically does not increase the number of negotiations, while the number of negotiations even for the group with 50 such participants does not exceed $m = 5$ (see Figure 5). A group of associate people quickly comes to a consensus, regardless of the size of the group. In addition, during the analysis of the simulation results, it was revealed that with an average authoritarianism tending to $p = 1/n$, the quality of the decision made becomes "equilibrium" for all types of groups. This means that the opinion of each member of the group counts equally in the consensus decision.

It is shown that in the group of lenient negotiators the average time to reach consensus is insignificant, but less than in the group of authoritarian negotiators with low authoritarianism (on the basis Figure 2). At the same time, the quality of the solution achieved in the latter case should be expected to be higher than in the former, when the responsibility for the decision is shifted to any other.

The simulation results showed that dyads the conditions of high leniency of the members can lead to a delay in the negotiation process. With the growth of authoritarianism, the delaying the decision-making can be expected in a group with a large number of participants.

In the introduction it was noted that a false consensus occurs when the members of the group are lenient, incompetent in the issue under consideration or indifferent, therefore, they try to give in to each other during the negotiations. As a result of discussions in the group, a decision is made that does not fit anyone in the group. Let's consider such an example in our model. Let the group consist of 4 people who discuss the requirements for general construction cement in the draft standard. The requirement for the share of clinker (in percentage) in Ordinary Portland Cement (CEM 1) cement is analyzed. Let the vector of opinions regarding the clinker content in cement be as follows: 1st expert – 95 %, 2nd expert – 96 %, 3rd expert – 20 %, 4th expert – 30 %. Let experts 1 and 2 be competent in the issue under consideration, but indifferent, experts 3 and 4 are inexperienced and lenient. As a result, the following trust matrix **P** can be formed:

$$\mathbf{P} = \begin{pmatrix} 0.1 & 0.2 & 0.2 & 0.5 \\ 0.3 & 0.1 & 0.2 & 0.4 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 & 0.1 \end{pmatrix}.$$

Consider the steps of discussions.

At *the first step*: S(0) = (95, 96, 20, 30) – the initial vector of opinions.

At *the second step*: after the second agreement, the confidence matrix will take the form,

$$\mathbf{P}^2 = \begin{pmatrix} 0.21 & 0.22 & 0.33 & 0.24 \\ 0.18 & 0.23 & 0.30 & 0.29 \\ 0.22 & 0.18 & 0.28 & 0.32 \\ 0.20 & 0.28 & 0.18 & 0.34 \end{pmatrix}.$$

Let's note that the opinions of the 3rd and 4th experts are characterized by higher probabilities, which indicates a shift of responsibility onto these experts (the first and second experts are indifferent).

At *the third step*: continuing to raise the matrices to a power, taking into account the rounding of probabilities, for example, to the second decimal place, after the sixth agreement we arrive at the following matrix:

$$\mathbf{P}^7 = \begin{pmatrix} 0.203 & 0.230 & 0.264 & 0.303 \\ 0.203 & 0.230 & 0.264 & 0.303 \\ 0.203 & 0.230 & 0.264 & 0.303 \\ 0.203 & 0.230 & 0.264 & 0.303 \end{pmatrix}.$$

This matrix can be considered as the final matrix **F** (all column-wise probabilities are equal).

At *the fourth step*: returning to the vector of expert opinions S(0) = (95, 96, 20, 30), taking into account the matrix **F**, we obtain the resulting consensus decision:

$$\mathbf{S} = \mathbf{S}(7) = 0.203 \cdot 95 + 0.230 \cdot 96 + 0.264 \cdot 20 + 0.303 \cdot 10 \approx 50\ \%.$$

It is clear that the resulting solution is a false consensus, since to ensure the strength of cement, the clinker content in it must be more than 95 %. This is how the consensus of the ignoramuses is formed according to the terminology of the mathematician Orlov (2017).

5. Results and Discussion

The consensus model built for lenient experts complements the model for a group of authoritarian members investigated by the authors earlier. Here we present some key results obtained for a group of authoritarian members.

(i) An increase in the number of group members negatively affects the time to reach consensus when the number of members in the group is more than five (Aronov et al., 2018; Zazhigalkin et al., 2019; Maksimova and Aronov, 2023).

(ii) With an increase in the authoritarianism of the group members (regardless of the group size), the time to reach consensus significantly increases (Zazhigalkin et al., 2019; Aronov et al., 2018). For an authoritarian group, decision-making based on the "consensus minus *k*" principle, where *k* is the number of experts in the group whose opinion is not taken into account, can dramatically reduce the time to reach consensus (Aronov and Maksimova, 2022).

