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Multivariate Exponentially Weighted Moving Average Control Chart under Neutrosophic Environment: A Bootstrap Approach

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Abstract
The Multivariate Exponentially Weighted Moving Average (MEWMA) control chart is an effective tool for monitoring small shifts in the mean vector of multiple correlated variables over time. The traditional MEWMA control charts are not appropriate when dealing with data that has indeterminacy. For the purpose of dealing with indeterminate data, we present a novel Neutrosophic MEWMA Control Chart that incorporates bootstrap control limits in this research. A simulation study is conducted to compare the performance of the proposed method with the neutrosophic Hotelling T^2 control chart. The study considered Alarm Rate (AR) and Average Run Length (ARL) have been used as the performance evaluation metrics. Finally, an illustrative example from the chemical industry was used to demonstrate the application of the proposed chart. It is considered that the proposed chart can be effectively applied to a wide range of manufacturing processes, providing significant benefits in process monitoring and control.

Keywords- SPC, Multivariate control charts, Hotelling's T^2 control chart, MEWMA, Neutrosophic logic, AR, ARL.

1. Introduction

In the modern world, updates are made on a daily basis and for the manufacturing industry, it is difficult to survive in the market. However, ensuring the quality of a product is necessary and the Statistical Process Control (SPC) is the method of quality control which is used to monitor and control entire production process. Control charts are the most prevalently employed tool in SPC. (Aslam et al., 2021), and they help to detect the sources of errors by identifying the special and common causes of variations. Mostly in a production process, we need to monitor and control more than one variable at a time. In such situations, it is mandatory to use multivariate control charts for better results. The multivariate approach to the control chart technique offers a more comprehensive understanding of the production process and helps to identify sources of variation over time (Chong et al., 2019).

Hotelling's T^2 control chart is a popular method used in multivariate control charts which measures the difference between the sample mean vector and the target mean vector, standardized by the sample covariance matrix. Since, Hotelling's T^2 control charts are not highly sensitive to small or moderate shifts in the mean vector as they only use information from the current sample (Tiryaki and Aydin, 2022). Researchers have suggested the development of the Multivariate Cumulative Sum (MCUSUM) (Haq, 2018) and MEWMA (Kim et al., 2017) control chart to improve the sensitivity of multivariate quality control problems to small shifts. These control charts, provide an efficient approach by monitoring the cumulative deviations of the sample mean from the target value or by applying weights to the observation depending on the time of occurrence. Since, they are capable of detecting minor changes to the process mean vector as soon as possible. As a result, they provide a major advantage where the early identification of process change is vital in ensuring high-quality products and maintaining customer satisfaction. The MEWMA control chart has the advantage over the MCUSUM control chart in that it considers variable correlation, whereas the MCUSUM chart assumes variable independence. This indicates that, compared to the MCUSUM chart, the MEWMA control chart is more sensitive to small changes in the process mean. Furthermore, the MEWMA control chart may accommodate variations in the process mean with greater flexibility. On the other hand, if there are modifications to the variable's correlation structure, the MCUSUM chart can require recalibration. A computer programme has been developed by Molnua et al. (2001) to compute ARLs for the MEWMA control chart. Moreover, Scranton et al. have demonstrated that by restricting its application to the primary components of the variables under observation, the ARL performance of MEWMA control chart can be further enhanced (Scranton et al., 1996).

Control limits of multivariate control chart are frequently determined using assumptions about the distribution of the data, such as normality. In actuality, though, the data might not be distributed normally. Without assuming anything about the underlying distribution, the bootstrap approach can estimate the distribution of the statistic of interest (such as the mean or standard deviation) from the sample data (Haydée Baranzano, 2011).The bootstrap method can be useful in control charts for improving the effectiveness of the control limits, mainly when dealing with small sample sizes or non-normal data. Mostajeran et al. (2016) proposed the percentile bootstrap method to obtain Hotelling's T^2 control limits under the non-multivariate normal distribution. However, this method had limitations as it only detected large shifts by collapsing the multivariate data into univariate. Furthermore, it did not address the out-of-control signal issue. For further development, Ikpotokin and Ishiekwene (2017) introduced the Bootstrap Multivariate Exponentially Weighted Moving Average (BMEWMA) method based on non-parametric control chart approach for addressing the problems of violating multivariate distributional assumptions and the inability to detect small to moderate shifts in the process mean vector. In this study, they adopted a bootstrap control limit for the MEWMA control chart by applying the bootstrap control limit, which offers a significant advantage over the traditional method in deriving the control limit in a complex manner.

