International Journal of Mathematical, Engineering and Management Sciences
Vol. 8, No. 4, 769-786, 2023
https://doi.org/10.33889/IJMEMS.2023.8.4.044

Performance Analysis of a Retrial Queueing System with Optional Service, Unreliable Server, Balking and Feedback

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(Received on February 21, 2023; Accepted on June 6, 2023)

Abstract
This paper considers a Markovian retrial queueing system with an optional service, unreliable server, balking and feedback. An arriving customer can avail of immediate service if the server is free. If the potential customer encounters a busy server, it may either join the orbit or balk the system. The customers may retry their request for service from the orbit after a random amount of time. Each customer gets the First Essential Service (FES). After the completion of FES, the customers may seek the Second Optional Service (SOS) or leave the system. In the event of unforeseen circumstances, the server may encounter a breakdown, at which point an immediate repair process will be initiated. After the service completion, the customer may leave the system or re-join the orbit if not satisfied and demand regular service as feedback. In this investigation, the stationary queue size distributions are framed using a recursive approach. Various system performance measures are derived. The effects induced by the system parameters on the performance metrics are numerically and graphically analysed.

Keywords- Retrial queue, Optional service, Feedback, Unreliable server.

1. Introduction

In our standard daily regimen, it is not always possible to get immediate services. When the demand for service is high, customers are expected to wait in a queue to receive service. It might be experienced in supermarkets, hospitals, banking sectors, ticket counters, etc. In retrial queueing, if the server is busy at the time of arrival, then the customer may enter into the orbit. From the orbit, the customers may check the server availability and retry their service after a certain period of time. For example, the router's transmitted packets are buffered when waiting for data transmission in a packet-switched network.

If a customer arrives when the server is busy and observes a long queue waiting for service, the customer may not join the queue and leave the service area. This phenomenon is termed as balking. For example, a customer arriving at a restaurant may observe a long queue due to unexpectedly high demand for service and not enter the restaurant. The arriving customers are all mandated to receive the first essential service.

On completion of first essential service, customers may request a second optional service or depart the system. For example, let us consider a fuel station. Every customer arriving needs to fill fuel in their vehicles. Apart from this, customers may require to fill air in their vehicle tyres, which is a second optional service; otherwise, the vehicles may leave the fuel station.

After completion of service, customers who are not satisfied with the service can re-join the orbit and receive regular service as feedback. For example, a user facing some issues with their mobile network may report to the customer care center for service. If, after receiving the service, the issue persists, the customer may report back to the call center and again request service. Due to unexpected circumstances, the server may be subject to breakdown. As the server becomes unable to provide service, the queue length blows up which provokes customers to balk the system. This has an adverse effect on the revenue of the system. To overcome this situation, an immediate repair process begins. Following this, the system resumes regular service. For example, consider the production of solar panels. When the trimming machine unexpectedly fails to work, the production process is temporarily stopped. To reactivate production, the machine is subject to immediate repair.

In the present era, a comprehensive service system is highly desirable to meet the diverse needs of customers. Such a system typically includes optional services and a feedback mechanism to ensure customer satisfaction. However, the real-world implementation of such a system is prone to service disruptions due to unreliable servers. Moreover, the busy lifestyles of customers may also result in impatience, causing them to avoid long queues and opt out of seeking services. Therefore, it is crucial to conduct a technical analysis of the proposed model's efficiency under these challenging conditions.

2. Literature Review

A single-server Markovian queueing system has been investigated by Engel and Hussin (2017) with two types of queues: a virtual queue, which is a regular type of queue, and a non-preemptive priority queue, which is known as a manual queue. The manual queue was found to be comparatively expensive. Dutta and Choudhary (2020) investigated a M/M/1 queueing system for its performance measures using simulation techniques. The performance of the system is effectively described with the use of substitution estimators and alternative estimators. Civelek et al. (2021) used an advanced simulation technique called the Vector-Auto-Regressive-to-Anything (VARTA) to analyse the impact of the time related dependencies in M/M/1 queueing systems. The simulation approach enables us to study the impact on non-exponentially distributed systems.

Retrial queueing system has been surveyed by Kim and Kim (2016). They have established the analytic findings on various performance metrics and briefed several real-life applications. Fiems (2022) reviewed the concept of retrial queues with a general retrial time. Different queueing systems are considered with general distribution of interarrival and service times. Dimitriou (2023) has studied the asymptotic behaviour of a retrial queue with increased retrial rates and varying arrival rates dependent on events.

A single server retrial queueing system with balking was inspected by Morozov et al. (2019), in which they considered a multiclass server. The Regenerative method is used to analyse the system, subject to generally distributed service times. The stability conditions and performance measures are established and validated through extensive simulation.

The concept of breakdown and repairs in a single server retrial queue was explored by Li et al. (2014). They have also incorporated observable balking strategies into their model. Chen and Zhou (2015) have studied an unreliable M/M/1 queue subject to setup times and balking strategies in both observable and nonobservable cases. Chang and Wang (2018) determined a M/M/1 retrial queue with unreliable server and a set-up time for reactivating an idle server. Two types of repair processes have been considered: perfectly repaired systems and partially repaired systems. Nazarov et al. (2019) have investigated a finite source single server Markovian retrial queue in which customer collisions were analysed. The server was considered to undergo unpredictable breakdowns and repairs based on its status.

Li and Wang (2021) have discussed a single-server Markovian retrial queueing system, subject to catastrophes. Following a repair process, a customer may re–join or balk the system subject to observable and unobservable levels. Lakaour et al. (2022) have discussed the effects of collisions, transmission errors, and unreliability in a single-server Markovian retrial queue. The steady state distribution has been derived by using the generating function method. The effectiveness of the system is validated through numerical illustration. Poongothai et al. (2022) have investigated two heterogeneous unreliable server retrial queueing models with customer discouragement. Using the recursive technique, the steady-state distribution and probabilities of the model have been obtained.

Customer feedback in unreliable single server retrial queues was studied by Chang et al. (2018). The impatience of customers with the concepts of balking and reneging has been taken into consideration. A comparison of the cost analysis between classical and constant retrial rates has been made. Bouchentouf et al. (2019) have analysed a single server Markovian finite capacity queueing system with multiple vacations, feedback, balking and reneging during busy periods. The steady-state solution of the model has been derived using a recursive approach. Akin and Ormeci (2022) have considered the Intensive Care Unit (ICU) as an example of a loss system with feedback.

The concept of optional service was explored by Kalidass and Kasturi (2014), who studied a single server queue with two service phases along with a finite number of immediate feedbacks. Arivudainambi and Godhandaraman (2015) determined a single server retrial queueing scenario with optional service, balking, and vacation. Using supplementary variable technique, explicit expressions for the probability generating functions of the server states, orbit length, and system length are established. The Stochastic decomposition law of the queueing system is also obtained.

Sundari and Srinivasan (2017) have considered a general service distribution with one essential and two optional services. Steady state as well as time-dependent solutions, along with various performance measures, have been established. Numerical examples were used to conduct a sensitivity analysis and determine how different parameters impact the system. Wang et al. (2017) have considered finite sources in single-server retrial queues with SOS. They have established some general results on queue size. Sasikala et al. (2018) have investigated the interactive telemedicine single server retrial queue model with optional service, breakdown, and multi-vacation. The steady state queueing system has been solved using the supplementary variable technique. Hoshur and Haji (2020) have considered a single-server queueing model with server breakdown and general service distribution of essential as well as optional services.

Zhang et al. (2018) have formed a queueing model for a dual-mode EV charging station and formulated a customer attrition minimization problem to effectively minimise the service drop rate of potential customers. Kumar et al. (2020) have come up with a solution to optimise the resource allocation subject to energy consumption and maxspan time. Mastoi et al. (2022) have carried out a deep analysis of planning the infrastructure for charging electric vehicles, considering the policies implemented. Future trends have also been taken into consideration.