(iii) If there are coalitions in the group, consensus is impossible. But the consensus can always be built at a unilateral concession (Maksimova and Aronov, 2021). An increase of the compromise size (other things being equal) results in a sharp decrease of the time until the consensus is built. It is shown that the time until the consensus is built is poorly dependent on the "strength" of the coalition. The study of the variability of the number of negotiations to reach consensus shows that the structure of the coalition accepting a small concession has an influence on the number of negotiations to reach consensus.

(iv) The presence of a leader in the group ensures a more harmonious consensus by taking into account the positions of all group members in the overall decision. The presence of a leader in the group (due to job responsibilities) ensures consensus based on the leader's opinion (Aronov and Maksimova, 2022).

The model for lenient experts that was built and analyzed in the present study produced unexpected results, especially in small groups. To analyze-the constructed model, simulations were carried out for groups with $\overline{2,5}; 10, 20, 50$ members. Let us give the main results obtained.

(i) It was found that in such groups, the growth of members' leniency (decrease in authoritarianism less than $1/n$) leads to an exponential growth in the number of negotiations (with a fixed number of members, more than two). This indicates the inconsistency of the group in the absence of a desire to take responsibility for the decision-making.

(ii) The growth of the group leads to the following results:

- With a decrease of leniency (approaching average authoritarianism of experts $p = 1/n$), there is a slow logarithmic increase in the number of negotiations;
- When approaching absolute leniency (average authoritarianism of experts $p \to 0$), the number of negotiations decreases.

(iii) The smaller the size of a group of participants with low authoritarianism, the greater the differences between the number of negotiations. This indicates the inconsistency of the group of lenient members in the absence of a desire to take responsibility for the decision being made.

(iv) The study proved that it is very difficult to reach consensus in dyads with low authoritarianism. The time to reach consensus can increase more than 10 (and even 100) times compared to a group of 5 members or more, since there is a mutual shifting of responsibility for making decision, which delays the negotiation process itself. On the contrary, in conditions of high authoritarianism of group members, a delay in the discussion can be expected with an increase in the number of members. The theoretically important result obtained can serve as evidence that the dynamics of small groups with the number of experts less than 5 people and low authoritarianism have significant specifics compared to large social groups of “ordinary” experts. This allows us to formulate a practical rule for forming, for example, a group of auditors: in conditions of uncertainty, the lower limit on the number of auditors is a triad (three people).

(v) In the group of participants with low authoritarianism but not lenient ones, the number of negotiations on average is slightly higher. As a result of the negotiations, discussion occurs rather than shifting responsibility to someone else, and a consensus decision is expected to be more balanced.

6. Conclusion

The paper builds a theoretical model of consensus for a group of participants with low authoritarianism, based on regular Markov chains.

One of the general practical recommendations for avoiding delay in the process of making a consensus decision-arising from the results of the study is that when increasing the number of members in a group with low authoritarianism, it is necessary to include fewer authoritarian auditors, and with a decrease in the number of members, it is required to include more authoritarian members.

All studies concerning consensus show that, along with the positive aspects, the consensual method also has significant costs associated, first of all, with delaying decision-making due to the need for multiple approvals. This factor should be the focus of the research. Consensus, as a method of decision-making, always requires significant time and resources. The general recommendation in conditions of uncertainty is to form a group of 5 to 20 people, which is consistent with the results of other negotiation models (Settle and Armstrong, 1979; Rowe and Wright, 2001; Zazhigalkin et al., 2019).

The DeGroot model has not exhausted itself and allows one to find correct solutions in various practical situations. In recent years, due to the development of research on various processes in information networks and the improvement of multi-agent systems, this model has become essential (Jackson and Golub, 2007; Buechel et al., 2015).

It can be assumed that this is due to both ease of interpretation and wide possibilities for adaptation to various practical cases. Currently, it is planned to develop a standardization document containing recommendations for achieving consensus in technical committees on standardization that reflect the provisions of this paper.

7. Future Direction of the Research

The model does not address the problem of correlating consensus and truth, when the deliberate formation of a group of "ignoramuses" leads to a consensus that is far from the truth. This situation is demonstrated and analyzed in one example in the study. Therefore, the problem of approaching the truth of a collective decision deserves special attention and requires further research. It is connected with the study of the structure of the vector of initial opinions: the proximity of its components to each other and the proximity to the truth. Consideration of this factor in the model will provide an opportunity for further improvement and study.

Unfortunately, there are currently no guidelines on how to generate a consensus model that would meet all of the requirements listed in the study (Taleb, 2020).

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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