Generally, the design of the classical approach to the control chart technique is based on the assumption of accurate data values, which may not be readily available. The underlying uncertainty in the parameters of control charts or inaccuracy in observed quality characteristics can lead to inaccurate data in practical situations. In such cases, it is not recommended to use traditional control charts, and it necessitates the development of a fuzzy control chart to handle data uncertainty (Shu and Wu, 2011). The literature reveals a growing trend among researchers to develop control charts based on fuzzy parameters (Ghosh et al., 2022; Giri et al., 2023a; Giri et al., 2023b; Maity and Roy, 2019; Mondal et al., 2023). The primary benefit of fuzzy control charts is that they are more sensitive than traditional control charts. These approaches to uncertainly provide an immense amount of flexibility for users to handle fuzzy data (Sentürk et al., 2014). In addition to offering considerable flexibility, this method allows users to manage fuzzy data encountered during the measurement process. According to Bradshaw (1983), fuzzy theory was introduced into the construction of control charts for product conformity to specified limits. This approach illustrates the realism of graded product criteria by utilizing graded product criteria in a fuzzy concept over a binary category. Further investigation of the fuzzy approach in designing control charts for linguistic variables has been attempted (El-Shal and Morris, 2000; Rowlands and Wang, 2000). Furthermore, Fuzzification has also been applied to construct traditional CUSUM and EWMA control charts (Tannock, 2003; Wang, 2006). The design of multivariate control charts in fuzzy mode has been explored in the work by Moheb Alizadeh et al. (2010).

In recent years, the theory of neutrosophic logic, a broader framework that incorporates fuzzy sets has gained recognition in a variety of fields. Neutrosophic logic, an extension of fuzzy logic, has developed for dealing with indeterminate data. This is the case since many real-world scenarios are neutrosophic, which means the data obtained may be incomplete, imprecise, or inconsistent. For instance, we can consider a plastic containers manufacturing process. The process is controlled by monitoring variables such as thickness and weight of the containers. However, the measurements of thickness and weight may be inaccurate and indeterminate due to changes in the manufacturing process. In such situations traditional control charts may not provide accurate decision to manage this data. While the indeterminacy is present in data we could not express in data terms of crisp values so we need to go for interval values instead of crisp numbers. Furthermore, including indeterminacy in to investigation, neutrosophic logic provides a flexible frame work for control chart technique. It permits the use of truth, falsehood, and indeterminacy to describe uncertainty (Smarandache, 2014). Control charts that use neutrosophic logic may take into consideration the uncertainty in the data, which makes the chart more sensitive and enhances the precision of process control. The effectiveness of neutrosophic-based analysis has been demonstrated by Chen et al. (2017a), through applying neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. The application of neutrosophic logic has been increasing in recent studies, such as in the chemical industry, health sector, rock measuring industry, etc. (Aslam, 2018; Aslam, 2019a; Aslam, 2019b; Aslam et al., 2018; Aslam and Albassam, 2019; Chen et al., 2017b; Giri and Roy, 2022).

In recent studies, Wibawati et al. (2022) have developed a multivariate Hotelling T^2 control chart for neutrosophic data for individual measurements using the beta distribution. For further developments, Saritha and Varadharajan (2023) proposed a novel approach to Hotelling T^2 control chart for sub grouped data in a neutrosophic environment by using F distribution. Additionally, to compare the effectiveness of the method, the Neutrosophic Alarm Rate (NARL) has been calculated. The Neutrosophic Hotelling T^2 control chart is an efficient tool for monitoring multivariate data in a neutrosophic environment but it is often inadequate for detecting small shifts in the process. To overcome this issue, mainly in non-parametric circumstances, we developed an NMEWMA control chart that combines the benefits of neutrosophic logic with the bootstrap approach.