However, to the best of our knowledge, there is no research work on single-server retrial queues with optional service, unreliable server, balking, or feedback. Several real-life scenarios require considering all the above-mentioned criteria when being modelled as a queueing system. This motivates us to develop such a retrial queueing model. Our concept is applicable to electrical vehicle charging stations.

The following sections of the paper are organised as follows: In Section 3, the real-life application of the proposed model is described. A detailed description of the working of the model is presented in Section 4. Section 5 derives the steady-state probabilities of queue size distribution by using a recursive approach. Various performance measures are presented in Section 6. Section 7 presents the results obtained through numerical analysis of the performance metrics. Also, the conclusion is provided in Section 8.

3. Model Implementation in Electrical Vehicle (EV) Charging Station

Electric Vehicle (EV) charging station is a potential application of the proposed model. as shown in Figure 1. EVs are completely eco-friendly, with zero percent toxic emissions, reduced noise pollution, and considerably low maintenance costs. With a large number of people switching to EVs, several EV charging stations are required. Electrical cars can be charged using level 2 and level 3 DC fast charging outlets. Level 2 charging outlets (208-240 Volts) are mainly used at homes, parking garages, malls, hotels, etc. Level 3 DC fast charging outlets (400-900 Volts) are intended for public and commercial areas.

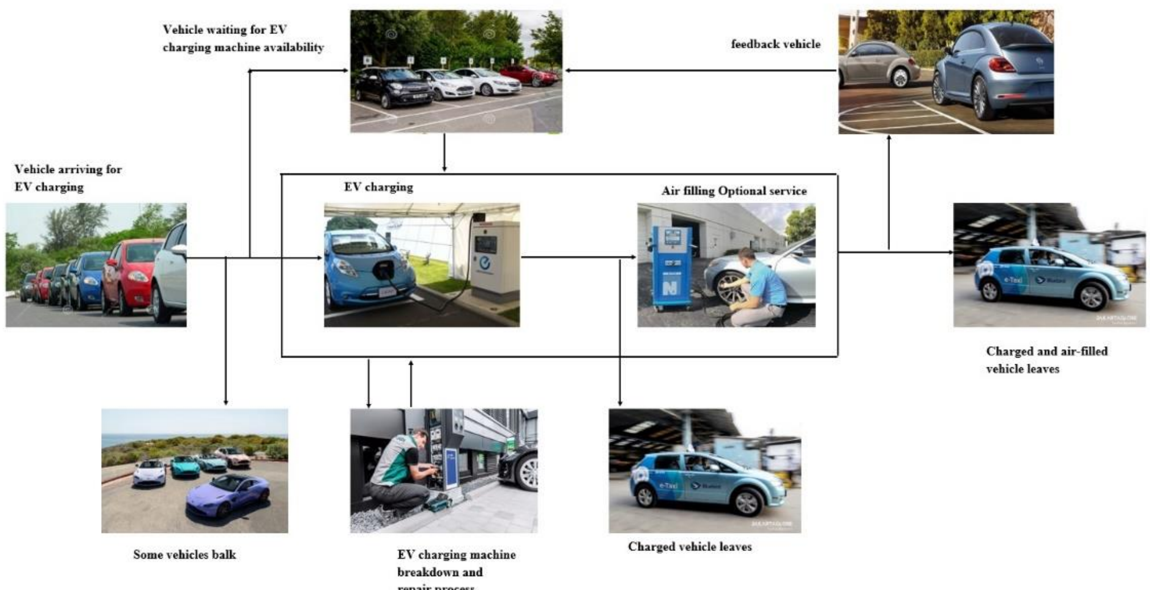

Figure 1. Electrical vehicle charging station.

In an EV charging station, an arriving EV is immediately served if the server is free. In the case of a busy server, the EV can join the waiting space or leave without getting service. From the waiting space, EVs can retry for service after checking the server’s availability. Level 3 DC fast charging is highly preferable for electric cars at public charging stations. After getting the vehicles charged, the EV may get their tyres air filled or leave the charging station. If the charging machine undergoes failure due to some unexpected reasons, an immediate repair process is started, and the system resumes service upon recovery. Charged

EVs that have departed the charging station, on finding the proposed range not being covered by the vehicle, may rejoin the waiting space for getting service.

4. Model Description

The transition diagram of a Markovian retrial queueing system with an optional service, unreliable server, balking and feedback is depicted in Figure 2. The primary customers arrive at a Poisson rate λ. All arriving customers enter the system to get the FES, which is provided at an exponential rate μ_1. Having received the FES, the customer may either leave the system with a probability $\bar{r}$ or join the SOS with probability r. The SOS is assumed to follow exponential distribution with a rate of μ_2.

If the server is busy, an arriving customer can either join the orbit with probability b or leave the system with probability $1-b$. After random amount of time, the customer may retry their request at an exponential rate γ. After the service completion, the customer either departs the system being fully satisfied with probability θ or joins the retrial orbit for repeated service as feedback with probability $1-\theta$. The server breakdown occurs at an exponential rate α. The failed server is then immediately subject to repair at an exponential rate β.

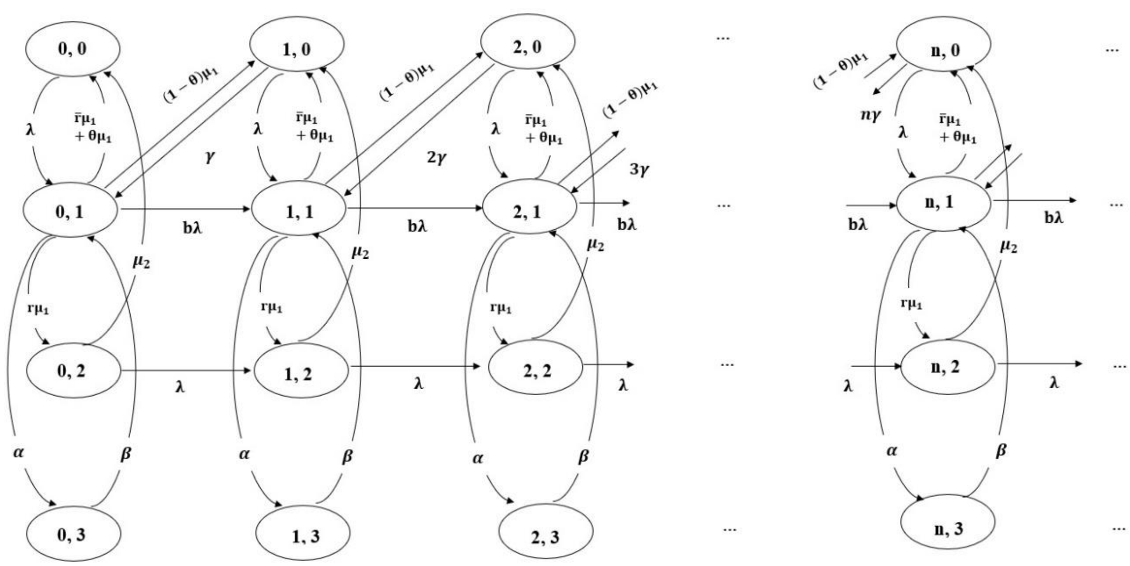

Figure 2. State transition diagram of a M/M/1 retrial queueing system with an optional service, unreliable server, balking and feedback.

Let $N(\tau), C(\tau)$ denote the number of customers present in the system and server's status at any time τ. We define,

$$C(\tau)=\begin{cases}0, & \text{idle state}\\ 1, & \text{FES state}\\ 2, & \text{SOS state}\\ 3, & \text{breakdown and repair state.}\end{cases}$$

Then $\{N(\tau), C(\tau): \tau \geq 0\}$ is a discrete state space Markov process in two variables. Let us denote the probability of n customers in the system with the server in state s by $P_{n,s}(\tau) = Prob\{N(\tau) = n, C(\tau) = s\}$. The various probabilities in steady state are denoted by $P_{n,s}(\tau) = \lim_{t\to\infty} P_{n,s}(\tau)$, where $n \geq 0, s = 0,1,2,3$.

