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Solving Redundancy Allocation Problems using Jaya Algorithm

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Abstract
Reliability-based design is related to the performance analysis of engineering systems. The redundancy allocation problem is one of the most common problems in the reliability-based design approach. The redundancy allocation problem determines the redundancy level of components in a system to maximize system reliability, subject to several constraints. In recent years, obtaining solutions to reliability-related redundancy allocation problems by means of evolving meta-heuristic algorithms has appealed to researchers due to the several drawbacks of classical mathematical methods. Meta-heuristics have shown the potential of obtaining precise solution in optimization problems and many techniques have been applied in the literature for optimal redundancy allocation. In this paper, a recently developed Jaya optimization algorithm is proposed to be applied for redundancy allocation to maximize system reliability. The Jaya algorithm is a simple, population-based intelligent meta-heuristic algorithm consisting of a single phase and an algorithm-specific parameter-less algorithm. This paper aims to present an application of the Jaya algorithm for searching the optimal solution of two redundancy allocation problems from the literature with nonlinear constraints so that system reliability is maximized. The first problem is the over speed protection system for a gas turbine, whose control system is modelled as a four-stage series system. The objective is to determine the optimal level of redundancy of the valves of the protection system under cost and weight constraints. The second one is the redundancy allocation problem of a five-stage series system with volume, weight, and cost constraints. The results are validated by comparing them with two other meta-heuristics.

Keywords- Reliability based design, Redundancy allocation, Series system, Jaya algorithm, Particle swarm optimization, Teaching learning based optimization.

1. Introduction

Reliability-based design is very important in today’s world, and considerable research efforts have been made in the last few decades to offer increasingly reliable products and systems. Therefore, the interest of the researchers is growing in the field of system reliability optimization to offer reliable solutions. Two prime strategies are applied to achieve higher system reliability. The first is to increase the reliability of the components of the system by using better materials, manufacturing processes, etc. The second strategy is to make use of redundancy, i.e., using identical components in a parallel configuration. In this case, the system designer needs to choose the optimal combination of components and redundancy levels. These problems typically involve various constraints related to cost, weight, volume, and other factors. They are known as reliability redundancy problems, where the choices of redundancy and component reliability greatly impact the maximization of system reliability. Redundancy allocation problems are widely employed in reliability-based design and are categorized as integer or mixed-integer problems within the domains of operations research and non-linear programming.

Many optimization methods have been presented in the literature for optimal reliability- redundancy allocation problems. These methods can be classified into three main groups: exact methods, heuristics, and meta-heuristics. Exact methods leverage the analytical properties of functions and encompass techniques such as dynamic programming, branch-and-bound integer programming, implicit searches, and Lagrangian methods. While these methods provide optimal solutions, their computational demands escalate as the problem size increases. When dealing with non-convex or large-scale optimization problems, locating the global optimum becomes exceedingly challenging. Heuristic methods have gained popularity due to their adaptability, efficiency compared to exact methods, and capacity to significantly reduce computational time in reaching optimal solutions. Heuristic methods are iterative approaches that are relatively straightforward to implement and involve a moderate level of computational complexity. While these methods often provide solutions within a reasonable time frame, they do not guarantee optimality. Since heuristics are dependent on the problem at hand or tailored to specific problem characteristics, the utilization of meta-heuristic optimization methods has become increasingly prevalent in recent times. Meta-heuristics are general methods that are not tailored to specific problems but instead draw inspiration from the principles of evolution theory or swarm behaviour. In meta-heuristic approaches, it is essential to fine-tune parameters such as population size, elite size, number of iterations, and other algorithm-specific parameters. The performance of these algorithms relies heavily on appropriately tuning these parameters. In the last two decades, there has been an influx of various meta-heuristic optimization algorithms and their variants. A few of those are the Genetic Algorithm (GA), Ant Colony Optimization (ACO), Spider Monkey Optimization (SMO), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC) algorithm, Differential Evolution (DE), Bat Algorithm (BA), Wolf Optimizer, Whale Optimization Algorithm (WOA), Grasshopper Optimization Algorithm (GOA), Biogeography-Based Optimization (BBO), Crow Search Algorithm (CSA), etc. Though the above-mentioned algorithms are mainly animal or nature inspired, there are a few other similar algorithms, such as the Harmony Search (HS) Algorithm, the Gravitational Search Algorithm (GSA), the Fireworks Algorithm (FA), the Grenade Explosion Method (GEM), and so on. These algorithms have demonstrated quite high performance by efficiently optimizing the system parameters.

The rest of the paper is organized as follows: Literature on reliability-redundancy allocation is presented in Section 2, followed by a brief description of the structure of the Jaya algorithm in Section 3. Two redundancy allocation problems are stated in Section 4. Solutions i.e., results of both problems using the Jaya algorithm and comparative results with Particle swarm optimization and Teaching learning-based optimization are provided in Section 5. The paper ends with a conclusion section, i.e., Section 6.