The current study is structured into five sections. Section 2 briefly outlines the design of the neutrosophic BMEWMA control chart. In Section 3, a simulation study is conducted and compared to Hotelling's T^2 control chart. An illustrative example is presented in Section 4. Finally, Section 5 provides the conclusion of this study.

2. Designing of the Control Chart

This section focuses on developing an expression for the control limit of the proposed neutrosophic BMEWMA control chart. This section assumes that the observations are taken from a neutrosophic multivariate normal distribution.

Let $\tilde{X}_{iN} \in [\tilde{X}_{iL}, \tilde{X}_{iU}] = \left[\begin{bmatrix} \tilde{X}_{i1L} \\ \tilde{X}_{i2L} \\ \vdots \\ \tilde{X}_{ipL} \end{bmatrix}, \begin{bmatrix} \tilde{X}_{i1U} \\ \tilde{X}_{i2U} \\ \vdots \\ \tilde{X}_{ipU} \end{bmatrix}\right]$ denote the i^{th} neutrosophic observation taken from a p-variate neutrosophic normal distribution. i.e.,

$$\tilde{X}_{iN} \in [\tilde{X}_{iL}, \tilde{X}_{iU}] \sim N_{pN}\big[[\mu_L \ , \ \mu_U] \ , \ [\Sigma_L \ , \ \Sigma_U]\big] \tag{1}$$

where, *i* varies from 1 to n and *j* varies from 1 to p.

The neutrosophic sample mean vector corresponding to i^{th} subgroup is

$$\tilde{\bar{\bar{X}}}_N \in \left[\left[\frac{1}{\mathrm{n}}\sum_{i=1}^{\mathrm{n}} \tilde{X}_L\right], \left[\frac{1}{\mathrm{n}}\sum_{i=1}^{\mathrm{n}} \tilde{X}_U\right]\right] = \left[\begin{bmatrix}\tilde{\bar{\bar{X}}}_{1L}\\ \tilde{\bar{\bar{X}}}_{2L}\\ \vdots\\ .\\ \tilde{\bar{\bar{X}}}_{pL}\end{bmatrix}, \begin{bmatrix}\tilde{\bar{\bar{X}}}_{1U}\\ \tilde{\bar{\bar{X}}}_{2U}\\ \vdots\\ .\\ \tilde{\bar{\bar{X}}}_{pU}\end{bmatrix}\right] \tag{2}$$

sample covariance matrix is

$$\tilde{S}_N \in [\tilde{S}_L\ ,\ \ \tilde{S}_U] = \left[\begin{bmatrix}\tilde{S}_{11L} & \tilde{S}_{12L} & \cdots & \tilde{S}_{1pL}\\ \tilde{S}_{21L} & \tilde{S}_{22L} & \cdots & \tilde{S}_{2pL}\\ . & . & \cdots & .\\ \tilde{S}_{p1L} & \tilde{S}_{p2L} & \cdots & \tilde{S}_{ppL}\end{bmatrix}, \begin{bmatrix}\tilde{S}_{11U} & \tilde{S}_{12U} & \cdots & \tilde{S}_{1pU}\\ \tilde{S}_{21U} & \tilde{S}_{22U} & \cdots & \tilde{S}_{2pU}\\ . & . & \cdots & .\\ \tilde{S}_{p1U} & \tilde{S}_{p2U} & \cdots & \tilde{S}_{ppU}\end{bmatrix}\right] \tag{3}$$

Then the Neutrosophic Hotelling's $T^2{}_N$ statistic is

$$T_{iN}^2 \in [T_{iL}^2\ ,\ T_{iU}^2] = \left[\left[\mathrm{n}\left(\tilde{X}_{iL} - \tilde{\bar{\bar{X}}}_L\right)^T \left(\tilde{S}_L\right)^{-1}\left(\tilde{X}_{iL} - \tilde{\bar{\bar{X}}}_L\right)\right], \left[\mathrm{n}\left(\tilde{X}_{iU} - \tilde{\bar{\bar{X}}}_U\right)^T \left(\tilde{S}_U\right)^{-1}\left(\tilde{X}_{iU} - \tilde{\bar{\bar{X}}}_U\right)\right]\right] \tag{4}$$