5. Governing Equations

The steady state probability for retrial queueing system with an optional service, unreliable server, balking and feedback balance equation is formulated. Chapman-Kolmogorov equations for the system states related to different server states are formulated below.

$$\lambda P_{0,0} = (\bar{r}\mu_1 + \theta\mu_1)P_{0,1} + \mu_2 P_{0,2} \tag{1}$$
$$(\lambda + n\gamma)P_{n,0} = (\bar{r}\mu_1 + \theta\mu_1)P_{n,1} + \bar{\theta}\mu_1 P_{n-1,1} + \mu_2 P_{n,2}; \quad n \geq 1 \tag{2}$$
$$(b\lambda + 2\mu_1 + \alpha)P_{0,1} = \lambda P_{0,0} + \beta P_{0,3} + \gamma P_{1,0}. \tag{3}$$
$$(b\lambda + 2\mu_1 + \alpha)P_{n,1} = \lambda P_{n,0} + \beta P_{n,3} + (n+1)\gamma P_{n+1,0} + b\lambda P_{n-1,1}; \quad n \geq 1 \tag{4}$$
$$(\lambda + \mu_2)P_{0,2} = r\mu_1 P_{0,1} \tag{5}$$
$$(\lambda + \mu_2)P_{n,2} = r\mu_1 P_{n,1} + \lambda P_{n-1,2}; \quad n \geq 1 \tag{6}$$
$$\beta P_{n,3} = \alpha P_{n,1}; \quad n \geq 0 \tag{7}$$

5.1 Steady-State Analysis

We derive the steady state solution for the retrial queueing system with an optional service, unreliable server, balking and feedback model by solving Equations (1) - (7) recursively. Recursive technique is employed to examine the steady state probabilities in terms of $P_{0,0}$.

Solving equation (5), we get,

$$P_{0,2} = \frac{r\mu_1}{\lambda + \mu_2} P_{0,1} \tag{8}$$

Using Equation (8) in (1), we get the probability of the first customer getting FES,

$$P_{0,1} = \frac{\xi_0}{\eta} P_{0,0}; \quad \text{where } \xi_0 = \lambda(\lambda + \mu_2) \text{ and } \eta = (\lambda + \mu_2)(\bar{r}\mu_1 + \theta\mu_1) + r\mu_1\mu_2 \tag{9}$$

Applying Equation (9) in (8), we get the probability of the first customer getting SOS,

$$P_{0,2} = \frac{\nu_0}{\eta} P_{0,0}; \quad \text{where } \nu_0 = \lambda r \mu_1 \tag{10}$$

Considering $n = 0$ in equation (7) and substituting in equation (9), we get the probability of the system's breakdown during the service of the first customer,

$$P_{0,3} = \left(\frac{\alpha}{\beta}\right)\frac{\xi_0}{\eta} P_{0,0} \tag{11}$$

Using Equations (7) and (9) in Equation (3), we get the probability of having one customer in the orbit,

$$P_{1,0} = \frac{\zeta_1}{\gamma\eta} P_{0,0}; \quad \text{where } \zeta_1 = (b\lambda + 2\mu_1)\xi_0 - \lambda\eta \tag{12}$$

Substituting $n = 1$ in equation (6), we get,

$$P_{1,2} = \frac{r\mu_1}{\lambda + \mu_2} P_{1,1} + \frac{\lambda}{\lambda + \mu_2} P_{0,2} \tag{13}$$

While $n = 1$ in Equation (2), the Equations (9), (10), (12) and (13) give the probability of the customer from the orbit receiving FES given that there is only one customer in the orbit,

$$P_{1,1} = \frac{\xi_1}{\gamma\eta^2} P_{0,0}; \quad \text{where } \xi_1 = (\lambda + \gamma)(\lambda + \mu_2)\zeta_1 - \bar{\theta}\gamma\mu_1(\lambda + \mu_2)\xi_0 - \lambda\mu_2\nu_0\gamma \tag{14}$$

On substituting Equations (10) and (14) in Equation (13), we get the probability of the customer from the orbit having received FES, opting for SOS, given that there is only one customer in the orbit,

$$P_{1,2} = \frac{\nu_1}{(\lambda+\mu_2)\gamma\eta^2} P_{0,0}; \qquad \text{where } \nu_1 = r\mu_1\xi_1 + \lambda\nu_0\gamma\eta \tag{15}$$

Applying Equation (14) in Equation (7) with $n = 1$ yields the probability that the system breakdown during the service of the only customer in the orbit,

$$P_{1,3} = \left(\frac{\alpha}{\beta}\right)\frac{\xi_1}{\gamma\eta^2} P_{0,0} \tag{16}$$

Taking $n = 1$ in Equation (4) and using equations (7), (9), (12) and (14), the probability of having two orbital customers is obtained as,

$$P_{2,0} = \frac{\zeta_2}{2\gamma^2\eta^2} P_{0,0}; \qquad \text{where } \zeta_2 = (b\lambda + 2\mu_1)\xi_1 - \lambda\zeta_1\eta - b\lambda\xi_0\gamma\eta \tag{17}$$

Substituting $n = 2$ in equation (6), we get,

$$P_{2,2} = \frac{r\mu_1}{\lambda+\mu_2} P_{2,1} + \frac{\lambda}{\lambda+\mu_2} P_{1,2} \tag{18}$$

Considering $n = 2$ in Equation (2) and using Equations (14), (15), (17) and (18), we get the probability of the second customer from the orbit receiving FES,

$$P_{2,1} = \frac{\xi_2}{2\gamma^2\eta^3(\lambda+\mu_2)} P_{0,0}; \qquad \text{where } \xi_2 = (\lambda + 2\gamma)(\lambda + \mu_2)^2\zeta_2 - 2\bar{\theta}\gamma\mu_1\xi_1(\lambda + \mu_2)^2 - 2\lambda\mu_2\gamma\nu_1 \tag{19}$$

On substituting Equations (15) and (19) in equation (18), we get the probability of the second customer from the orbit who has received the FES and is opting for the SOS,

$$P_{2,2} = \frac{\nu_2}{2\gamma^2\eta^3(\lambda+\mu_2)^2} P_{0,0}; \qquad \text{where } \nu_2 = r\mu_1\xi_2 + 2\lambda\nu_1\gamma\eta \tag{20}$$

Taking $n = 2$ in Equation (7) and using Equation (19), we get the probability that the system breaks down during the service of the second customer in the orbit,

$$P_{2,3} = \left(\frac{\alpha}{\beta}\right)\frac{\xi_2}{2\gamma^2\eta^3(\lambda+\mu_2)} P_{0,0} \tag{21}$$

Similarly solving Equation (4), the probability of having n orbital customers is obtained as,

$$P_{n,0} = \frac{\zeta_n}{n!\gamma^n\eta^n(\lambda+\mu_2)^{n-2}} P_{0,0} \tag{22}$$

where, $\zeta_n = (b\lambda + 2\mu_1)\xi_{n-1} - \lambda\zeta_{n-1}\eta(\lambda + \mu_2) - (n-1)b\lambda\xi_{n-2}\gamma\eta(\lambda + \mu_2)$.

Solving Equation (2) in a similar way, we get the probability of the n^{th} customer from the orbit receiving First Essential Service,

$$P_{n,1} = \frac{\xi_n}{n!\gamma^n\eta^{n+1}(\lambda+\mu_2)^{n-1}} P_{0,0} \tag{23}$$

where $\xi_n = (\lambda + n\gamma)(\lambda + \mu_2)^2\zeta_n - n\gamma(\lambda + \mu_2)^2\bar{\theta}\mu_1\xi_{n-1} - n\lambda\mu_2\gamma\nu_{n-1}$.