2. Literature on Redundancy Allocation

In reliability-redundancy allocation problems, a number of scholars, including Kuo et al. (2001), used exact approaches, heuristics, and meta-heuristics. Aggarwal (1976) provided a method for the optimal allocation of redundancies in general systems in one of the early papers on redundancy optimisation. A technique for restricted reliability optimisation problems was developed by Kuo et al. (1987). The branch-and-bound approach and the Lagrange multiplier method were both used in the procedure. For the reliability-redundancy allocation problem, they demonstrated a 4-stage series system with two linear constraints and a 5-stage series system with three non-linear constraints. A technique for resolving restricted redundancy optimisation issues in complicated systems was described by the authors in Kim and Yum (1993). Their approach allowed for excursions over a limited, unfeasible zone, reducing the likelihood of being stuck at a local optimum. Jianping (1996) suggested putting boundaries on the region. He proposed a non-linear integer programming method to arrive at the optimal solution for the allocation of redundant components. Kuo and Prasad (2000), in their review paper, presented the methods that have been researched and developed since 1977 to solve several reliability optimization problems. They also presented applications of those methods to various types of problems in the design field: heuristics, metaheuristic algorithms, exact methods, reliability-redundancy allocation, multi-objective optimization and assignment for solving optimal redundancy-allocation problems. A workable method for the joint reliability-redundancy optimisation of multi-state series-parallel systems was developed by Tian et al. (2009). The authors claimed that their approach is a multi-state version of the joint system reliability-redundancy optimization methods.

In the last two decades, many researchers have applied different metaheuristics and their variants to redundancy allocation problems. Nonlinear mixed-integer reliability design problems were investigated by Chen (2006) by proposing a penalty-guided artificial immune algorithm. In this paper, the author numerically illustrated system reliability maximization considering both the number of redundant components that are added and the reliability of those redundant components. Sahoo et al. (2010) solved the constrained redundancy allocation problem of series-parallel, parallel-series, and complex systems with the interval-valued reliability of each component. They illustrated the proposed method with some numerical examples solved using a genetic algorithm. Yeh and Hsieh (2011) proposed a method of solving the reliability redundancy allocation problem using the artificial bee colony algorithm. They investigated nonlinear mixed-integer reliability design problems from the literature. The authors varied the number of scout bees and the limit values and concluded better results compared to the traditional method of using a single scout bee and limit value. Dos Santos et al. (2011) introduced a modified firefly algorithm approach combined with chaotic sequences applied to reliability-redundancy optimization. An example of mixed integer programming in the reliability-redundancy design of an over-speed protection system for a gas turbine was evaluated. Wang and Li (2012) introduced an alternative hybrid algorithm that combines co-evolutionary differential evolution with a harmony search algorithm. This algorithm aims to address the reliability redundancy optimization problem by dividing it into a continuous and an integer component. By comparing the simulation results of three representative problems with several existing algorithms, the authors demonstrated that their proposed method exhibits superior effectiveness, efficiency, and robustness. In 2013, Kanagaraj et al. (2013) hybridized cuckoo search meta-heuristic optimization algorithm with a genetic algorithm to solve the reliability and redundancy allocation problem. The experimental results were compared with other evolutionary optimization methods, and it was found that the algorithm proposed by the authors turned out to be exceptionally effective and efficient. Chmabari et al. (2013) addressed the issue of redundancy allocation to optimize system reliability by selecting suitable redundancy strategies, components, and redundancy levels for each subsystem. Given the challenging nature of this non-linear problem, they proposed the utilization of a highly efficient simulated annealing algorithm as a solution. In a separate study, Afonso et al. (2013) proposed an enhancement to the imperialist competitive algorithm by incorporating the concepts of attraction and repulsion during the search for improved solutions.

The authors demonstrated through simulation results that their methods are more efficient and promising for the four reliability redundancy allocation problems from the literature. In the paper by Liu and Qin (2014a), the authors presented an algorithm in which the differential evolution algorithm was clubbed with the Tabu search algorithm. A hybrid algorithm was proposed to solve the reliability-redundancy optimization problem. Kim and Kim (2017) considered an advanced reliability-redundancy allocation problem with the optimal redundancy strategy and active or cold standby mode. In addition, an imperfect detector/ switch case was explored. The authors developed a variant of the genetic algorithm for solving the mixed-integer nonlinear programming model.