Now we can define the neutrosophic MEWMA statistic as

$$\tilde{Z}_{iN} = r\tilde{X}_{iN} + (1-r)\tilde{Z}_{(i-1)N} \in \left[r\tilde{X}_{iL} + (1-r)\tilde{Z}_{(i-1)L}, r\tilde{X}_{iU} + (1-r)\tilde{Z}_{(i-1)U}\right] \tag{5}$$

By considering the $Z_{0N} = [0]$ and with an optimal choice of the weight parameter r.

And the corresponding variance is

$$V(\tilde{Z}_{iN}) = \tilde{S}_{iN} = \frac{r\left(1-(1-r)^{2i}\right)}{2-r}\tilde{S}_N \tag{6}$$

As the sample size becomes large

$$\tilde{S}_{Z_iN} \to \frac{r}{2-r}\tilde{S}_N \tag{7}$$

The MEWMA statistics to be plotted is,

$$T_{iN}^2 = Z_{iN}^T\tilde{S}_{Z_iN}^{-1}Z_{iN} \in \left[Z_{iL}^T\tilde{S}_{Z_iL}^{-1}Z_{iL},\ Z_{iU}^T\tilde{S}_{Z_iU}^{-1}Z_{iU}\right] \tag{8}$$

This reduces to Equation (4) neutrosophic Hotelling's T^2 statistic when the weight parameter $r = 1$.

We have employed the bootstrap method to determine the control limits for the proposed NMEWMA control chart. This technique involves repeatedly drawing a sample of size *k*, where *k* represents the number of observations in the dataset without replacement. Then we calculated the upper control limit (UCL_N) for each sample point using the formula $T^2_{N\,(1-\alpha)}$, 100(1-α)th percentile. This process was repeated a large number of times, denoted as G, to obtain a distribution of UCL_N values. Generally, for each iteration *j* of the bootstrap process, we obtained a corresponding UCL_N value denoted as $UCL_N^{(j)}$. To derive the control limit for the bootstrap sample, we computed the average of the bootstrap UCL_N values, and it is given as,

$$UCL_N(1-\alpha) = \frac{1}{G}\sum_{j=1}^{G} UCL_N^{(j)} \in \left[\frac{1}{G}\sum_{j=1}^{G} UCL_L^{(j)}, \frac{1}{G}\sum_{j=1}^{G} UCL_U^{(j)}\right] \tag{9}$$

which provides a reliable estimate of the true UCL_N. Using the bootstrap approach, we can estimate the distribution of UCL_N values and give a more accurate control limit for the NMEWMA control chart.

3. Simulation Study

In this section, we have conducted a simulation study to evaluate the effectiveness of the proposed method in detecting small shifts in the process. The study was conducted by Microsoft R v4.0.2. The objective of the study is to compare the performance of the Neutrosiophic MEWMA control chart with the neutrosophic Hotelling's T^2 chart. In order to generate the simulation data, we have created multivariate neutrosophic normal data by using the numbers of components P = 3, 5, and 10. The control limits were obtained by using the bootstrap approach and been discussed in detail in Section 2 of this article. Equivalent to a 0.005 false alarm rate, we set the ARL under control at 200. The primary focus of this simulation study is to investigate the effectiveness of the proposed NMEWMA control chart in identifying small shifts by introducing different levels of shifts to the process mean, such as 0.25, 0.5, 0.75, and 1. The NAR_1 and $NARL_1$were estimated based on 1000 iterations and with tuning parameters of 0.1, 0.2, 0,3, 0.8. **Table 1** and **Table 2** demonstrates the results of the simulation study. The combination of multivariate neutrosophic normal data and the bootstrap method to determine control limits improves the adaptability and accuracy of the analysis. The comparison with the neutrosophic Hotelling's T^2 chart provides a foundation to evaluate the proposed chart's effectiveness for detecting small shifts.

Table 1. NAR and NARL for hotelling's T^2 control chart with bootstrap control limits.