After getting the FES, the probability of the n^{th} customer from the orbit receiving SOS can be obtained using Equation (6),

$$P_{n,2} = \frac{\nu_n}{n!\gamma^n\eta^{n+1}(\lambda+\mu_2)^n} P_{0,0}; \qquad \text{where } \nu_n = r\mu_1\xi_n + n\lambda\gamma\eta\nu_{n-1} \tag{24}$$

Equation (7) yields the probability that the system breaks down during the service of the n^{th} customer in the orbit,

$$P_{n,3} = \left(\frac{\alpha}{\beta}\right)\frac{\xi_n}{n!\gamma^n\eta^{n+1}(\lambda+\mu_2)^{n-1}}P_{0,0} \tag{25}$$

To obtaine $P_{0,0}$, the normalized expression is given below,

$$\sum_{n=0}^{\infty}\sum_{s=0}^{3} P_{n,s} = 1 \tag{26}$$

Further few algebraic manipulations give the probability value of $P_{0,0}$ as,

$$P_{0,0} = \left(\frac{1}{1+A+B+CD+\sum_{n=3}^{\infty}(E_n+F_n+G_nD)}\right) \tag{27}$$

where, $A = \frac{\zeta_1}{\gamma\eta} + \frac{\zeta_2}{2\gamma^2\eta^2}$, $B = \frac{\nu_0}{\eta} + \frac{\nu_1}{\gamma\eta^2(\lambda+\mu_2)} + \frac{\nu_2}{2\gamma^2\eta^3(\lambda+\mu_2)^2}$, $C = \frac{\xi_0}{\eta} + \frac{\xi_1}{\gamma\eta^2} + \frac{\xi_2}{2\gamma^2\eta^3(\lambda+\mu_2)}$.
$D = 1 + \frac{\alpha}{\beta}$, $E_n = \frac{\zeta_n}{n!\gamma^n\eta^n(\lambda+\mu_2)^{n-2}}$, $F_n = \frac{\nu_n}{n!\gamma^n\eta^{n+1}(\lambda+\mu_2)^n}$ and $G_n = \frac{\xi_n}{n!\gamma^n\eta^{n+1}(\lambda+\mu_2)^{n-1}}$.

5.2 Stability Condition

The existence of finite long run expectations of a queueing system is essential to deem the system as stable. To guarantee the stability of our proposed model, we derive an essential condition. Considering the lexicographical sequence for the states, we get the infinitesimal generator Q of the process as follows:

$$Q = \begin{bmatrix} X_0 & Y & & & & & \\ Z_1 & X_1 & Y & & & & \\ & Z_2 & X_2 & Y & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & Z_N & X_N & Y & \\ & & & & Z_N & X_N & Y \\ & & & & & \ddots & \ddots & \ddots \end{bmatrix}; \text{ where } Y = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \bar{\theta}\mu_1 & b\lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$X_N = \begin{bmatrix} -(\lambda+N\gamma) & \lambda & 0 & 0 \\ (\bar{r}\mu_1+\theta\mu_1) & -(b\lambda+2\mu_1+\alpha) & r\mu_1 & \alpha \\ \mu_2 & 0 & -(\lambda+\mu_2) & 0 \\ 0 & \beta & 0 & -\beta \end{bmatrix} \text{ and } Z_N = \begin{bmatrix} 0 & N\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Representing the long run probability vector as $x = [x_1, x_2, x_3, x_4]$, we have $x(X_N + Y + Z_N) = 0$ and $xe_4 = 1$, where 0 is a row vector of zeros with dimension 4 and e_4 is a column vector of unity with dimension 4. According to the stability condition obtained in Neuts (1981), the steady state probabilities exist if and only if $xYe < xZ_Ne$.

i.e., $$\frac{\lambda+n\gamma}{2n\gamma}\left[\frac{\lambda b}{\mu_1} + \frac{\lambda r}{\mu_2} + 1\right] < 1 \tag{28}$$

6. Performance Indices

To investigate the proposed model, certain measures are essential to evaluate its performance. Based on the steady-state queue size distribution, the long-run probabilities of the server being in different states, the expected number of customers, and their corresponding average waiting times are evaluated in the following manner.

Probability that the server is idle P_I,

$$P_I = \sum_{n=0}^{\infty} P_{n,0} = \left(1 + A + \sum_{n=3}^{\infty} \frac{\zeta_n}{n!\gamma^n \eta^n (\lambda+\mu_2)^{n-2}}\right) P_{0,0} \tag{29}$$

Probability that the server is busy P_B,

$$P_B = \sum_{n=0}^{\infty} \left(P_{n,1} + P_{n,2}\right) = \left[B + C + \sum_{n=3}^{\infty} \frac{(\lambda+\mu_2)\xi_n + \nu_n}{n!\gamma^n \eta^{n+1} (\lambda+\mu_2)^n}\right] P_{0,0} \tag{30}$$

Probability that the server is in breakdown state P_{BD},

$$P_{BD} = \sum_{n=0}^{\infty} P_{n,3} = \frac{\alpha}{\beta}\left[C + \sum_{n=3}^{\infty} \frac{\xi_n}{n!\gamma^n \eta^{n+1} (\lambda+\mu_2)^{n-1}}\right] P_{0,0} \tag{31}$$

Fraction of lost customers due to balking (LCB),

$$LCB = \sum_{n=0}^{\infty} \lambda(1-b)\left(P_{n,0} + P_{n,1}\right) = \lambda(1-b)\left[1 + A + C + \sum_{n=3}^{\infty}(E_n + G_n)\right] P_{0,0} \tag{32}$$

Fraction of lost customers due to breakdown (LCBD),

$$LCBD = \sum_{n=0}^{\infty} \lambda(1-b) P_{n,3} = \frac{\lambda(1-b)\alpha}{\beta}\left[C + \sum_{n=3}^{\infty} G_n\right] P_{0,0} \tag{33}$$

The mean number of customers in the orbit $E(N_O)$,

$$E(N_O) = \sum_{n=0}^{\infty} n P_{n,0} = \left[\frac{\zeta_1 \gamma\eta + \zeta_2}{\gamma^2 \eta^2} + \sum_{n=3}^{\infty} n E_n\right] P_{0,0} \tag{34}$$

The mean number of customers in the system $E(N_S)$,

$$\begin{aligned} E(N_S) &= \sum_{n=0}^{\infty} n\left(P_{n,0} + P_{n,1} + P_{n,2} + P_{n,3}\right) \\ &= E(N_O) + \left\{\left[\frac{\left((\lambda+\mu_2)^2 \gamma\eta\xi_1 + (\lambda+\mu_2)\xi_2\right)D + (\lambda+\mu_2)\gamma\eta\nu_1 + \nu_2}{\gamma^2 \eta^3 (\lambda+\mu_2)^2}\right] + \sum_{n=3}^{\infty} n(E_n + D G_n)\right\} P_{0,0} \end{aligned} \tag{35}$$

Expected waiting time of customer in the orbit (W_O),

$$W_O = \frac{E(N_O)}{\lambda_{eff}} = \frac{1}{\lambda_{eff}}\left[\frac{\zeta_1 \gamma\eta + \zeta_2}{\gamma^2 \eta^2} + \sum_{n=3}^{\infty} n E_n\right] P_{0,0} \tag{36}$$

Expected waiting time of customer in the system (W_S),

$$W_S = \frac{E(N_S)}{\lambda_{eff}} = \frac{E(N_O)}{\lambda_{eff}} + \frac{1}{\lambda_{eff}}\left\{\left[\frac{\left((\lambda+\mu_2)^2 \gamma\eta\xi_1 + (\lambda+\mu_2)\xi_2\right)D + (\lambda+\mu_2)\gamma\eta\nu_1 + \nu_2}{\gamma^2 \eta^3 (\lambda+\mu_2)^2}\right] + \sum_{n=3}^{\infty} n(E_n + D G_n)\}\right\} P_{0,0} \tag{37}$$

where, $\lambda_{eff} = b\lambda$.

7. Numerical Results

Consider an EV charging station. Suppose these EVs arrive at the EV charging station at a rate λ. An arriving EV is immediately charged at a rate μ_1, if the EV charging machine is free at the time of arrival. After being charged, the EVs can get their tyres air-filled or leave the charging station, with a probability r and $\bar{r}$, respectively. An EV gets their tyres air filling at a rate μ_2. In case, the EV charging machine is busy, an arriving EV can join the waiting space or leave without getting service, with probability b and $1-b$, respectively.

