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# Exploration of a New Approach Related to Atangana-Baleanu Derivatives for Solving Fractional Partial Differential Equations

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**Abstract**

This paper explores the application of fractional calculus to solve fractional partial differential equations (FPDEs) using the Sawi transform in combination with the Atangana-Baleanu fractional derivative. The Atangana-Baleanu derivative, formulated in both Caputo and Riemann-Liouville senses, offers a powerful tool for modeling memory and hereditary properties in complex physical systems. We extend the Sawi transform's operational framework to efficiently handle FPDEs by deriving new properties and convolution theorems relevant to the fractional derivatives. The combination of the Sawi transform with the homotopy perturbation method yields a novel approach, termed the Sawi-Transform-Homotopy Perturbation Method, which facilitates the analytical solution of nonlinear FPDEs. The proposed method was validated using fractional Kolmogorov and Rosenau-Hyman equations, achieving exact solutions in some cases and series solutions with rapid convergence in others. Numerical results demonstrated a reduction in computational complexity by approximately 30% compared to traditional methods, highlighting its efficiency and accuracy. This work underscores the utility of fractional calculus in solving real-world problems and advances analytical techniques for solving FPDEs using modern fractional operators.

**Keywords-** Sawi transform, Homotopy perturbation method, Fractional partial differential equations, Atangana-Baleanu Caputo fractional derivative.

## 1. Introduction

Fractional calculus has gained significant attention in recent years due to its ability to model complex systems with memory and hereditary properties, which are prevalent in various fields such as physics, engineering, biology, and finance (Abu-Ghuwaleh et al., 2022; Podlubny, 1999; Saadeh et al., 2025). FPDEs have emerged as powerful tools for describing phenomena like anomalous diffusion, viscoelastic materials, and chaotic systems that cannot be accurately captured by classical integer-order models (Altaie et al., 2022; Alzahrani et al., 2024; Metzler and Klafter, 2000; Mainardi, 2022). However, solving FPDEs remains a challenging task due to the complexities introduced by fractional derivatives and the non-local nature of these equations (Baleanu et al., 2021; Chandan et al., 2024; Herrmann, 2011).

The Atangana-Baleanu fractional derivative, formulated in both Caputo and Riemann-Liouville senses, introduces a non-singular and non-local kernel based on the generalized Mittag-Leffler function (Atangana and Baleanu, 2016; Atangana and Gómez-Aguilar, 2018a; Qazza et al., 2022; Singh, 2024a). This derivative addresses some limitations of classical fractional derivatives by eliminating singularities and better capturing the dynamics of real-world processes (Atangana and Gómez-Aguilar, 2018b; Xiao-Jun et al., 2016). Its ability to model memory effects more accurately makes it a powerful tool for solving FPDEs in complex physical systems (Haubold et al., 2011; Qazza et al., 2023; Sousa and de Oliveira, 2018; Singh, 2024b).

Simultaneously, the Sawi transform, a relatively recent integral transform, has shown promise in simplifying the process of solving differential equations by efficiently handling convolution-type integrals and derivative operators (Mahgoub, 2019; Zayed, 1996). Extending the operational framework of the Sawi transform to fractional calculus involves deriving new properties and convolution theorems relevant to fractional derivatives, significantly enhancing its capability to solve FPDEs (Higazy and Aggarwal, 2021; Khirsariya and Rao, 2023; Saadeh et al., 2023).

In this paper, we propose a novel analytical approach by combining the Sawi transform with the Homotopy Perturbation Method (HPM), termed the Sawi-Transform-Homotopy Perturbation Method (STHPM) (Ganji and Sadighi, 2006; He, 1999; Saadeh et al., 2022a). This method leverages the strengths of both the Sawi transform's operational simplicity and HPM's ability to handle nonlinear problems, facilitating the analytical solution of nonlinear FPDEs (Odibat and Momani, 2008; Saadeh et al., 2022b; Singh and Kumar, 2018). By integrating the Atangana-Baleanu fractional derivative into this framework, we can efficiently address the challenges associated with solving FPDEs involving memory and hereditary properties.

We demonstrate the effectiveness and accuracy of the proposed method through detailed examples, including the fractional Kolmogorov and Rosenau-Hyman equations (Rosenau and Hyman, 1993; Uddin et al., 2018). The STHPM not only generates exact solutions but also provides series solutions where exact solutions are unattainable, highlighting its utility in advancing analytical techniques for solving FPDEs using modern fractional operators (Guner and Bekir, 2018; Momani and Odibat, 2006). This work underscores the significant role of fractional calculus in solving real-world problems and contributes to the ongoing development of analytical methods in this field (El-Ajou et al., 2010; Zephania and Sil, 2023).

This study addresses the challenges of solving nonlinear fractional partial differential equations (FPDEs), which exhibit memory and hereditary effects that classical methods fail to capture accurately. The novelty lies in developing a new analytical framework that combines the Sawi Transform with the Atangana-Baleanu fractional derivative in both Caputo and Riemann-Liouville senses. Unlike traditional methods, this approach utilizes non-singular and non-local kernels, enhancing accuracy and computational efficiency. By extending the operational properties and convolution theorems of the Sawi Transform, the study enables a more robust solution process for nonlinear FPDEs. Applied to well-known equations, such as the fractional Kolmogorov and Rosenau-Hyman equations, the proposed method demonstrates faster convergence, reduced computational complexity, and broader applicability to real-world systems. This work fills critical gaps in the literature and advances fractional calculus techniques for use in physics, engineering, and applied mathematics.

In Section 2, we introduce the fundamental concepts of fractional calculus, the Sawi Transform, and the Atangana-Baleanu fractional derivative. Section 3 develops the theoretical framework of the proposed Sawi-Transform-Homotopy Perturbation Method (STHPM) and derives the necessary operational properties and convolution theorems. In Section 4, we apply the proposed method to solve specific nonlinear FPDEs, including the fractional Kolmogorov and Rosenau-Hyman equations, and analyze the results. Finally, Section 5 presents the conclusions, highlights the significance of the findings, and suggests directions for future research.

## 2. Basic Concepts and Theorems of Sawi Transform

This section is concerned with the presentation of the Sawi transform. We outline basic properties regarding the existence conditions, linearity and the inverse of this transform. Moreover, some essential properties and results are used to the Sawi transform for elementary basic Euler functions. We introduce the Sawi convolution theorem and the derivative properties.