The Particle swarm optimization algorithm has attracted many researchers, and diverse versions of the same have been applied to redundancy allocation problems. In 2009, das Santos (2009) proposed a nature-inspired Particle swarm optimization algorithm that is based on Gaussian distribution and chaotic order to solve redundancy optimization problems. The author assessed two problems of reliability redundancy design and revealed that the solutions obtained by his method are superior to those of the previously known optimal solutions available in the literature. A modified Particle swarm optimization algorithm was suggested to solve reliability problems in the paper by Zao et al. (2010). They introduced dynamic probability and new inertia weight calculation features. Wu et al. (2011) proposed an improved Particle swarm optimization to solve reliability problems and demonstrated that the solutions obtained by their proposed method are much better than the previously reported optimal solutions in the literature. In a related study, Garg and Sharma (2013a) proposed an efficient penalty-based Particle swarm optimization algorithm for reliability-redundancy optimization problems. Their research focused on applying the PSO algorithm, to find optimal solutions for reliability-redundancy allocation problems with nonlinear resource constraints in a pharmaceutical plant. The same authors, Garg and Sharma (2013b), extended their work to consider a multi-objective reliability redundancy allocation problem in a series system. They treated the system's reliability and the corresponding design cost as separate objectives. To handle the problem, they formulated a fuzzy multi-objective optimization problem derived from the original crisp optimization problem. They then proposed a Particle swarm optimization approach to solve the fuzzified problem while considering various constraints. To illustrate the efficacy of their method, they presented a case study involving a pharmaceutical plant. A new dynamic self-adaptive multi-objective Particle swarm optimization method is proposed by Damghani et al. (2013), to solve binary-state multi-objective reliability redundancy allocation problems. A combination of a penalty function and modification strategies was used to handle the constraints. Zhang et al. (2014) united two approaches, namely bare-bones PSO and sensitivity-based clustering, for solving multiple objective reliability redundancy allocation problems. Their two-stage process aimed to identify optimistic solutions. A modified Particle swarm optimization algorithm was proposed by Liu and Qin (2014b) to solve the reliability redundancy optimization problem. In this paper, an adaptive parameter is employed to promote a range of viable solutions. Huang (2015) introduced a novel swarm intelligence method called the Particle-based simplified swarm optimization algorithm. This approach involved modifying the updating mechanism to effectively address reliability-redundancy allocation problems. An improvised methodology titled ‘Boundary simplified swarm optimization’ was suggested by Yeh (2019) by integrating self-boundary search and a two-variable update mechanism in solving mixed-integer programming problems that included discrete as well as continuous variables. In the paper by Ouyang et al. (2019), the reliability–redundancy allocation problem was extensively researched to obtain the optimal system parameters considering several constraints, to maximize the system reliability to maximum extent possible. The optimization problem was solved by an improved Particle swarm optimization algorithm with stochastic perturbation nature.

In recent years, a few other researchers have published their findings in the reliability allocation domain. In the paper (Sun et al., 2019), a multi-state series–parallel system redundancy allocation problem was investigated. For each multi-state component, epistemic uncertainty was assumed. The authors proposed a linear-time algorithm, and the results were compared with standard NSGA-II on multiple benchmarks. In this model proposed by Hadipour et al. (2019), each subsystem in the problem simultaneously uses active as well as warm-standby strategies. The authors applied a simulation approach and neural network techniques, considering the complexity of the model and reparability of the components. They introduced a multi-objective water flow algorithm and employed Taguchi-design of experiments to fine-tune the parameters of the meta-heuristics. The study encompassed the resolution of 32 test problems, and a comparative analysis of the outcomes was conducted against alternative algorithms within the paper. Huang et al. (2019) simplified the optimization model for the reliability-redundancy allocation problem by incorporating the theory of survival signature. To convert the constrained optimization problem into an unconstrained one, the authors proposed an innovative adaptive penalty function. Subsequently, a heuristic algorithm called stochastic fractal search was employed to solve the unconstrained optimization problem. A notable aspect of this approach is its capability to reduce the dimensionality of the optimization problem. Coit and Zio (2019) discussed all possible types of system reliability optimization problems, namely the redundancy allocation problem, the reliability allocation problem, and the combined reliability-redundancy allocation problem. They analysed the evolution of the various methods for obtaining optimal solutions and also derived benefits from improved optimization methods.

3. Jaya Algorithm

Jaya is a newly developed algorithm, proposed by Rao (2016). It's a simple, easy-to-apply method and has major features, i.e., it does not need any algorithm-specific control parameters. It has successfully adapted to a wide variety of problems. Like other meta-heuristics, Jaya is a population-based heuristic method. The algorithm works by searching for solutions continuously in each iteration by moving closer to the target (victory, i.e., Jaya) while simultaneously avoiding the worst solution. The position of the point(s) (solution) is updated by considering only the best position and specifically disregarding the worst position. Raikar and Gaonkar (2021) effectively applied the Jaya algorithm for estimation of P[X>Y] for a two- parameter Weibull distribution.