From the waiting space, EVs can retry for the availability of the EV charging machine at a rate γ. If the charging machine undergoes failure at a rate α due to some unexpected reasons, an immediate repair process is initiated at a rate of β. The charging machine resumes service provision upon recovery. After getting charged, an EV that finds the proposed range not being attained may rejoin the waiting space or leave the charging station with probability θ or $1-\theta$ respectively.

The influence of system parameters on performance metrics is validated through numerical analysis. The effects of variations in parameters on the probabilities of the EV charging machine being idle $[P_I]$, busy $[P_B]$, breakdown $[P_{BD}]$ on the Expected number of EVs in the charging station $[E(N_S)]$ are analysed. By varying these parameters, their influence on the performance metrics is measured in the following cases using MATLAB: The choice of the parameters is subject to satisfying the stability condition established.

Case 1: For the following choice of parameters $\mu_1 = 10, \mu_2 = 8, \gamma = 2, \alpha = 0.5, \beta = 4, r = 0.3,$ and $\bar{\theta} = 0.2$, the results of variations in λ and b are depicted in Figure 3. It is observed that irrespective of the choice of b, when the arrival rate increases, the expected number of EVs in the EV charging station also increases proportionally. Further, the effect of the same choice of parameters along with a fixed value of $b = 0.8$ is depicted in Figure 4 against variations in λ. This indicates that the probability of the EV charging machine being busy and being subject to breakdown increases, whereas the probability of the EV charging machine being idle decreases, corresponding to an increase in the arrival rate of EVs.

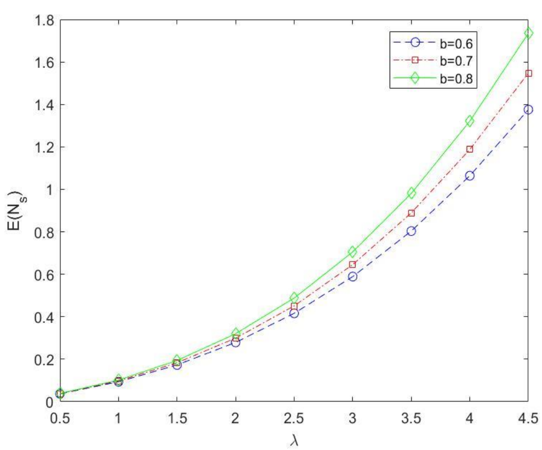

Figure 3. λ for various in b vs. $E(N_S)$.

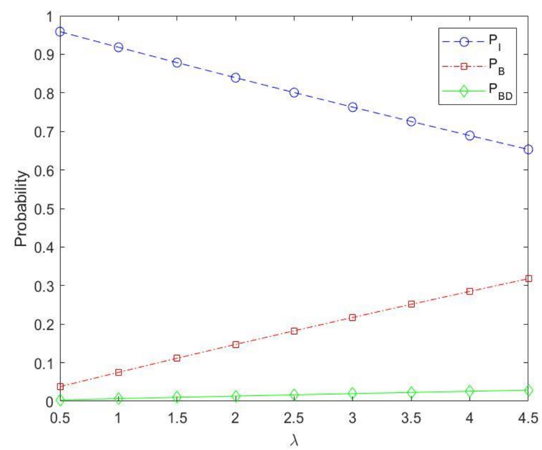

Figure 4. λ vs. P_I, P_B and P_{BD}.

Case 2: For $\lambda = 3, \mu_2 = 8, \gamma = 2, \alpha = 0.5, \beta = 4, r = 0.3,$ and $\bar{\theta} = 0.2$, the effects of changes in μ_1 and b are shown in Figure 5. It can be seen that by increasing μ_1, the expected number of EVs in the EV charging station considerably decreases, irrespective of the choice of b. Moreover, the effect of the same set of values along with a particular choice of $b = 0.8$ is depicted in Figure 6 against variations in μ_1. It is observable that the probability of the EV charging machine being idle considerably increases, whereas the probability of the EV charging machine being busy and being subject to breakdown declines with an increase in the service rate of the EV charging machine.

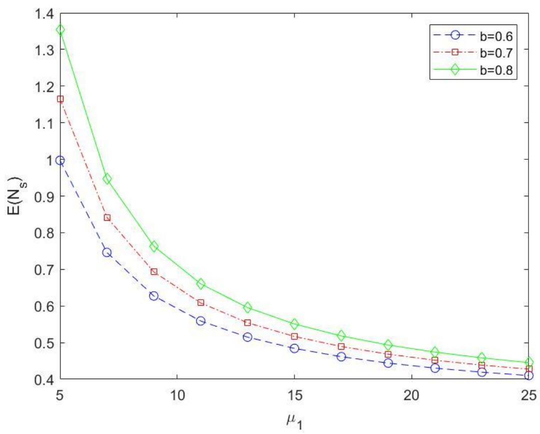

Figure 5. μ_1 for various in b vs. $E(N_S)$.

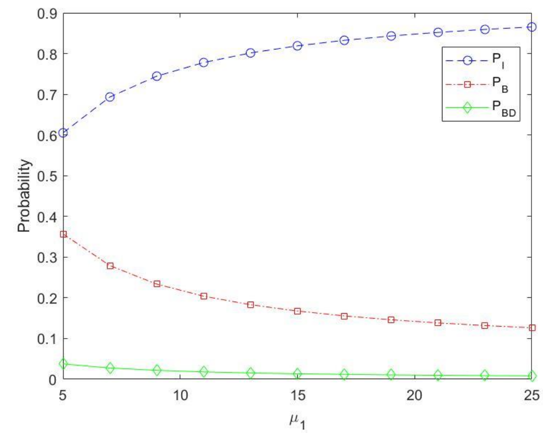

Figure 6. μ_1 vs. P_I, P_B and P_{BD}.

Case 3: By choosing $\lambda = 3, \mu_1 = 10, \gamma = 2, \alpha = 0.5, \beta = 4, b = 0.8,$ and $\bar{\theta} = 0.2$, the effects of variations in μ_2 and r are shown in Figure 7. It is observed that an increase in μ_2 results in an effective decrease in the expected number of EVs in the EV charging station irrespective of the choice of r. Further, the effect of the same choice of parameters along with a fixed value of $r = 0.3$ is plotted in Figure 8 against variations in μ_2. This indicates that the chances of the air filling machine at the EV charging station being idle increases, and the chances of the machine being busy decrease with an increase in the service rate of the air filling machine. Further, this rate has no influence over the chances of the machine being subject to breakdown.

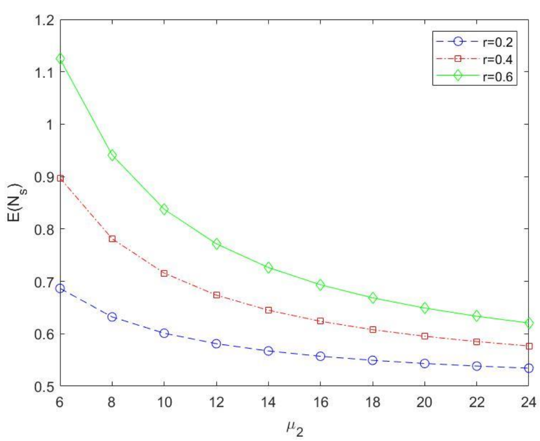

Figure 7. μ_2 for various in r vs. $E(N_S)$.