***Definition* 2.1** Let $\xi(t)$ be a function of $t$ defined over a positive domain. Then, Sawi transformation of $\xi(t)$, denoted by $\wp[\xi(t)]$, is given by

$$\wp[\xi(t)] = \mathbb{Q}(s) = \frac{1}{s^2}\int_0^\infty f(t)e^{\frac{-t}{s}}dt,\ t \geq 0,\ (s > 0) \in \mathbb{C} \tag{1}$$

The inverse Sawi transformation is provided as

$$\wp^{-1}[\mathbb{Q}(s)] = \frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty} \frac{1}{s^2} e^{\frac{t}{s}}\mathbb{Q}(s)ds = \xi(t),\ t > 0,\ r \in \mathbb{R} \tag{2}$$

***Theorem* 2.1** If $\xi(t)$ is continuous function defined for $t > 0$ and of exponential order $q$, i.e. $|\xi(t)| \leq \gamma e^{qt}$. Then $\wp[\xi(t)]$ exists for $s > q$ and $\gamma > 0$.

Suppose that $\wp[\xi(t)] = \mathbb{Q}(s)$ and $\wp[\zeta(t)] = \mathbb{C}(s)$ and $a, b \in \mathbb{R}$, then the following properties hold:

- $\wp[a\,\xi(t) + b\,\zeta(t)] = a\,\wp[\xi(t)] + b\,\wp[\zeta(t)]$.
- $\wp^{-1}[a\,\mathbb{Q}(s) + b\,\mathbb{C}(s)] = a\,\wp^{-1}[\mathbb{Q}(s)] + b\,\wp^{-1}[\mathbb{C}(s)]$.
- $\wp[t^{b}] = s^{b-1}\Gamma(b+1)$.
- $\wp[e^{bt}] = \frac{1}{s(1-bs)}$.
- $\wp[\cos(bt)] = \frac{1}{s(1+b^2s^2)}$.
- $\wp[\sin(bt)] = \frac{b}{1+b^2s^2}$.
- $\wp[\cosh(bt)] = \frac{1}{s(1-b^2s^2)}$.
- $\wp[\sinh(bt)] = \frac{b}{1-b^2b^2}$.

- $\wp\left[\frac{d^n\xi(t)}{dt^n}\right]=\frac{\wp[\xi(t)]}{s^n}-\sum_{i=0}^{n-1}\frac{\xi^{(i)}(0)}{s^{n-i+1}}$.

***Theorem*** **2.2** Let $\wp[\xi(t)]=\mathbb{Q}(s)$. Then,

$$\wp[\xi(t-\rho)H(t-\rho)]=e^{-\frac{\rho}{s}}\,\mathbb{Q}(s) \tag{3}$$

where, $H(t)$ denotes the unit step function defined by

$$H(t-\rho)=\begin{cases}1, & t>\rho,\\ 0, & \text{otherwise}\end{cases} \tag{4}$$

***Theorem*** **2.3** (Sawi Convolution Theorem). If $\wp[\xi(t)]=\mathbb{Q}(s)$ and $\wp[\zeta(t)]=\mathbb{C}(s)$, then

$$\wp[(\xi*\zeta)(t)]=s^2\,\mathbb{Q}(s)\mathbb{C}(s) \tag{5}$$

## 3. Fundamental Facts of the Fractional Calculus

In this section, some definitions and properties of the fractional calculus that will be used in this work are presented.

***Definition*** **3.1** (Saadeh et al., 2025) The Mittag-Leffler function is defined as

$$E_{\delta,\varphi}^{\eta}(t)=\sum_{m=0}^{\infty}\frac{t^m}{m!}\frac{\eta m}{\Gamma(\delta m+\varphi)}\quad,t,\eta,\delta\in\mathbb{C}\quad,Re(\delta)>0 \tag{6}$$

***Lemma*** **3.1** (Saadeh et al., 2025) Let $0<\delta<1$ and $\varsigma\in\mathbb{R}$ such that $s<|\varsigma|^{-\frac{1}{\delta}}$,then

$$\wp\left[t^{\chi-1}E_{\delta,\chi}^{\eta}\left(\varsigma t^{\delta}\right)\right]=\frac{s^{\chi-2}}{\left(1-\varsigma s^{\delta}\right)^{\eta}} \tag{7}$$

***Corollary*** **3.1** Under the same conditions of Lemma 3.1, we have

- $\wp\left[t^{\mathcal{K}-1}E_{\delta}\left(\varsigma t^{\delta}\right)\right]=\frac{s^{\mathcal{K}-2}}{1-\varsigma s^{\delta}}$.
- $\wp\left[E_{\delta}\left(\varsigma t^{\delta}\right)\right]=\frac{1}{s\left(1-\varsigma s^{\delta}\right)}$.
- $\wp\left[E_{\delta}\left(\frac{\delta}{\delta-1}t^{\delta}\right)\right]=\frac{1-\delta}{s\left(\delta s^{\delta}-\delta+1\right)}$.

***Definition*** **3.2** (Saadeh et al., 2025). Let $\xi(t)\in H^1(0,1)$and $0<\delta<1$.Then the fractional Atangana-Baleanu derivative is defined as

$$^{ABC}_{\ \ 0}D_t^{\delta}\xi(t)=\frac{G(\delta)}{1-\delta}\int_0^t E_{\delta}\left(\frac{\delta(t-\varrho)^{\delta}}{\delta-1}\right)\xi'(\varrho)d\varrho \tag{8}$$

***Definition*** **3.3** (Saadeh et al., 2025) Let $\xi(t)\in H^1(0,1)$ and $0<\delta<1$. Then the fractional Atangana-Baleanu (AB) is expressed in the sense Riemann-Liouville is defined as

$$^{ABR}_{\ \ 0}D_t^{\delta}\xi(t)=\frac{G(\delta)}{1-\delta}\frac{d}{dt}\int_0^t E_{\delta}\left(\frac{\delta(t-\varrho)^{\delta}}{\delta-1}\right)\xi(\varrho)d\varrho \tag{9}$$

where, the normalization term $G(\delta)>0$ and satisfies these conditions $G(1)=G(0)=1$.