	***P* = 3**		***P* = 5**		***P* = 10**	
Shift	**NAR**	**NARL**	**NAR**	**NARL**	**NAR**	**NARL**
0.00	[0.0037, 0.0038]	[263.15, 270.27]	[0.0044, 0.0046]	[217.39, 227.27]	[0.0036, 0.0049]	[204.08, 277.77]
0.25	[0.0065, 0.0082]	[121.95, 153.85]	[0.0065, 0.0094]	[106.38, 153.85]	[0.0067, 0.0068]	[147.06, 149.25]
0.50	[0.0104, 0.0128]	[78.12, 96.16]	[0.0092, 0.0162]	[61.73, 108.69]	[0.0121, 0.0183]	[54.65, 82.64]
0.75	[0.0228, 0.0303]	[33.01, 43.86]	[0.0181, 0.0311]	[32.15, 55.25]	[0.0269, 0.0368]	[27.17, 37.17]
1.00	[0.039, 0.049]	[20.41, 25.64]	[0.0365, 0.0613]	[16.31, 27.39]	[0.0559, 0.0692]	[14.45, 17.89]

Table 2. NAR and NARL for NMEWMA control chart with bootstrap control limits.

		***P* = 3**		***P* = 5**		***P* = 10**	
r	**Shift**	**NAR**	**NARL**	**NAR**	**NARL**	**NAR**	**NARL**
0.1	0.00	[0.0036, 0.005]	[200, 270.27]	[0.0053, 0.008]	[121.9512, 227.272]	[0.004, 0.0042]	[238.0952, 277.777]
	0.25	[0.044, 0.094]	[10.56, 153.84]	[0.051, 0.094]	[10.6157, 153.846]	[0.044, 0.169]	[5.88582, 149.253]
	0.50	[0.322, 0.483]	[2.07, 96.15]	[0.394, 0.534]	[1.870, 108.695]	[0.574, 0.812]	[1.231, 82.644]
	0.75	[0.824, 0.896]	[1.12, 43.86]	[0.913, 0.944]	[1.058, 55.248]	[0.985, 0.996]	[1.0037, 37.174]
	1.00	[0.988, 0.996]	[1.00, 25.64]	[0.996, 0.998]	[1.002, 27.397]	[0.994, 0.995]	[1.005, 17.889]
0.2	0.00	[0.004, 0.005]	[185.18, 263.16]	[0.004, 0.046]	[217.391, 217.393]	[0.047, 0.005]	[200, 277.777]
	0.25	[0.027, 0.05]	[19.88, 166.67]	[0.023, 0.039]	[25.38, 185.185]	[0.021, 0.066]	[14.992, 153.846]
	0.50	[0.129, 0.208]	[4.79, 90.09]	[0.149, 0.223]	[4.474, 104.166]	[0.174, 0.361]	[2.769, 80.645]
	0.75	[0.416, 0.526]	[1.89, 51.02]	[0.464, 0.570]	[1.754, 55.248]	[0.678, 0.824]	[1.213, 37.174]
	1.00	[0.768, 0.830]	[1.20, 23.92]	[0.861, 0.897]	[1.114, 27.397]	[0.955, 0.981]	[1.0187, 19.193]
0.3	0.00	[0.004, 0.005]	[188.68, 250]	[0.003, 0.005]	[181.818, 263.157]	[0.004, 0.005]	[185.185, 212.766]
	0.25	[0.016, 0.027]	[36.76, 138.88]	[0.014, 0.023]	[43.103, 185.185]	[0.015, 0.037]	[26.666, 149.253]
	0.50	[0.076, 0.117]	[8.51, 80]	[0.077, 0.125]	[7.974, 99.009]	[0.085, 0.206]	[4.837, 71.428]
	0.75	[0.218, 0.296]	[3.378, 47.619]	[0.268, 0.360]	[2.771, 48.780]	[0.346, 0.524]	[1.907, 39.840]
	1.00	[0.499, 0.581]	[1.719, 25.445]	[0.578, 0.647]	[1.544, 26.809]	[0.767, 0.860]	[1.162, 17.889]
0.8	0.00	[0.004, 0.005]	[178.571, 270.270]	[0.004, 0.005]	[200, 285.714]	[0.004, 0.004]	[208.333, 277.777]
	0.25	[0.008, 0.010]	[91.743, 153.846]	[0.009, 0.012]	[79.365, 163.934]	[0.007, 0.012]	[78.740, 153.846]
	0.50	[0.019, 0.027]	[35.971, 79.365]	[0.019, 0.029]	[34.364, 94.339]	[0.017, 0.026]	[37.735, 80.645]
	0.75	[0.036, 0.054]	[18.382, 48.309]	[0.040, 0.060]	[16.528, 54.347]	[0.044, 0.067]	[14.727, 39.840]
	1.00	[0.074, 0.102]	[9.765, 24.390]	[0.0816, 0.116]	[8.598, 25.906]	[0.105, 0.154]	[6.468, 17.889]
0.9	0.00	[0.004, 0.005]	[200, 238.095]	[0.004, 0.005]	[222.222, 263.157]	[0.004, 0.005]	[232.558, 277.777]
	0.25	[0.007, 0.010]	[96.153, 147.058]	[0.007, 0.011]	[87.719, 163.934]	[0.006, 0.010]	[95.238, 153.846]