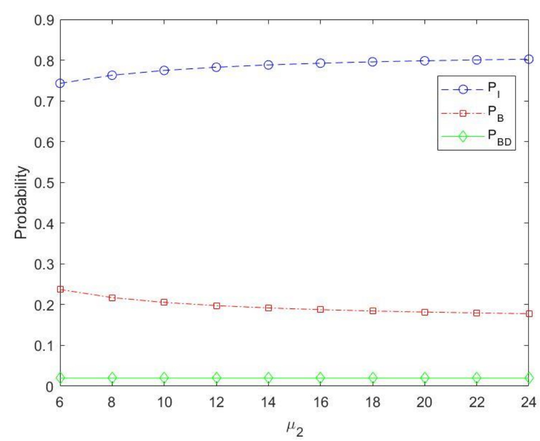

Figure 8. μ_2 vs. P_I, P_B and P_{BD}.

Case 4: Taking $\lambda = 3, \mu_1 = 10, \mu_2 = 8, \alpha = 0.5, \beta = 4, r = 0.3,$ and $\bar{\theta} = 0.2$, the results of changes in γ and b are depicted in Figure 9. It is observed that when the retrial rate increases, the expected number of EVs in the EV charging station considerably decreases, irrespective of the choice of b. Further, the effect of the same parameter set along with a choice of $b = 0.8$ is depicted in Figure 10 against variations in γ. It can be seen that changes in retrial rate have no effect on the status of the EV charging machine.

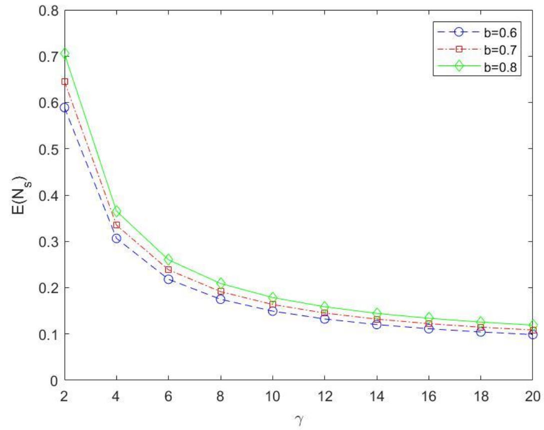

Figure 9. γ for various in b vs. $E(N_S)$.

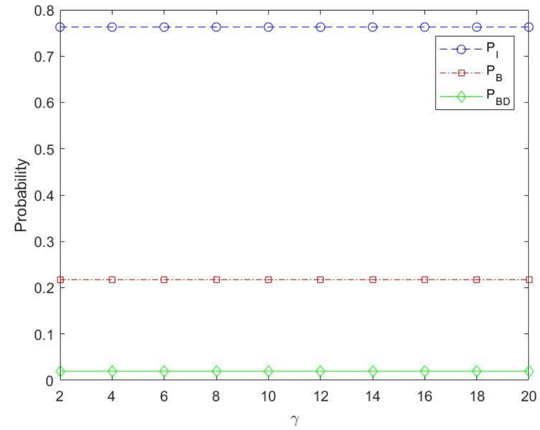

Figure 10. γ vs. P_I, P_B and P_{BD}.

Case 5: For $\lambda = 3, \mu_1 = 10, \mu_2 = 8, \gamma = 2, \beta = 4, r = 0.3,$ and $\bar{\theta} = 0.2$, the effects of various choices of α and b are shown in Figure 11. It is observed that with an increase in the breakdown rate, the expected number of EVs in the EV charging station considerably increases, irrespective of different choices of b. By fixing the value of b as 0.8, the effect of the above parameter set is shown in Figure 12 against various choices of α. It shows that the chances of the EV charging machine becoming idle or busy decrease, whereas the machine is highly likely to breakdown with an increase in the breakdown rate of the EV charging machine.

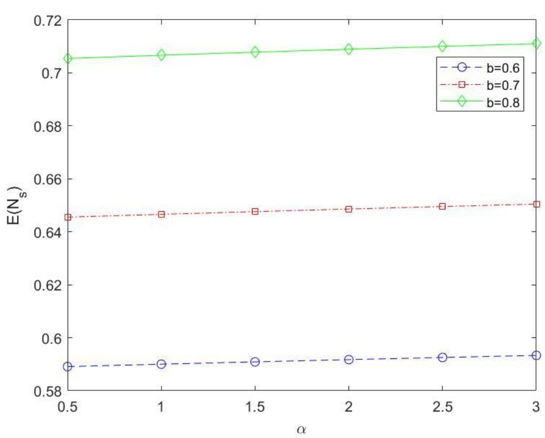

Figure 11. α for various in b vs. $E(N_S)$.

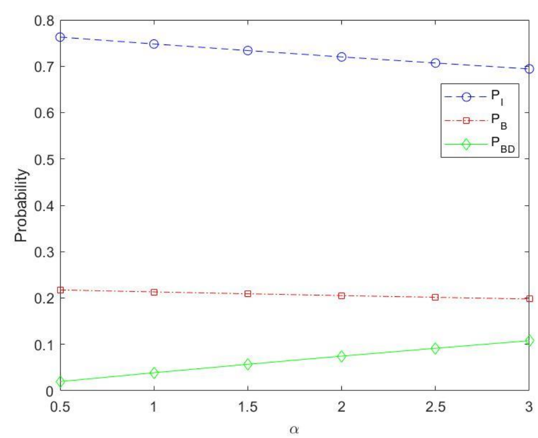

Figure 12. α vs. P_I, P_B and P_{BD}.

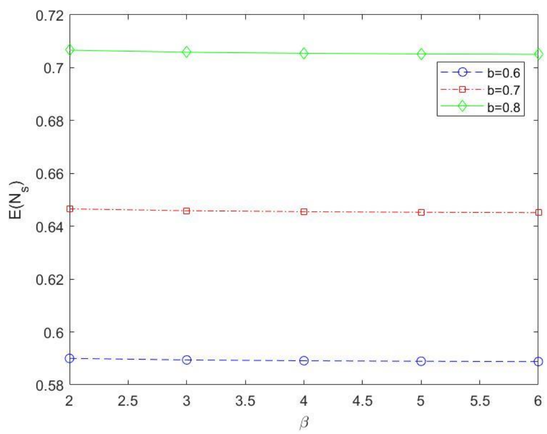

Figure 13. β for various in b vs. $E(N_S)$.

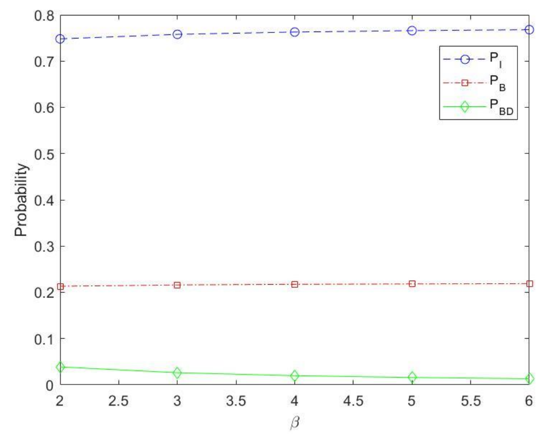

Figure 14. β vs. P_I, P_B and P_{BD}.

Case 6: Taking $\lambda = 3, \mu_1 = 10, \mu_2 = 8, \gamma = 2, \alpha = 0.5, r = 0.3,$ and $\bar{\theta} = 0.2$, the effects of various values of β and b are plotted in Figure 13. It is evident that when the EV charging machine has a high repair rate in times of failure, the expected number of EVs in the EV charging station shows a declining trend, irrespective of the choice of b. Further, for a choice of $b = 0.8$, the effect of the above parameter set is plotted in Figure 14 against various choices of β. It is observed that the chances of the EV charging machine becoming idle or busy increase with an increase in repair rates. Moreover, there is a considerable decline in the chance of the machine break down.

From Table 1, it is observed that P_B and P_{BD} increase and P_I decrease, with an increase in λ. Moreover, for various choices of b, an increase in λ produces an increase in $E(N_S)$. Table 2 displays an increase in P_I and a decrease in P_B and P_{BD} for increasing values of μ_1. Further, it also shows that for various choices of b, an increase in μ_1 results in a decrease in $E(N_S)$.