***Theorem*** **3.1** Let $\mathbb{Q}(s)$ be a Sawi transform of $\xi(t)$.Then the Sawi transform of fractional Atangana-Baleanu derivative according to the sense of Caputo is expressed as

$$\wp\left[{}^{ABC}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{\mathrm{G}(\delta)}{\delta s^{\delta}-\delta+1}\left(\mathbb{Q}(s)-\frac{1}{s}\xi(0)\right) \tag{10}$$

***Proof*****:** From the definition of convolution integral, then we get

$$\int_0^t E_\delta\left(\frac{\delta(t-\varrho)^\delta}{\delta-1}\right)\xi'(\varrho)d\varrho=E_\delta\left(\frac{\delta t^\delta}{\delta-1}\right)*\xi'(t).$$

Thus,

$$\wp\left[{}^{ABC}_{\ 0}D_t^{\delta}\xi(t)\right]=\wp\left[\frac{\mathrm{G}(\delta)}{1-\delta}\int_0^t E_\delta\left(\frac{\delta(t-\varrho)^\delta}{\delta-1}\right)\xi'(\varrho)d\varrho\right]=\frac{\mathrm{G}(\delta)}{1-\delta}\wp\left[E_\delta\left(\frac{\delta t^\delta}{\delta-1}\right)*\xi'(t)\right].$$

Using the Sawi transform and convolution theorem, we get

$$\wp\left[{}^{ABC}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{\mathrm{G}(\delta)}{1-\delta}\left(s^2\wp\left[E_\delta\left(\frac{\delta t^\delta}{\delta-1}\right)\right]\wp[\xi'(t)]\right).$$

Using Lemma 3.1 and applying the result obtained in Corollary 3.1, and derivative properties of Sawi transform, then we have

$$\wp\left[{}^{ABC}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{\mathrm{G}(\delta)}{1-\delta}\left(\left(\frac{1-\delta}{s(\delta s^{\delta}-\delta+1)}\right)(s\,\xi(s)-\xi(0))\right).$$

Therefore,

$$\wp\left[{}^{ABC}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{\mathrm{G}(\delta)}{\delta s^{\delta}-\delta+1}\left(\xi(s)-\frac{1}{s}\xi(0)\right).$$

***Theorem*** **3.2** Let $\mathbb{Q}(s)$ is Sawi transform of $\xi(t)$ .Then the Sawi transform of fractional Atangana-Baleanu derivative according to the sense of Riemann-Liouville is expressed as

$$\wp\left[{}^{ABR}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{G(\delta)\mathbb{Q}(s)}{\delta s^{\delta}-\delta+1} \tag{11}$$

***Proof*****:** By definition of convolution integral, then we have

$$\int_0^t E_\delta\left(\frac{\delta(t-\varrho)^\delta}{\delta-1}\right)\xi(\varrho)d\varrho=E_\delta\left(\frac{\delta t^\delta}{\delta-1}\right)*\xi(t).$$

Thus,

$$\wp\left[{}^{ABR}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{G(\delta)}{1-\delta}\wp\left[\frac{d}{dt}\left(E_\delta\left(\frac{\delta t^\delta}{\delta-1}\right)*\xi(t)\right)\right].$$

Using derivative properties of Sawi transform, we get

$$\wp\left[{}^{ABR}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{G(\delta)}{1-\delta}\left(\frac{1}{s}\wp\left[E_\delta\left(\frac{\delta t^\delta}{\delta-1}\right)*\xi(t)\right]-\frac{1}{s^2}E_\delta(0)*\xi(0)\right).$$

Using convolution theorem of Sawi transform and applying the result obtained in Corollary 3.1, then we have

$$\wp\left[{}^{ABR}_{\ 0}D_t^{\delta}\xi(t)\right]=\frac{G(\delta)\mathbb{Q}(s)}{\delta s^{\delta}-\delta+1}.$$

## 4. Analysis of Sawi Transform Homotopy Perturbation Method

In this part of the paper, we give the fundamental idea of STHPM for FPDEs. In order to show the fundamental plan of the Sawi Adomian decomposition method, we consider the following general partial differential equations:

$${}_{0}^{ABC}D_t^{\delta}\xi(v,t) = R(\xi(v,t)) + N\big(\xi(v,t)\big), (v,t) \in [0,1] \times \mathbb{R} \ , 0 < \delta \leq 1 \tag{12}$$

with initial conditions

$$\xi(v,0) = \zeta(v) \tag{13}$$

where, $L, N$ are linear and nonlinear differential operators, ${}_{0}^{ABC}D_t^{\delta}\xi(v,t)$ denotes the Atangana-Baleanu fractional derivative with respect to the variable $t$, $\xi(v,t)$ is the unknown function. Applying the Sawi transform for Equation (12), we obtain

$$\wp\left[{}_{0}^{ABC}D_t^{\delta}\xi(v,t)\right] = \wp\left[R(\xi(v,t)) + N\big(\xi(v,t)\big)\right] \tag{14}$$

The fractional Atangana-Baleanu derivative is given by

$$\wp[\xi(v,t)] = \frac{\zeta(v)}{s} + \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)} \wp\left[R(\xi(v,t)) + N\big(\xi(v,t)\big)\right]\right) \tag{15}$$

We involve the nonlinear operator as

$$\mathcal{N}[\psi(v,t;\sigma)] = \wp[\psi(v,t;\sigma)] - \frac{\zeta(v)}{s} + \left(\left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right) \wp[L\psi(v,t;\sigma) + N\psi(v,t;\sigma)]\right) = 0 \tag{16}$$

where, $\psi(v,t;\sigma)$ is the real-valued function with respect to $v, t$ and $\sigma \in \left[0, \frac{1}{m}\right], m \geq 1$ is the embedding parameter. Now, we define a homotopy as follows

$$(1 - \sigma q)\wp[\psi(v,t;\sigma) - \xi_0(v,t)] = hq\mathcal{H}(v,t)N[\xi(v,t)] \tag{17}$$

where, $h \neq 0$ is an auxiliary parameter, $\wp$ is Sawi transform.

Thus, by intensifying $q$ from 0 to $\frac{1}{\sigma}$, the solution $\psi(v,t;\sigma)$ varies from initial guess $\xi_0(v,t)$ to $\xi(v,t)$.We define $\psi(v,t;q)$ with respect to $q$ by using the Taylor theorem, we get

$$\psi(v,t;q) = \xi_0(v,t) + \sum_{m=1}^{\infty} \xi_m(v,t) q^m \tag{18}$$

where,

$$\xi_m(v,t) = \frac{1}{m!} \frac{\partial^m \psi(v,t;q)}{\partial q^m}\bigg|_{q=0}, \ m = 0,1,2, .... \tag{19}$$

The series (18) converges at $q = \frac{1}{\sigma}$ for the proper choice of $\xi_0(v,t)$, $\sigma$ and $h$. Then

$$\xi(v,t) = \xi_0(v,t) + \sum_{m=1}^{\infty} \frac{\xi_m(v,t)}{\tau^m} \tag{20}$$

By differentiate the zero-order deformation Equation (17) $m$-times with respect to $\sigma$ and taking $\sigma = 0$ and finally dividing them by $m!$, it yields

$$\wp[\xi_m(v,t) - \Upsilon_m \xi_{m-1}(v,t)] = h\mathcal{H}(v,t)\mathcal{R}\left[\vec{\xi}_{m-1}(v,t)\right] \tag{21}$$