Table 2 continued…

	0.50	[0.015, 0.022]	[45.454, 72.463]	[0.013, 0.022]	[44.247, 94.339]	[0.013, 0.020]	[49.7512, 80.645]
	0.75	[0.026, 0.041]	[23.866, 50.761]	[0.028, 0.045]	[22.026, 54.347]	[0.033, 0.052]	[19.2307, 37.174]
	1.00	[0.052, 0.076]	[13.054, 26.178]	[0.055, 0.086]	[11.600, 25.906]	[0.068, 0.105]	[9.460, 19.193]

The results of this simulation study reveal that the proposed NMEWMA control chart, based on a bootstrap approach, outperforms the bootstrap-based neutrosophic Hotelling T^2 control chart in terms of NAR and NARL at different levels of shifts in various numbers of components. The proposed NMEWMA control chart based on the bootstrap approach provides a higher response rate than the Neutrosophic Hotelling T^2 control chart. Additionally, we have investigated the effects of the NMEWMA smoothing parameters and observed that the smoothing parameter r increased to 1. Then the NMEWMA control chart performed similarly to the neutrosophic Hotelling T^2 control chart. From the results obtained from the simulation study, we can infer that the proposed NMEWMA control chart based on the bootstrap approach performs efficiently in monitoring small shifts in the neutosophic environment.

4. Illustrative Example

To assist in understanding the preceding idea of the proposed chart, neutrosophic data have been generated according to the procedure defined in (Škrabánek and Martinková, 2019) by taking the mean vector and covariance matrix provided in (Tracy et al., 1992). Here we have utilized data from a chemical industry with three variables such as X_1, X_2, and X_3 where they denote the percentage of impurities, temperature, and concentration strength, respectively. The bootstrap control limits for the proposed control chart were determined as $[UCL_L, UCL_U]$ = [4.29, 5.60]. **Table 3** provides the generated data and the corresponding NMEWMA statistic for the control chart, which is illustrated in **Figure 1**.

Table 3. The neutrosophic data generated and the corresponding NMEWMA statistic.