From Table 3, it is observed that P_I increases, P_B decreases, and P_{BD} remains unaffected by an increasing trait of μ_2. Also, it shows that $E(N_S)$ decreases with an increase in μ_2 for different choices of r. Table 4 shows that P_I, P_B and P_{BD} remain unaltered by changes in γ. However, it is observed that $E(N_S)$ decreases with an increase in γ for various values of b. From Table 5, it can be seen that P_I and P_B decrease and P_{BD} increase with an increase in α. For different choices of b, an increase in α results in an increase in $E(N_S)$. It is evident from Table 6 that P_I and P_B increase, whereas P_{BD} decreases with an increase in β. Also, it is observed that an increase in β over various values of b results in a decrease in $E(N_S)$.

Table 1. Effects of λ on performance metrics for different values of b.

λ	$E(N_S)$			P_I	P_B	P_{BD}
	$b = 0.6$	$b = 0.7$	$b = 0.8$			
0.5	0.0369	0.0379	0.0389	0.9587	0.0379	0.0034
1	0.0937	0.0978	0.1020	0.9181	0.0750	0.0068
1.5	0.1732	0.1834	0.1939	0.8783	0.1116	0.0101
2	0.2791	0.2992	0.3202	0.8391	0.1475	0.0134
2.5	0.4161	0.4512	0.4880	0.8007	0.1827	0.0166
3	0.5891	0.6455	0.7055	0.7628	0.2174	0.0198
3.5	0.8034	0.8890	0.9807	0.7257	0.2515	0.0229
4	1.0640	1.1878	1.3215	0.6891	0.2850	0.0259
4.5	1.3754	1.5476	1.7347	0.6532	0.3179	0.0289

Table 2. Effects of μ_1 on performance metrics for different values of b.

μ_1	$E(N_S)$			P_I	P_B	P_{BD}
	$b = 0.6$	$b = 0.7$	$b = 0.8$			
5	0.9973	1.1650	1.3534	0.6053	0.3571	0.0376
7	0.7460	0.8420	0.9464	0.6934	0.2790	0.0276
9	0.6274	0.6930	0.7632	0.7445	0.2336	0.0218
11	0.5590	0.6084	0.6606	0.7780	0.2040	0.0181
13	0.5149	0.5543	0.5955	0.8015	0.1831	0.0154
15	0.4841	0.5167	0.5507	0.8190	0.1676	0.0134
17	0.4614	0.4893	0.5181	0.8325	0.1556	0.0119
19	0.4441	0.4683	0.4934	0.8433	0.1461	0.0107
21	0.4303	0.4518	0.4739	0.8520	0.1383	0.0097
23	0.4192	0.4385	0.4583	0.8593	0.1319	0.0088
25	0.4101	0.4275	0.4454	0.8654	0.1264	0.0082

Table 3. Effects of μ_2 on performance metrics for different values of r.

μ_2	$E(N_S)$			P_I	P_B	P_{BD}
	$r = 0.2$	$r = 0.4$	$r = 0.6$			
6	0.6866	0.8971	1.1246	0.7431	0.2372	0.0198
8	0.6321	0.7814	0.9412	0.7628	0.2174	0.0198
10	0.6010	0.7157	0.8375	0.7747	0.2055	0.0198
12	0.5810	0.6738	0.7717	0.7826	0.1976	0.0198
14	0.5672	0.6449	0.7264	0.7883	0.1920	0.0198
16	0.5571	0.6238	0.6935	0.7925	0.1877	0.0198
18	0.5493	0.6078	0.6686	0.7958	0.1845	0.0198
20	0.5433	0.5952	0.6491	0.7984	0.1818	0.0198
22	0.5384	0.5851	0.6334	0.8006	0.1797	0.0198
24	0.5343	0.5768	0.6206	0.8024	0.1779	0.0198

Table 4. Effects of γ on performance metrics for different values of b.

γ	$E(N_S)$			P_I	P_B	P_{BD}
	$b = 0.6$	$b = 0.7$	$b = 0.8$			
2	0.5891	0.6455	0.7055	0.7628	0.2174	0.0198
4	0.3064	0.3350	0.3652	0.7628	0.2174	0.0198
6	0.2181	0.2385	0.2601	0.7628	0.2174	0.0198
8	0.1749	0.1915	0.2090	0.7628	0.2174	0.0198
10	0.1493	0.1636	0.1787	0.7628	0.2174	0.0198
12	0.1324	0.1452	0.1587	0.7628	0.2174	0.0198
14	0.1203	0.1321	0.1445	0.7628	0.2174	0.0198
16	0.1113	0.1223	0.1339	0.7628	0.2174	0.0198
18	0.1043	0.1147	0.1256	0.7628	0.2174	0.0198
20	0.0987	0.1086	0.1191	0.7628	0.2174	0.0198

Table 5. Effects of α on performance metrics for different values of b.

α	$E(N_S)$			P_I	P_B	P_{BD}
	$b = 0.6$	$b = 0.7$	$b = 0.8$			
0.5	0.5891	0.6455	0.7055	0.7628	0.2174	0.0198
1	0.5900	0.6466	0.7067	0.7481	0.2132	0.0388
1.5	0.5909	0.6476	0.7078	0.7338	0.2091	0.0570
2	0.5917	0.6486	0.7089	0.7201	0.2052	0.0746
2.5	0.5925	0.6495	0.7100	0.7070	0.2015	0.0916
3	0.5933	0.6505	0.7110	0.6942	0.1978	0.1079

Table 6. Effects of β on performance metrics for different values of b.

β	$E(N_S)$			P_I	P_B	P_{BD}
	$b=0.6$	$b=0.7$	$b=0.8$			
2	0.5900	0.6466	0.7067	0.7481	0.2132	0.0388
3	0.5894	0.6459	0.7059	0.7579	0.2160	0.0262
4	0.5891	0.6455	0.7055	0.7628	0.2174	0.0198
5	0.5889	0.6453	0.7052	0.7659	0.2183	0.0159
6	0.5888	0.6452	0.7050	0.7679	0.2188	0.0133

The impact of different system parameters on the main performance metrics is portrayed using three-dimensional figures. While studying the effects, the values of λ, μ_1 and μ_2 are arbitrarily varied, whereas the choice of other parameters such as α, γ, b and $\bar{\theta}$ is subject to the stability condition. The surface depicted in Figure 15 shows an increasing trend of $E(N_S)$ against rising values of λ and μ_1. In particular, it is observed that $E(N_S)$ increases with an increase in λ and decreases with an increase in μ_1. Figure 16 depicts a surface showing an upward trend of $E(N_S)$ against increasing values of λ and μ_2. Specifically, $E(N_S)$ increases with an increase in λ and decreases with an increase in μ_2.

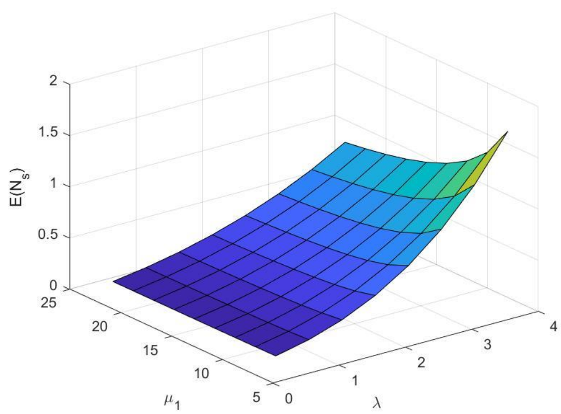

Figure 15. λ vs. μ_1 vs. $E(N_S)$.

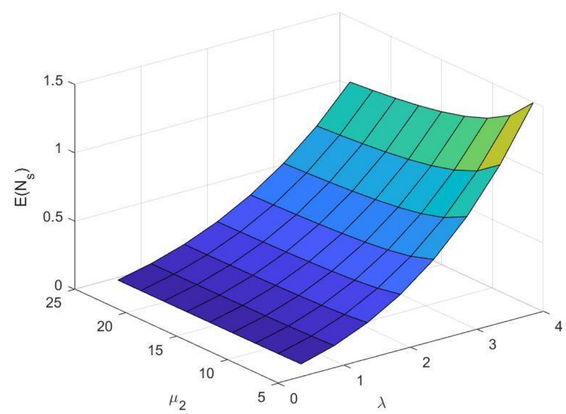

Figure 16. λ vs. μ_2 vs. $E(N_S)$.