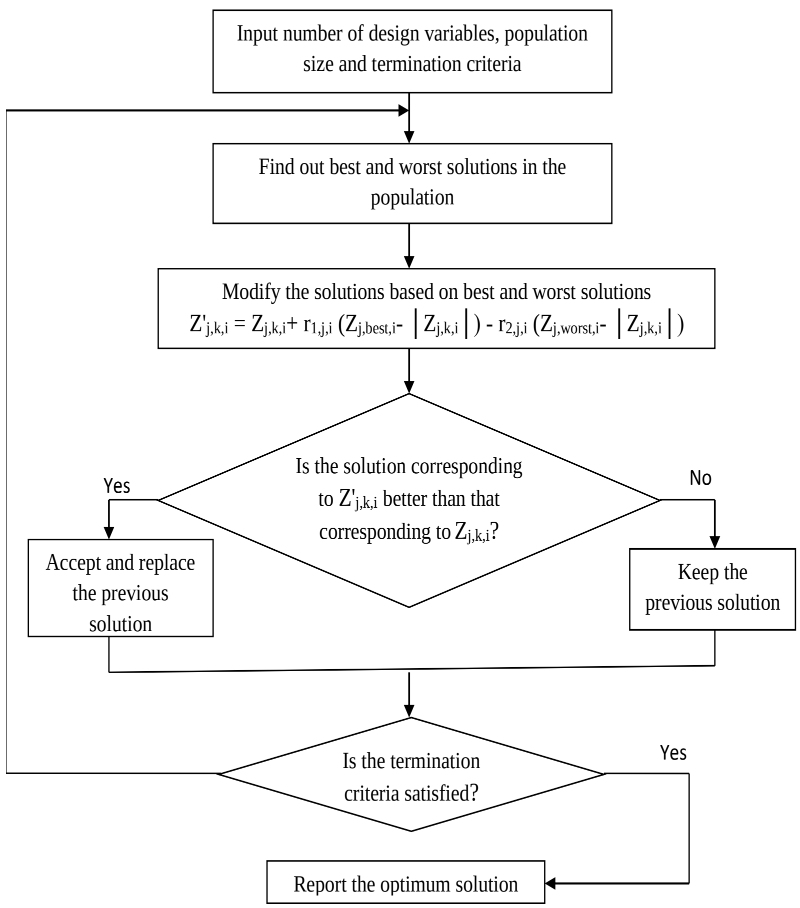

Figure 1. Flowchart of Jaya algorithm (Rao, 2016).

In the Jaya algorithm, the initial population for upper and lower bounds of the parameters is generated using pseudo random number generator. Then, each candidate in the population is updated based on the equation:

$$Z^{!}_{j,k,i} = Z_{j,k,i} + r_{1,j,i}\left(Z_{j,best,i} - \left|Z_{j,k,i}\right|\right) - r_{2,j,i}\left(Z_{j,worst,i} - \left|Z_{j,k,i}\right|\right) \quad (1)$$

where, $Z_{j,k,i}$ is the previous value of variable k and $Z'_{j,k,i}$ is the new value of variable k for candidate solution j. i is the iteration number. $r_{1,j,i}$ and $r_{2,j,i}$ are the random variates generated between 0 and 1 using pseudo random number generator. The striking point of the Jaya algorithm is that it constantly takes the candidate solution towards the best solution by the term ($Z_{j,best,i}$- $|Z_{j,k,i}|$) and goes away from the worst solution by the term ($Z_{j,worst,i}$ - $|Z_{j,k,i}|$). Depending on the best and worst function value, the candidate solutions are updated in each iteration. Figure 1 shows the flowchart of the Jaya algorithm.

4. Redundancy Allocation Problems

Two problems from the literature are considered in this paper (Kuo et al., 2001).

***Problem* 1.** This redundancy allocation problem is meant for an over speed protection of a gas turbine system. Over speed protection is continuously provided to the gas turbine by the electrical and mechanical systems. Cutting off the fuel supply is a basic necessity in case over-speed occurs. For this purpose, four control valves, namely V1, V2, V3 and V4 must close. The control system is modelled as a 4-stage series system, as shown in Figure 2. It is assumed that all the components are identical, i.e., have a constant failure rate.