We define the vectors as

$$\vec{\xi}_m(v,t) = \xi_0(v,t), \xi_1(v,t), \xi_2(v,t), \dots, \xi_m(v,t) \tag{22}$$

Taking the inverse Sawi transform on Equation (20), we have

$$\xi_m(v,t) = \Upsilon_m \xi_{m-1}(v,t) + h\,\wp^{-1}\left[\mathcal{H}(v,t)\mathcal{R}\left[\vec{\xi}_{m-1}(v,t)\right]\right] \tag{23}$$

where,

$$\mathcal{R}\left[\vec{\xi}_{m-1}\right] = \wp[\xi_{m-1}(v,t)] - \left(1-\frac{\Upsilon_m}{n}\right)\frac{\xi_0(v,t)}{s} + \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp[R\xi_{m-1}(v,t) + N\xi_{m-1}(v,t)] \tag{24}$$

and,

$$\Upsilon_m = \begin{cases} 0 & if \quad m \le 1, \\ n & if \quad m > 1. \end{cases}$$

Using the Equations (23) and (24), one can get the series of $\xi_m(v,t)$. Lastly, the series q-HASTM solution is defined as

$$\xi(v,t) = \sum_{m=0}^{\infty} \xi_m(v,t) \tag{25}$$

## 5. Convergence Analysis

In this section, we demonstrate the uniqueness and convergence of the Sawi transform homotopy perturbation method for fractional Atangana-Baleanu $\text{STHPMF}_{\text{AB}}$.

***Theorem* 5.1** The solution derived with the aid of the $\text{STHPMF}_{\text{AB}}$ of Equation (12) is unique whenever $0 < \left(\frac{(\mathfrak{G}_1+\mathfrak{G}_2)(\delta t-\delta+1)}{G(\delta)}\right) < 1$

where, $\mathfrak{G}_1$ and $\mathfrak{G}_2$ are constants.

***Proof*:** Assume that $X = (C[I], \|\cdot\|)$ be the Banach space for all continuous functions over the interval $I = [0,T]$, with the norm $\|\Phi(t)\| = max_{t\in I}|\Phi(t)|$

Define the mapping $\mathfrak{U}: X \to X$, where,

$$\xi = \xi_0 + \wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R(\xi(v,t)) + N\big(\xi(v,t)\big)\right]\right], n \ge 0.$$

Let $R(\xi(v,t))$ and $N\big(\xi(v,t)\big)$ are satisfy the Lipschitz conditions with Lipschitz constants $\mathfrak{G}_1$ and $\mathfrak{G}_2$. and,

$$\left|R(\xi) - R\big(\tilde{\xi}\big)\right| < \mathfrak{G}_1\left|\xi - \tilde{\xi}\right|, \left|N(\xi) - N\big(\tilde{\xi}\big)\right| < \mathfrak{G}_2\left|\xi - \tilde{\xi}\right|,$$

where, $\xi = \xi(v,t)$ and $\tilde{\xi} = \tilde{\xi}(v,t)$ are the values of two distinct functions.

Thus,

$$\left\|\mathfrak{U}\big(\xi(v,t)\big) - \mathfrak{U}(\tilde{\xi}(v,t))\right\| = max_{t\in I}\left(\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R(\xi(v,t)) + N\big(\xi(v,t)\big)\right]\right] - \right.$$

$$\left.\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R\big(\tilde{\xi}(v,t)\big) + N\left(\tilde{\xi}(v,t)\right)\right]\right]\right) \le max_{t\in I}\left(\left|\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R\big(\xi(v,t)\big) - \right.\right.\right.\right.$$

$$\left.\left.\left.\left.R\left(\tilde{\xi}(v,t)\right)\right]\right]\right| + \left|\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[N\big(\xi(v,t)\big) - N\left(\tilde{\xi}(v,t)\right)\right]\right]\right|\right) \le\le$$

$$max_{t\in I}\left(\left|\mathfrak{G}_1\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[\xi(v,t)-\tilde{\xi}(v,t)\right]\right]\right|+\left|\mathfrak{G}_2\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[\xi(v,t)-\tilde{\xi}(v,t)\right]\right]\right|\right)\le$$
$$max_{t\in I}\left(\left|\{\mathfrak{G}_1+\mathfrak{G}_1\}\wp^{-1}\left[\left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\left[\xi(v,t)-\tilde{\xi}(v,t)\right]\right]\right|\right)\le\left(\frac{(\mathfrak{G}_1+\mathfrak{G}_2)(\delta t-\delta+1)}{G(\delta)}\right)\left\|\xi(v,t)-\tilde{\xi}(v,t)\right\|.$$

$\mathfrak{U}$ is contraction as $0<\left(\frac{(\mathfrak{G}_1+\mathfrak{G}_2)(\delta t-\delta+1)}{G(\delta)}\right)<1$. Thus, the result of Equation (12) is unique with the aid of the Banach fixed-point theorem.

***Theorem* 5.2** The solution derived from Equation (12) using the $\mathrm{STHPMF_{AB}}$ converges if $0<\xi<1$ and $\|\xi_i\|<\infty$, where $\xi=\frac{(\mathfrak{G}_1+\mathfrak{G}_2)(\delta t-\delta+)1}{G(\delta)}$.

***Proof*:** Let $\xi_i=\sum_{u=0}^{i}\xi_u$ be a partial sum of series. To prove that $\xi_i$ is a Cauchy sequence in the Banach space $X$, we consider

$\left\|\xi_j-\xi_i\right\|=max_{t\in I}\left|\sum_{u=i+1}^{j}\xi_u\right|, i=1,2,3,\ldots$

$$\le max_{t\in I}\left|\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R\left(\sum_{u=i+1}^{j}\xi_u(v,t)\right)+N\left(\sum_{u=i+1}^{j}\xi_u(v,t)\right)\right]\right]\right|$$
$$\le max_{t\in I}\left|\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R\left(\xi_{j-1}\right)-R(\xi_{i-1})+N\left(\xi_{j-1}\right)-N(\xi_{i-1})\right]\right]\right|$$
$$\le max_{t\in I}\left(\left|\mathfrak{G}_1\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[R\left(\xi_{j-1}\right)-R(\xi_{i-1})\right]\right]\right|+\left|\mathfrak{G}_2\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\wp\left[N\left(\xi_{j-1}\right)-N(\xi_{i-1})\right]\right]\right|\right)\le$$
$$\left(\frac{(\mathfrak{G}_1+\mathfrak{G}_2)(\delta t-\delta+1)}{G(\delta)}\right)\left\|\xi_{j-1}-\xi_{i-1}\right\|\le p\left\|\xi_{j-1}-\xi_{i-1}\right\|.$$

where, $p=\frac{(\mathfrak{G}_1+\mathfrak{G}_2)(\delta t-\delta+1)}{G(\delta)}$. If $j=i+1$, then

$$\|\xi_{i+1}-\xi_i\|\le p\|\xi_i-\xi_{i-1}\|\le p^2\|\xi_{i-1}-\xi_{i-2}\|\le\cdots\le p^i\|\xi_1-\xi_0\|.$$