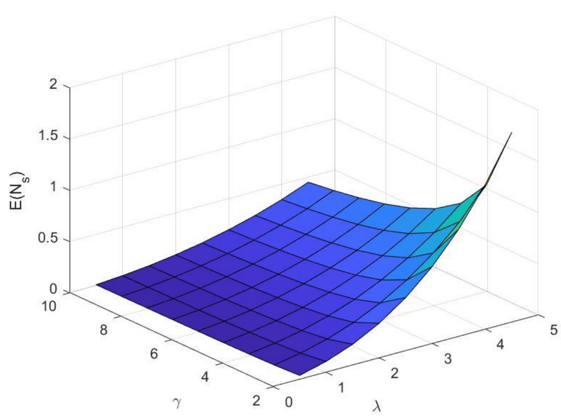

Figure 17. λ vs. γ vs. $E(N_S)$.

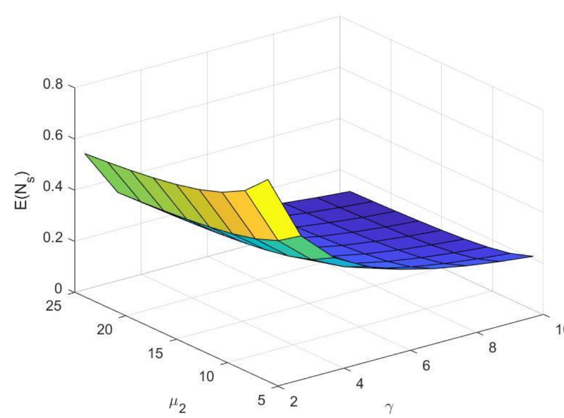

Figure 18. γ vs. μ_2 vs. $E(N_S)$.

An upward trend of $E(N_S)$ against increasing values of λ and γ is seen in the surface shown in Figure 17, which more specifically implies that $E(N_S)$ increases with an increase in λ and decreases with an increase in γ. Figure 18 depicts a surface showing a downward trend of $E(N_S)$ against increasing values of γ and μ_2. In particular, it is seen that $E(N_S)$ decreases with an increase in γ and μ_2. The surface in Figure 19 displays a downward trend of $E(N_S)$ against increasing values of μ_2 and b. Specifically, $E(N_S)$ decreases with an increase in μ_2 and increases with an increase in b. A downward trend of $E(N_S)$ against increasing values of μ_1 and $\bar{\theta}$ is displayed in the surface depicted by Figure 20. It shows that $E(N_S)$ decreases with an increase in μ_1 and increases with an increase in $\bar{\theta}$ in particular. Figure 21 depicts a surface showing a downward trend of $E(N_S)$ against increasing values of μ_1 and b. In particular, it is seen that $E(N_S)$ decreases with an increase in μ_1 and increases with an increase in b. Figure 22 shows that $E(N_S)$ decreases with an increase in γ and increases with an increase in α, as displayed in the surface depicted.

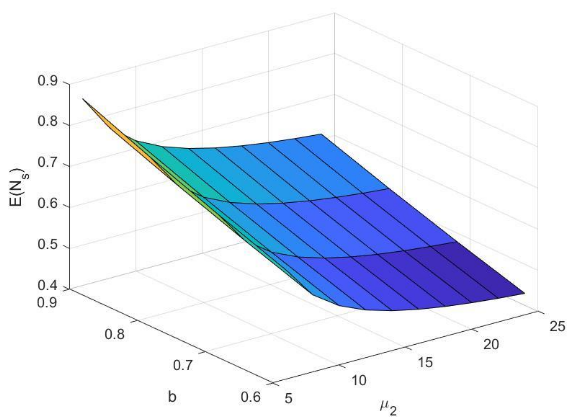

Figure 19. μ_2 vs. b vs. $E(N_S)$.

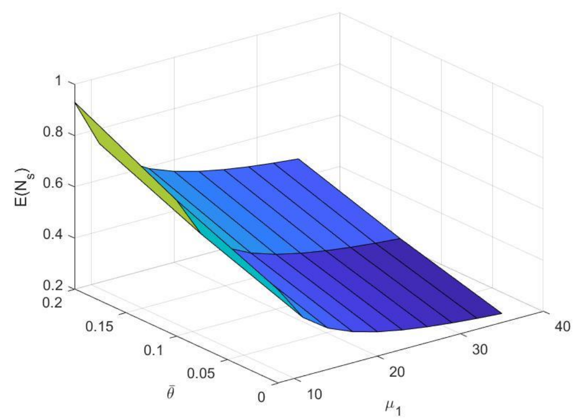

Figure 20. μ_1 vs. $\bar{\theta}$ vs. $E(N_S)$.

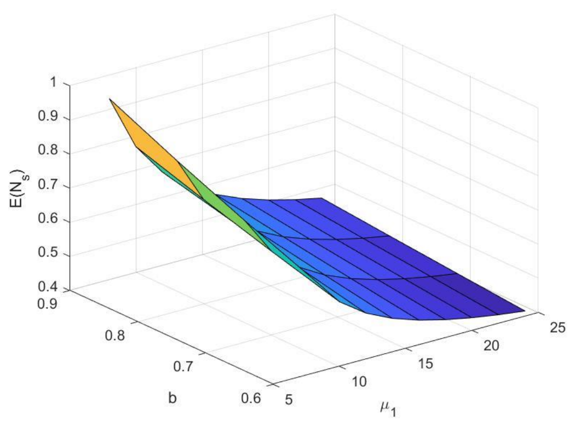

Figure 21. μ_1 vs. b vs. $E(N_S)$.

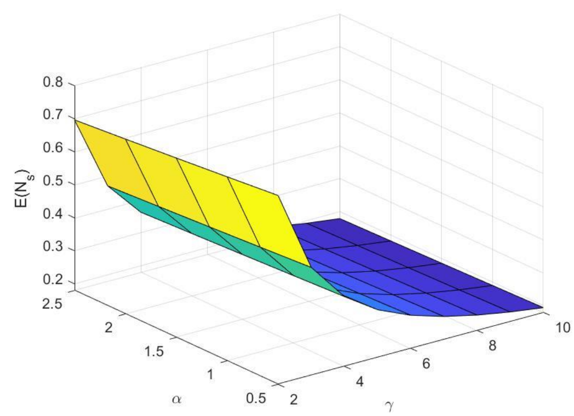

Figure 22. γ vs. α vs. $E(N_S)$.

The efficiency of the system in terms of queue size has been investigated under varying rates of FES, SOS, retrial, breakdown, and repair. With the investigation carried out, we infer that a speedy recovery rate of the impaired server and possible high FES, SOS and retrial rates enhance the system's performance. This follows due to the resultant drastic fall in the balking of likely customers. Further, it also facilitates a good customer satisfaction level and takes care of the system's revenue.

8. Conclusion

In this paper, we have investigated a M/M/1 retrial queueing system with an optional service, unreliable server, balking and feedback. By employing recursive techniques, the system in steady state was analysed to obtain the queue size distribution. Various performance measures for analysing the effectiveness of the proposed system have been evaluated. A model implementation with a detailed illustration of the working of an electrical vehicle charging station has been provided. The effects produced by the various system parameters on the performance metrics have been numerically and graphically illustrated. The investigation provides a means to enhance the system's efficiency and reduce the likely balking of potential customers by controlling the relevant parameters accordingly. The study can be extended to a system with a working breakdown. Further, it can also be investigated for an unreliable multi server retrial queue.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The authors would like to thank the editor and anonymous reviewers for their comments that help improve the quality of this work.

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