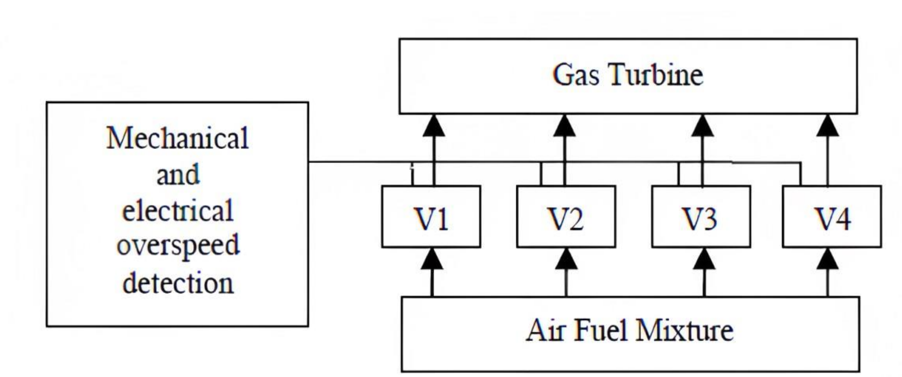

Figure 2. Schematic diagram of an over speed protection system of a gas turbine (Kuo et al., 2001).

$$Maximize\ \ Rs = \prod_{i=1}^{4}[1-(1-r_i)^{X_i}] \quad (2)$$

subject to:

$$\sum_{i=1}^{4} c_i x_i \leq 44.6 \quad (3)$$

$$\sum_{i=1}^{4} w_i x_i \leq 96 \quad (4)$$

The value 44.6 denotes the budgetary cost of the system, and the value 96 is the limiting weight of the system.

X_i being a non-negative integer, for i = 1, 2, 3, 4 that denotes the level of redundancy, i.e., the number of redundant components at stage i. The data value of r_i, c_i, and w_i are provided in Table 1.

Table 1. Data values for problem 1.

i	r_i	c_i	w_i
1	0.80	1.2	5
2	0.70	2.3	4
3	0.75	3.4	8
4	0.85	4.5	7

***Problem* 2.** This is a 5 stage series system redundancy allocation problem (Kuo et al., 2001) in which component redundancy is to be provided at each stage so that system reliability is maximised. The constraints are related to volume, cost, and weight. Redundant components need to be added to the system shown in Figure 3 so that all the constraints are satisfied and the objective function is maximized.

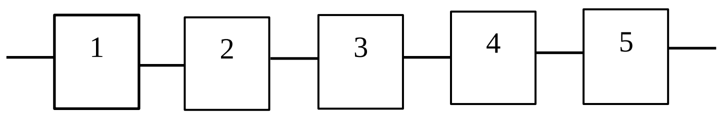

Figure 3. Series system.

$$Maximize\ R_s = \prod_{i=1}^{5} R_i \quad (5)$$

subject to the constraints:

$$g_1 = \sum_{i=1}^{5} p_i x_i^2 \leq P \quad (6)$$

$$g_2 = \sum_{i=1}^{5} c_i \left[x_i + e^{\frac{x_i}{4}}\right] \leq C \quad (7)$$

$$g_3 = \sum_{i=1}^{5} w_i x_i e^{x_i/4} \leq W \quad (8)$$

P is the maximum allowable total volume of the system, *C* is the upper limit of the total cost of the system and *W* is the upper limit of the total weight of the system.

X_i is a positive integer and $R_i(x_i) = 1 - (1 - r_i)^{x_i}$ is the reliability stage *i*. This 5-stage series redundant system is with X_i redundant components, each with reliability r_i that are arranged in *i* number stages. *P*, *C* and *W* are given limiting available resources as given in Table 2.

Table 2. Data values for problem 2.

i	r_i	p_i	*P*	c_i	*C*	w_i	*W*
1	0.80	1	110	7	175	7	200
2	0.85	2	110	7	175	8	200
3	0.90	3	110	5	175	8	200
4	0.65	4	110	9	175	6	200
5	0.75	2	110	4	175	9	200

5. Discussion of Results

Both the problems stated in the earlier section are solved using the Jaya algorithm. The results are shown in Table 3. For the problem of over speed protection system of a gas turbine, X_1, X_2, X_3, and X_4 are the number of redundant components to be added for valves V1, V2, V3 and V4, respectively. The optimal value of the objective function obtained, i.e., maximum system reliability value is 0.99750. For the second problem, i.e., a five-stage series system problem, X_1, X_2, X_3, X_4 and X_5 are the number of redundant components with a maximum system reliability value of 0.9045.

Graphical representations of the convergence of optimal values for both problems using the Jaya algorithm are shown in Figures 4 and 5, respectively. The convergence time are 0.0211 and 0.0236 for problems 1 and 2, respectively.