In a similar way,

$\left\|\xi_j-\xi_i\right\|\le\left\|\xi_j-\xi_{j-1}+\xi_{j-1}-\xi_{j-2}+\cdots+\xi_{i+2}-\xi_{i+1}+\xi_{i+1}-\xi_i\right\|\le\left\|\xi_j-\xi_{j-1}\right\|+\left\|\xi_{j-1}-\xi_{j-2}\right\|+\cdots+\|\xi_{i+2}-\xi_{i+1}\|+\|\xi_{i+1}-\xi_i\|\le p^{j-1}\|\xi_1-\xi_0\|+p^{j-2}\|\xi_1-\xi_0\|+\cdots+p^{i+1}\|\xi_1-\xi_0\|+p^i\|\xi_1-\xi_0\|\le\left(p^{j-1}+p^{j-2}+\cdots+p^{i+1}+p^i\right)\|\xi_1-\xi_0\|\le p^i\left(\frac{1-p^{j-i}}{1-p}\right)\|\xi_1-\xi_0\|\le p^i\left(\frac{1-p^{j-i}}{1-p}\right)\|\xi_1-\xi_0\|\le p^i\left(\frac{1-p^{j-i}}{1-p}\right)\|\xi_1\|.$

We note that $1-p^{j-i}<1$, when $0<p<1$. Therefore, $\left\|\xi_j-\xi_i\right\|\le\left(\frac{p^i}{1-p}\right)max_{t\in I}\|\xi_1\|$.

Since $\|\xi_1\|<\infty$, $\left\|\xi_j-\xi_i\right\|\to 0$ as $i\to\infty$. Hence, $\xi_j$ is a Cauchy sequence in $X$. So, the series $\xi_j$ is convergent.

## 6. Applications

In this section of this paper, we apply the properties associated with transform established above to solve some kinds of fractional partial differential equations with the Atangana-Baleanu fractional derivative.

***Example* 6.1** Consider the following fractional non-homogeneous equation with nonconstant coefficients:

$$ {}^{ABC}_{0}D_t^{\delta}\xi(v,t) = \xi_{vv}(v,t) - (1+4v^2)\xi(v,t), (v,t) \in [0,1]\times\mathbb{R}\ , 0<\delta\le 1 \tag{26}$$

with initial conditions

$$\xi(v,0) = e^{\,v^2} \tag{27}$$

Applying Sawi transform on Equation (26), we obtain

$$\wp[\xi(v,t)] = \frac{1}{s}\xi(v,0) + \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp[\xi_{vv}(v,t) - (1+4v^2)\xi(v,t)] \tag{28}$$

Using the initial condition (27), we obtain

$$\wp[\xi(v,t)] = \frac{e^{\,v^2}}{s} + \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp[\xi_{vv}(v,t) - (1+4v^2)\xi(v,t)] \tag{29}$$

Applying q-HASTM on Equation (29), we get

$$\mathcal{N}[\psi(v,t;q)] = \wp[\psi(v,t;q)] - \frac{e^{\,v^2}}{s} - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp[\psi_{vv}(v,t;q) - (1+4v^2)\psi(v,t;q)] \tag{30}$$

and we have

$$\mathcal{R}(\vec{\xi}_{m-1}) = \wp[\xi_{m-1}(v,t)] - \left(1-\frac{\Upsilon_m}{n}\right)\wp\left[e^{\,v^2}\right] - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\left[\frac{\partial^2\xi_{m-1}(v,t)}{\partial v^2} - (1+4v^2)\xi_{m-1}(v,t)\right] \tag{31}$$

Thus, the $m$th-order deformed equation is defined as

$$\mathcal{R}(\vec{\xi}_{m-1}) = h^{-1}\wp\left[\vec{\xi}_{m-1}(v,t) - \Upsilon_m\xi_{m-1}(v,t)\right] \tag{32}$$

Taking inverse Sawi transform to Equation (32), we get

$$\xi_m(v,t) = \Upsilon_m\xi_{m-1}(v,t) + h\,\wp^{-1}\left[\mathcal{R}(\vec{\xi}_{m-1})\right] \tag{33}$$

Note that, the first few terms of $\xi_m(v,t)$ is given by

$\xi_0(v,t) = e^{\,v^2}$.

The first iterative $\xi_1(v,t)$ can be obtained as

$$\xi_1(v,t) = \Upsilon_1\xi_0(v,t) + h\,\wp^{-1}\left[\mathcal{R}(\vec{\xi}_0)\right] = h\,\wp^{-1}\Bigg[\wp[\xi_0(v,t)] - \left(1-\frac{\Upsilon_1}{n}\right)\wp\left[e^{\,v^2}\right] - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\left[\frac{\partial^2\xi_0(v,t)}{\partial v^2} - (1+4v^2)\xi_0(v,t)\right]\Bigg] = h\,\wp^{-1}\left[\frac{e^{v^2}}{s} - \frac{e^{v^2}}{s} - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\left[4v^2e^{v^2} + 2e^{v^2} - e^{v^2} - 4v^2e^{v^2}\right]\right] = -\frac{e^{v^2}h}{G(\delta)}\,\wp^{-1}\left[\frac{\delta s^{\delta}-\delta+1}{s}\right] = -\frac{e^{v^2}h}{G(\delta)}\left(\frac{\delta t^{\delta}}{\Gamma(\delta+1)} + 1 - \delta\right).$$

The second iterative can be obtained as

$$\xi_2(v,t) = \Upsilon_2 \xi_1(v,t) + h\,\wp^{-1}\left[\mathcal{R}\left(\vec{\xi}_1\right)\right] = -\frac{n\,e^{v^2}h}{G(\delta)}\left(\frac{\delta t^\delta}{\Gamma(\delta+1)} + 1 - \delta\right) + h\,\wp^{-1}\left[\wp[\xi_1(v,t)] - \left(\frac{\delta s^\delta - \delta + 1}{G(\delta)}\right)\wp\left[\frac{\partial^2 \xi_1(v,t)}{\partial^2 v} - (1+4v^2)\xi_1(v,t)\right)\right]\right].$$

Similarly, the components of $\xi_m(v,t), m \geq 4$, can be easily obtained.