$[X_{1L}, X_{1U}]$	$[X_{2L}, X_{2U}]$	$[X_{3L}, X_{3U}]$	$[T_L, T_U]$
[1.83, 1.75]	[1.66, 1.58]	[0.77, 0.69]	[3.26, 3.62]
[-1.04, -1.17]	[-1.3, -1.44]	[-1.5, -1.63]	[2.28, 2.52]
[-0.33, -0.38]	[-0.78, -0.82]	[-1.53, -1.58]	[2.91, 2.94]
[-1.59, -1.97]	[-1.24, -1.63]	[0.21, -0.16]	[6.3, 6.46]
[-0.56, -0.46]	[-0.99, -0.9]	[-1.69, -1.59]	[2.83, 3.22]
[-1.01, -1.3]	[-1.06, -1.35]	[-0.94, -1.23]	[1.26, 1.88]
[0.82, 0.97]	[0.77, 0.92]	[0, 0.14]	[2.93, 3.09]
[-0.02, -0.47]	[-0.09, -0.54]	[-0.54, -0.99]	[1.4, 2.35]
[-0.65, -0.64]	[-0.25, -0.23]	[0.56, 0.58]	[1.89, 1.91]
[1.21, 0.71]	[1.39, 0.89]	[1.4, 0.9]	[0.98, 2.2]
[-0.57, -0.31]	[-0.28, -0.02]	[0.52, 0.78]	[1.67, 1.89]
[1.73, 1.9]	[1.37, 1.54]	[0.83, 1]	[5.53, 5.73]
[1.28, 1.45]	[1.23, 1.4]	[1.07, 1.24]	[2.5, 2.66]
[0.28, 0.17]	[0.61, 0.51]	[0.39, 0.28]	[4.88, 4.98]
[0.9, 1.17]	[1.11, 1.38]	[1.29, 1.56]	[1.71, 2.32]
[-0.16, -0.11]	[0.13, 0.18]	[0.53, 0.58]	[0.9, 0.91]
[-0.95, -0.66]	[-1.48, -1.19]	[-1.71, -1.42]	[4.67, 4.85]
[-0.95, -0.75]	[-0.44, -0.24]	[0.43, 0.63]	[2.76, 3.13]
[-0.38, -0.23]	[-0.26, -0.11]	[-0.07, 0.07]	[0.17, 0.32]
[0.17, 0.37]	[-0.09, 0.1]	[-0.05, 0.13]	[2.91, 3.07]

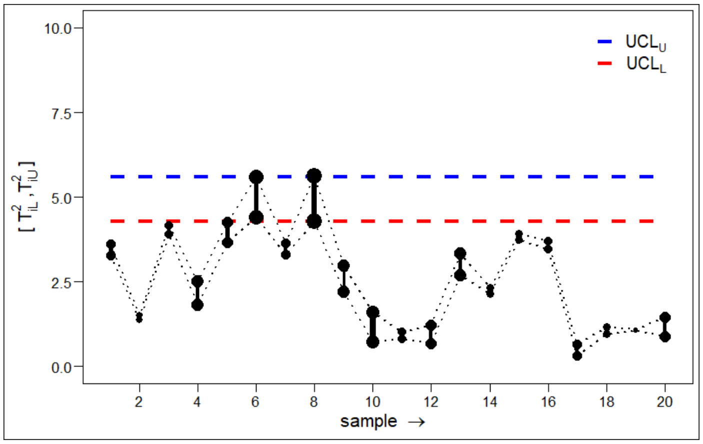

Figure 1. Neutrosophic BMEWMA control chart was obtained for the data given in **Table 3**.

The NEWMA control chart with bootstrap limit is explained in **Figure 1**. Furthermore, the results obtained from the illustrative example reveal that the two points (corresponding to sample numbers 6 and 8) lie outside the control limits. Thus, we can infer that the process is in an out-of-control state. The thickness of the points on the graph represents the level of indeterminacy in the process.

5. Conclusion

This research proposes the adoption of the NMEWMA control chart based on the bootstrap method as a robust tool for detecting subtle shifts in multivariate processes operating within a neutrosophic environment. This is particularly relevant in situations where there are multiple characteristics are need to be controlled. Based on the simulation study, our findings suggest that the NMEWMA control chart outperforms Neutrosophic Hotelling's T^2 chart regarding response rate, as evidenced by higher neutrosophic AR and ARL for detecting more minor shifts. In addition to highlighting the effectiveness of the proposed method, it is imperative to point out the importance of using neutrosophic logic when dealing with real-world issues that are characterized by uncertain, incomplete, or imprecise information. We considered an example from the chemical industry to demonstrate how our suggested approach might be used in practice to manage data in a neutrosophic environment. The limitation of this study is that the indeterminacy level in data is not uniform; it varies across different data points. To overcome this, we could give equal importance to each data point and assign weights based on the range of indeterminacy for each. This approach could be investigated in future research to strengthening our findings and make existing methods more widely applicable.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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