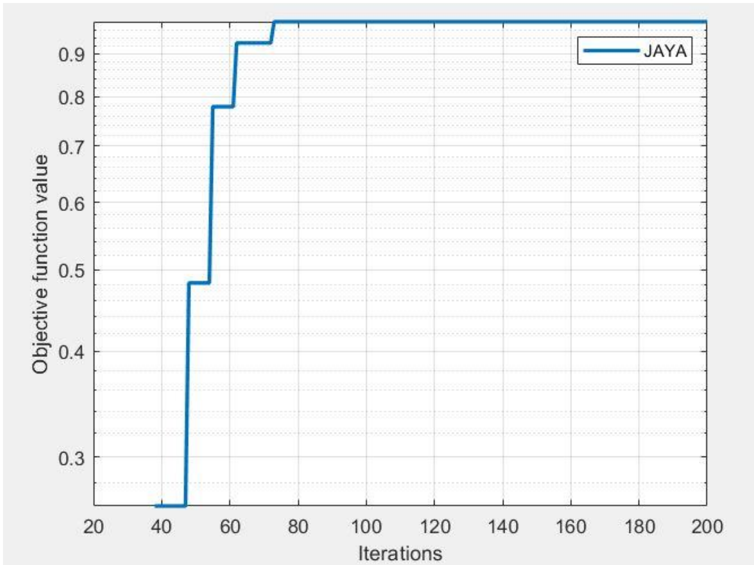

Figure 4. Graphical representation of convergence of optimal value for problem 1 (Jaya algorithm).

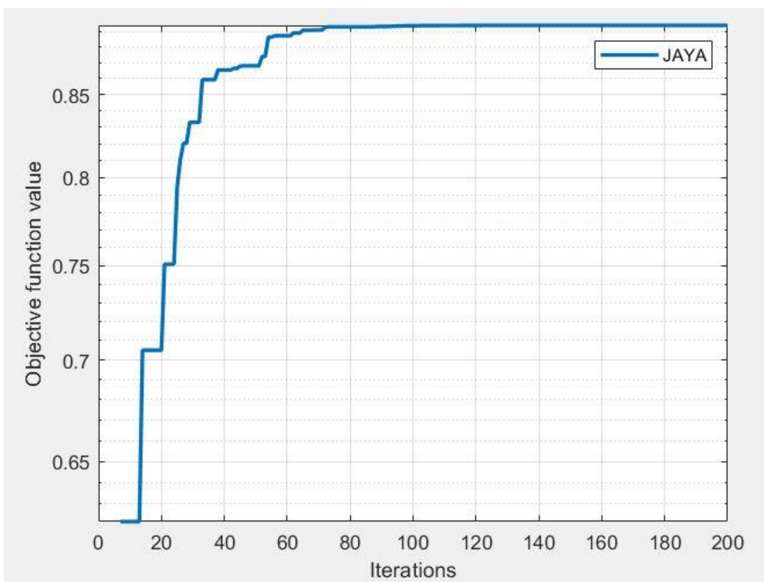

Figure 5. Graphical representation of convergence of optimal value for problem 2 (Jaya algorithm).

6. Comparison with Other Meta-Heuristics

For validation and confirmation of results, both problems were solved using the Particle swarm optimization algorithm and the Teaching learning-based optimization algorithm.

Particle swarm optimization (PSO) is a meta-heuristic optimization method that has many similarities with evolutionary algorithms like the Genetic Algorithm (GA) and its variants. It generates a pool of initial solutions and updates those based on a definite strategy. The initial set of solutions and updated solutions use a probabilistic strategy of random number variants. Particle swarm optimization took center stage in evolutionary computation at the end of the last century and in the initial decade of this century. The popularity of PSO can be attributed to its excellent performance and simple implementation.

The Teaching Learning Based Optimization (TLBO) algorithm was initially introduced by Rao et al. (2011) specifically for optimizing mechanical design problems. This approach operates by leveraging the influence of a teacher on learners. Like other nature-inspired algorithms, TLBO is a population-based method that seeks a collection of solutions during its execution, gradually converging towards a global solution. In this method, the population is represented by a group or class of learners. The TLBO process can be divided into two distinct phases: the first phase is known as the 'Teacher Phase,' where learners acquire knowledge from the teacher, and the second phase is referred to as the 'Learner Phase,' wherein learning occurs through interactions among the learners.

As shown in Table 3, a number of redundant components exactly matched in the case of all three optimization algorithms. However, the convergence time taken by the Jaya algorithm turned out to be the least compared to the other two algorithms. Teaching learning-based algorithm took the maximum time for convergence, while the Particle swarm optimization algorithm took almost double the time of the Jaya algorithm.

Figures 6 and 7 show a graphical representation of the convergence of optimal values for both the problems using the Particle swarm optimization algorithm. Whereas, Figures 8 and 9 depict a graphical representation of the convergence of optimal values using the particle swarm optimization algorithm for both problems under investigation.