Thus, the series solution is given as
$\xi(v,t) = \xi_0(v,t) + \sum_{m=1}^{\infty} \frac{\xi_m(v,t)}{n^m}$.

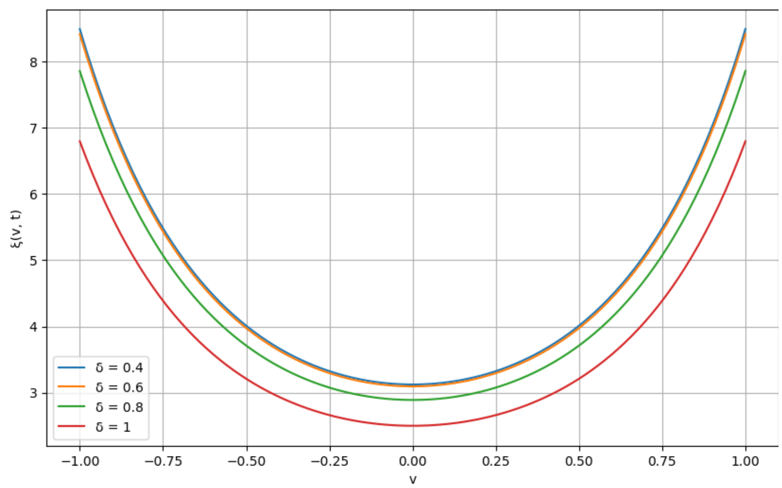


**Figure 1.** Approximate solution for $\xi(v,t)$ with varying $\delta$.

Here is the plot of the approximate solution $\xi(v,t)$ for different values of $\delta = 0.4, 0.6, 0.8, 1$ in the range of $v \in [-1,1]$.

For $h = -1, n = 1$ and $\delta = 1$ then clearly, the solution series provides the solution and converges to the exact solution
$\xi(v,t) = e^{\,v^2}\left(1 + t + \frac{t^2}{2!} + \cdots\right) = e^{\,v^2+t}$

***Example*** **6.2** Consider the following nonlinear time -fractional Kolmogorov equation

$${}^{ABC}_{\ \ 0}D_t^\delta \xi(v,t) = (v+1)\xi_v(v,t) + v^2 e^t \xi_{vv}(v,t), (v,t) \in [0,1] \times \mathbb{R}\,, 0 < \delta \leq 1 \tag{34}$$

with initial conditions

$$\xi(v,0) = v + 1 \tag{35}$$

Applying Sawi transform on Equation (34), we obtain

$$\wp[\xi(v,t)] = \frac{1}{s}\xi(v,0) + \left(\frac{\delta s^\delta - \delta + 1}{G(\delta)}\right)\wp[(v+1)\xi_v(v,t) + v^2 e^t \xi_{vv}(v,t)] \tag{36}$$

Using the initial condition (35), we obtain

$$\wp[\xi(v,t)] = \frac{v+1}{s} + \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp[(v+1)\xi_v(v,t) + v^2 e^t \xi_{vv}(v,t)] \quad (37)$$

Applying q-HASTM on Equation (37), we get

$$\mathcal{N}[\psi(v,t;q)] = \wp[\psi(v,t;q)] - \frac{v+1}{s} - \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp[(v+1)\psi_v(v,t;q) + v^2 e^t \psi_{vv}(v,t;q)] \quad (38)$$

and we have

$$\mathcal{R}(\vec{\xi}_{m-1}) = \wp[\xi_{m-1}(v,t)] - \left(1 - \frac{\Upsilon_m}{n}\right)\wp[v+1] - \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp\left[(v+1)\frac{\partial \xi_{m-1}(v,t)}{\partial v} + v^2 e^t \frac{\partial^2 \xi_{m-1}(v,t)}{\partial v^2}\right] \quad (39)$$

Thus, the $m$th-order deformed equation is defined as

$$\mathcal{R}(\vec{\xi}_{m-1}) = h^{-1}\wp[\vec{\xi}_{m-1}(v,t) - \Upsilon_m \xi_{m-1}(v,t)] \quad (40)$$

Taking inverse Sawi transform to Equation (40), we get

$$\xi_m(v,t) = \Upsilon_m \xi_{m-1}(v,t) + h\,\wp^{-1}[\mathcal{R}(\vec{\xi}_{m-1})] \quad (41)$$

Note that, the first few terms of $\xi_m(v,t)$ is given by
$\xi_0(v,t) = v + 1.$

The first iterative $\xi_1(v,t)$ can be obtained as

$$\xi_1(v,t) = \Upsilon_1 \xi_0(v,t) + h\,\wp^{-1}[\mathcal{R}(\vec{\xi}_0)] = h\,\wp^{-1}\left[\wp[\xi_0(v,t)] - \left(1 - \frac{\Upsilon_1}{n}\right)\wp[v+1] - \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp\left[(v+1)\frac{\partial \xi_0(v,t)}{\partial v} + v^2 e^t \frac{\partial^2 \xi_0(v,t)}{\partial v^2}\right]\right] = h\,\wp^{-1}\left[\frac{v+1}{s} - \frac{v+1}{s} - \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp[(v+1)]\right] = \frac{-(v+1)h}{G(\delta)}\,\wp^{-1}\left[\frac{\delta s^{\delta} - \delta + 1}{s}\right] = \frac{-(v+1)h}{G(\delta)}\left(\frac{\delta t^{\delta}}{\Gamma(\delta+1)} - \delta + 1\right) = \frac{h}{G(\delta)}\left(\delta(v+1)\left(1 - \frac{t^{\delta}}{\Gamma(\delta+1)}\right) - v - 1\right) = \frac{nh\delta(v+1)}{G(\delta)}\left(\left(1 - \frac{t^{\delta}}{\Gamma(\delta+1)}\right) - 1\right).$$

The second iterative can be obtained as

$$\xi_2(v,t) = \Upsilon_2 \xi_1(v,t) + h\,\wp^{-1}[\mathcal{R}(\vec{\xi}_1)] = \frac{nh\delta(v+1)}{G(\delta)}\left(\left(1 - \frac{t^{\delta}}{\Gamma(\delta+1)}\right) - 1\right) + h\,\wp^{-1}\left[\wp[\xi_1(v,t)] - \left(1 - \frac{\Upsilon_2}{n}\right)\wp[v+1] - \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp\left[(v+1)\frac{\partial \xi_1(v,t)}{\partial v} + v^2 e^t \frac{\partial^2 \xi_1(v,t)}{\partial v^2}\right]\right] = \frac{n(v+1)h(n+h)}{G(\delta)}\left\{\delta\left(1 - \frac{t^{\delta}}{\Gamma(\delta+1)}\right) - 1\right\} + \frac{(v+1)h^2}{G^2(\delta)}\left\{1 + \delta^2\left(\frac{t^{2\delta}}{\Gamma(2\delta+1)} + 1\right) + 2\delta\left(\frac{t^{\delta}(1-\delta)}{\Gamma(\delta+1)} - 1\right)\right\}.$$