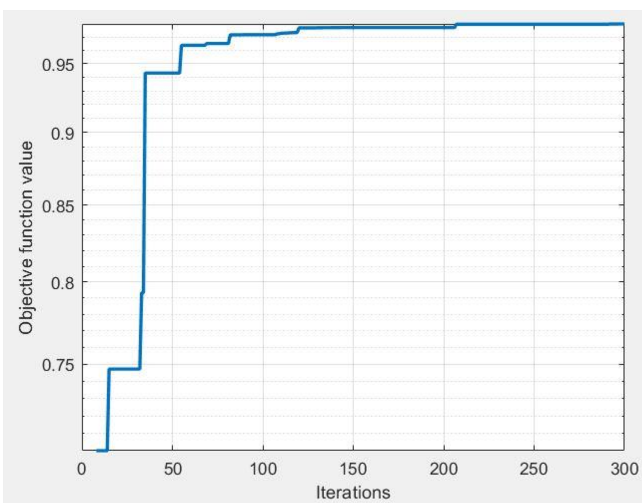

Figure 6. Graphical representation of convergence of optimal value for problem 1 (Particle swarm optimization algorithm).

Table 3. Results obtained.

Optimization algorithm	Problem number	X_1	X_2	X_3	X_4	X_5	Convergence time (sec)
Jaya algorithm	1	4	5	4	3	--	0.0211
	2	3	2	2	3	3	0.0236
Particle swarm optimization algorithm	1	4	5	4	3	--	0.0455
	2	3	2	2	3	3	0.0469
Teaching learning-based optimization	1	4	5	4	3	--	0.2314
	2	3	2	2	3	3	0.2019

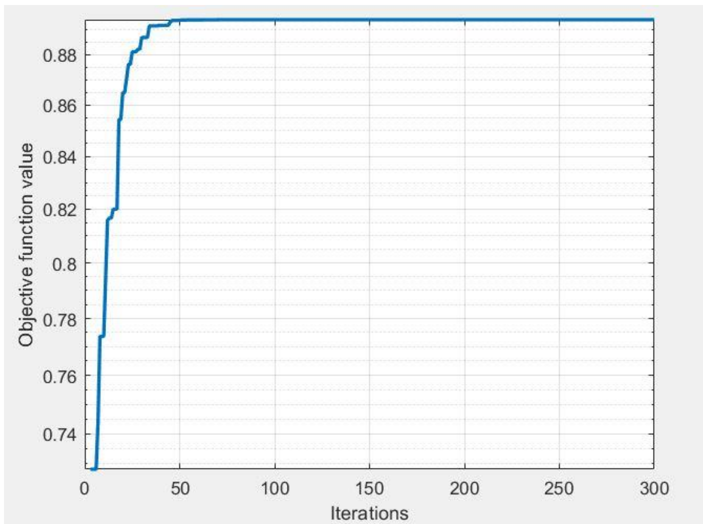

Figure 7. Graphical representation of convergence of optimal value for problem 2 (Particle swarm optimization).

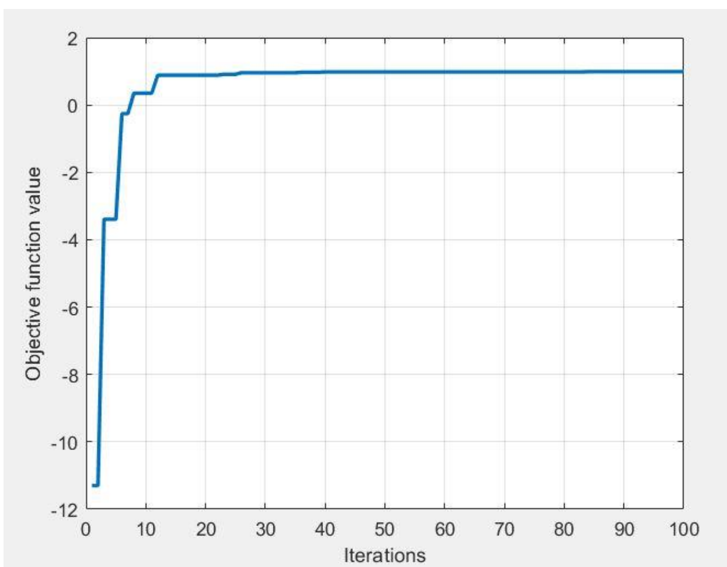

Figure 8. Graphical representation of convergence of optimal value for problem 1 (Teaching learning-based optimization).