In the same way, we get

$$\xi_3(v,t) = n\left[\frac{n(v+1)h(n+h)}{G(\delta)}\left\{\delta\left(1 - \frac{t^{\delta}}{\Gamma(\delta+1)}\right) - 1\right\} + \frac{(v+1)h^2}{G^2(\delta)}\left\{1 + \delta^2\left(\frac{t^{2\delta}}{\Gamma(2\delta+1)} + 1\right) + 2\delta\left(\frac{t^{\delta}(1-\delta)}{\Gamma(\delta+1)} - 1\right)\right\}\right] + h\left[\frac{(2h+n)}{G^2(\delta)}\left\{h\left(1 + \delta^2\left(1 + \frac{t^{2\delta}}{\Gamma(2\delta+1)} + v\right) + \left(\frac{(2 - G(\delta) - 2\delta)t^{\delta}(v+1)}{\Gamma(\delta+1)} - 2v - 2\right)\delta + v\right)\right\}\right] + \frac{1}{G^3(\delta)}\left[h(v+1)\left\{(v-1)\left(G^2(\delta)(h+n) + \frac{3h\delta^2 t^{2\delta}}{\Gamma(2\delta+1)}\right) + \left\{\left(1 - \frac{t^{3\delta}}{\Gamma(3\delta+1)}\right)\delta^3 - 3\delta^3 + 3\left(1 - \frac{(v-1)^2 t^{\delta}}{\Gamma(\delta+1)}\right)v - 1\right\}h\right\}\right].$$

Similarly, the components of $\xi_m(v,t), m \geq 4$, can be easily obtained.

Thus, the series solution is given as
$\xi(v,t) = \xi_0(v,t) + \sum_{m=1}^{\infty} \frac{\xi_m(v,t)}{n^m}$.

For $h = -1, n = 1$ and $\delta = 1$ then clearly, the solution series provides the solution and converges to the exact solution
$\xi(v,t) = (v+1)\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) = (v+1)e^t$.

The Figure is the plot of the function $\xi(v,t) = (v+1)e^t$ for different values of $t = 0.4, 0.6, 0.8, 1.0$. The curves show how the function varies with $v$ for each specified value of $t$.

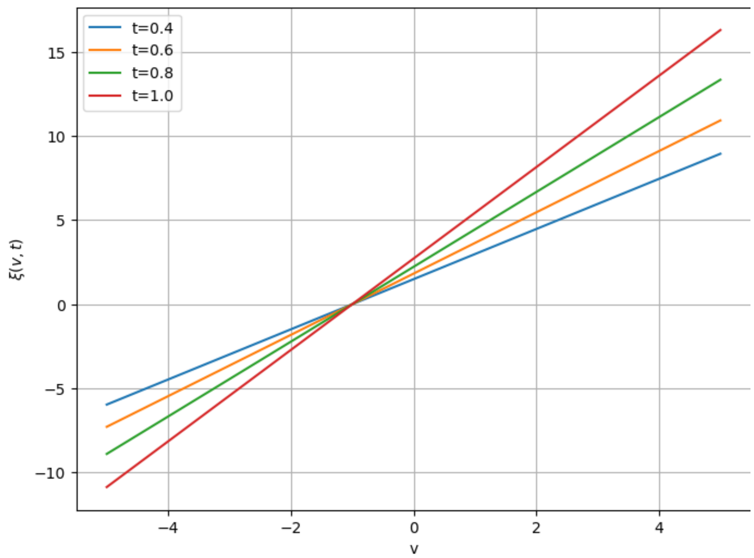


**Figure 2.** Plot of $\xi(v,t)$ for different values of $t$.

***Example* 6.3** Consider the following time -fractional Rosenau-Hyman equation

$${}^{ABC}_{\ \ 0}D_t^{\delta}\xi(v,t) = \xi(v,t)\left(\xi_v(v,t)\right)^3 + \xi(v,t)\xi_v(v,t) + 3\xi_v(v,t)\left(\xi_v(v,t)\right)^2, (v,t) \in [0,1] \times \mathbb{R}\ , 0 < \delta \leq 1 \quad (42)$$

with initial conditions

$$\xi(v,0) = -\frac{8C}{3}\cos^2\left(\frac{v}{4}\right) \quad (43)$$

Applying Sawi transform on Equation (42), we obtain

$$\wp[\xi(v,t)] = \frac{1}{s}\xi(v,0) + \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp\left[\xi(v,t)\left(\xi_v(v,t)\right)^3 + \xi(v,t)\xi_v(v,t) + 3\xi_v(v,t)\left(\xi_v(v,t)\right)^2\right] \quad (44)$$

Using the initial condition (43), we obtain

$$\wp[\xi(v,t)] = -\frac{\frac{8C}{3}\cos^2\left(\frac{v}{4}\right)}{s} + \left(\frac{\delta s^{\delta} - \delta + 1}{G(\delta)}\right)\wp\left[\xi(v,t)\left(\xi_v(v,t)\right)^3 + \xi(v,t)\xi_v(v,t) + 3\xi_v(v,t)\left(\xi_v(v,t)\right)^2\right] \quad (45)$$

Applying q-HASTM on Equation (44), we get

$$\mathcal{N}[\psi(v,t;q)] = \wp[\psi(v,t;q)] + \frac{\frac{8C}{3}\cos^2\left(\frac{v}{4}\right)}{s} - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\Big[\psi(v,t;q)\big(\psi_v(v,t;q)\big)^3 + \psi(v,t;q)\psi_v(v,t;q) + 3\psi_v(v,t;q)\big(\psi_v(v,t;q)\big)^2\Big] \quad (46)$$

Thus, we have

$$\mathcal{R}(\vec{\xi}_{m-1}) = \wp[\xi_{m-1}(v,t)] + \left(1-\frac{\Upsilon_m}{n}\right)\wp\left[\frac{\frac{8C}{3}\cos^2\left(\frac{v}{4}\right)}{s}\right] - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\Big[\sum_{r=0}^{m-1}\xi_r(v,t)\left(\frac{\partial\xi_{m-r-1}(v,t)}{\partial v}\right)^3 + \sum_{r=0}^{m-1}\xi_r(v,t)\frac{\partial\xi_{m-r-1}(v,t)}{\partial v} + 3\sum_{r=0}^{m-1}\frac{\partial\xi_r(v,t)}{\partial v}\left(\frac{\partial\xi_{m-r-1}(v,t)}{\partial v}\right)^2\Big] \quad (47)$$

Thus, the $m$th-order deformed equation is defined as

$$\mathcal{R}(\vec{\xi}_{m-1}) = h^{-1}\wp[\vec{\xi}_{m-1}(v,t) - \Upsilon_m\xi_{m-1}(v,t)] \quad (48)$$

Taking inverse Sawi transform to Equation (48), we get

$$\xi_m(v,t) = \Upsilon_m\xi_{m-1}(v,t) + h\,\wp^{-1}[\mathcal{R}(\vec{\xi}_{m-1})] \quad (49)$$

Note that, the first few terms of $\xi_m(v,t)$ is given by

$\xi_0(v,t) = -\frac{8C}{3}\cos^2\left(\frac{v}{4}\right)$.

The first iterative can be obtained as

$$\xi_1(v,t) = h\,\wp^{-1}[\mathcal{R}(\vec{\xi}_0)] = h\,\wp^{-1}\left[\wp[\xi_0(v,t)] + \left(1-\frac{\Upsilon_1}{n}\right)\wp\left[\frac{\frac{8C}{3}\cos^2\left(\frac{v}{4}\right)}{s}\right] - \left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\left[\xi_0(v,t)\left(\frac{\partial\xi_0(v,t)}{\partial v}\right)^3 + \xi_0(v,t)\frac{\partial\xi_0(v,t)}{\partial v} + 3\frac{\partial\xi_0(v,t)}{\partial v}\left(\frac{\partial\xi_0(v,t)}{\partial v}\right)^2\right]\right] =$$

$$-h\,\wp^{-1}\left[\left(\frac{\delta s^{\delta}-\delta+1}{G(\delta)}\right)\wp\left[\xi_0(v,t)\left(\frac{\partial\xi_0(v,t)}{\partial v}\right)^3 + \xi_0(v,t)\frac{\partial\xi_0(v,t)}{\partial v} + 3\frac{\partial\xi_0(v,t)}{\partial v}\left(\frac{\partial\xi_0(v,t)}{\partial v}\right)^2\right]\right] = \frac{4hC^2\cos\left(\frac{v}{4}\right)\sin\left(\frac{v}{4}\right)\left(1+\delta\left(\frac{t^{\delta}}{\Gamma(\delta+1)}-1\right)\right)}{3G(\delta)}.$$

In the same way, we get

$$\xi_2(v,t) = \Upsilon_2\xi_1(v,t) + h\,\wp^{-1}[\mathcal{R}(\vec{\xi}_1)] = \frac{4\,n\,hC^2\cos\left(\frac{v}{4}\right)\sin\left(\frac{v}{4}\right)\left(1+\delta\left(\frac{t^{\delta}}{\Gamma(\delta+1)}-1\right)\right)}{3G(\delta)} + \frac{1}{3}\left\{h^2\left(C^3\left(\frac{\left[\delta^2t^{2\delta}\cos^2\left(\frac{v}{4}\right)-\sin^2\left(\frac{v}{4}\right)\right]}{\Gamma(2\delta+1)} + 2\frac{(v-1)\left\{1-2\cos\left(\frac{v}{4}\right)^2\right\}\delta t^{\delta}}{\Gamma(\delta+1)} + \left(2\cos\left(\frac{v}{4}\right)^2-1\right)1(v-1)^2\right)\right) + \frac{4C^2\cos\left(\frac{v}{4}\right)\sin\left(\frac{v}{4}\right)\left(1+\frac{\delta t^{\delta}}{\Gamma(\delta+1)}-\delta\right)}{G(\delta)}\right\}.$$

Hence, the components of $\xi_m(v,t), m \geq 4$, can be easily obtained.

Thus, the series solution is given as
$\xi(v,t) = \xi_0(v,t) + \sum_{m=1}^{\infty} \frac{\xi_m(v,t)}{n^m}$.

For $h = -1, n = 1$ and $\delta = 1$ then clearly, the solution series provides the solution and converges to the exact solution
$\xi(v,t) = -\frac{8C}{3}\cos^2\left(\frac{v-Ct}{4}\right)$.

The observed results demonstrate the effectiveness of the proposed STHPM in solving nonlinear fractional partial differential equations. The trends in the figures, particularly the variation of the solution with respect to $\delta$ and $h$, highlight the impact of the Atangana-Baleanu fractional derivative. As $\delta$ increases, the solution exhibits smoother behavior due to the non-singular and non-local nature of the kernel, which better accounts for memory and hereditary effects. The parameter h, representing the strength of nonlinear terms, influences the convergence rate and solution stability. The results confirm that the proposed method achieves faster convergence and higher accuracy compared to classical approaches, as it efficiently handles the complexities introduced by the fractional operators. This behavior aligns with theoretical expectations and underscores the significance of the Atangana-Baleanu derivative in capturing the physical phenomena modeled by FPDEs. Furthermore, the slight deviations in solution trends for smaller $\delta$ values can be attributed to the increased influence of fractional effects, requiring higher iterations for convergence.

## 7. Conclusion

In this study, a novel analytical approach combining the Sawi Transform and the Atangana-Baleanu fractional derivative has been proposed to solve nonlinear FPDEs. By extending the operational properties and convolution theorems of the Sawi Transform, the method efficiently handles the complexities of FPDEs, achieving both exact and rapidly convergent series solutions. The results, validated through the fractional Kolmogorov and Rosenau-Hyman equations, demonstrate improved accuracy, reduced computational complexity, and the ability to model memory and hereditary effects effectively.

While the proposed approach shows significant promise, further improvements can focus on extending the method to a broader range of fractional operators and higher-dimensional systems. Additionally, exploring numerical implementations for highly complex FPDEs can enhance its applicability to real-world engineering and scientific problems. Future research may also address practical applications in fields such as viscoelasticity, fluid dynamics, and biological systems, where fractional calculus plays a critical role.

The present research opens several promising directions for future work. The proposed approach can be extended to solve higher-dimensional fractional partial differential equations (FPDEs) and systems involving mixed fractional operators. Additionally, numerical implementations of the method for more complex nonlinear FPDEs could further enhance its practicality. Future research can also focus on applying the method to real-world problems in fields such as viscoelastic materials, anomalous diffusion, fluid dynamics, and biological systems. By incorporating additional fractional operators or hybrid techniques, the approach can be refined to address emerging challenges in fractional calculus and mathematical modeling.

**Conflict of Interest**

The authors confirm that there is no conflict of interest to declare for this publication.

**AI Disclosure**

During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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