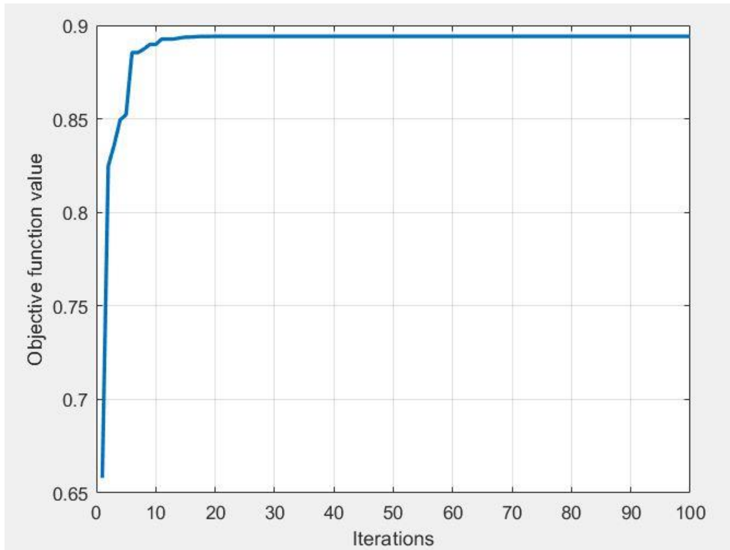

Figure 9. Graphical representation of convergence of optimal value for problem 2 (Teaching learning-based optimization).

7. Conclusion

In this paper, a popular Jaya optimization algorithm is applied for redundancy allocation to maximize system reliability. Two well-known redundancy allocation problems (four-stage series and five-stage series) with nonlinear constraints are considered in the literature for searching for the optimal solution, so that system reliability is maximized. Both problems are solved by the Jaya optimization algorithm, and the obtained results are validated by comparing them with the solutions obtained by using two other meta-heuristics namely, the Particle swarm optimization algorithm, and the teaching learning-based optimization algorithm. It is seen that the Jaya algorithm converges much faster than other meta-heuristics and looks to have a promising future in the domain of optimization in years to come. In its future scope in the sphere of reliability optimization, other categories of complex, real-life redundancy optimization problems involving linked system configurations may be attempted using the Jaya algorithm.

Conflict of Interest

The authors declare no conflict of interest.

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References

Afonso, L.D., Mariani, V.C., & dos Santos, L.C. (2013). Modified imperialist competitive algorithm based on attraction and repulsion concepts for reliability-redundancy optimization. *Expert Systems with Applications*, *40*(9), 3794-3802.

Aggarwal, K.K. (1976). Redundancy optimization in general systems. *IEEE Transactions on Reliability*, *25*(5), 330-332.

Chambari, A., Najafi, A.A., Rahmati, S.H.A., & Karimi, A. (2013). An efficient simulated annealing algorithm for the redundancy allocation problem with a choice of redundancy strategies. *Reliability Engineering & System Safety*, *119*, 158-164.

Chen, T.C. (2006). IAs based approach for reliability redundancy allocation problems. *Applied Mathematics and Computation*, *182*, 1556-1567.

Coit, D.W., & Zio, E. (2019). The evolution of system reliability optimization. *Reliability Engineering & System Safety*, *192*, 106259. https://doi.org/10.1016/j.ress.2018.09.008.

Damghani, K.K., Abtahi, A.R., & Tavana, M. (2013). A new multi-objective particle swarm optimization method for solving reliability redundancy allocation problems. *Reliability Engineering & System Safety*, *111*, 58-75.

dos Santos, L.C. (2009). An efficient particle swarm approach for mixed-integer programming in reliability–redundancy optimization applications. *Reliability Engineering & System Safety*, *94*(4), 830-837.

dos Santos, L.C., de Andrade, D.L.B., & Mariani, V.C. (2011). A chaotic firefly algorithm applied to reliability-redundancy optimization. In *2011 IEEE Congress of Evolutionary Computation* (pp. 517-521). IEEE. New Orleans, LA, USA.

Garg, H., & Sharma, S.P. (2013a). Multi-objective reliability-redundancy allocation problem using particle swarm optimization. *Computers & Industrial Engineering*, *64*(1), 247-255.

Garg, H., & Sharma, S.P. (2013b). Reliability-redundancy allocation problem of pharmaceutical plant. *Journal of Engineering Science and Technology*, *8*(2), 190-198.

Hadipour, H., Amiri, M., & Sharifi, M. (2019). Redundancy allocation in series-parallel systems under warm standby and active components in repairable subsystems. *Reliability Engineering & System Safety*, *192*, 106048. https://doi.org/10.1016/j.ress.2018.01.007.

Huang, C.L. (2015). A particle-based simplified swarm optimization algorithm for reliability redundancy allocation problems. *Reliability Engineering & System Safety*, *142*, 221-230.

Huang, X., Coolen, F.P., & Coolen, T.M. (2019). A heuristic survival signature based approach for reliability-redundancy allocation. *Reliability Engineering & System Safety*, *185*, 511-517.

Jianping, L. (1996). A bound heuristic algorithm for solving reliability redundancy optimization. *Microelectronics Reliability*, *36*(3), 335-339.

Kanagaraj, G., Ponnambalam, S.G., & Jawahar, N. (2013). A hybrid cuckoo search and genetic algorithm for reliability–redundancy allocation problems. *Computers & Industrial Engineering*, *66*(4), 1115-1124.

Kim, H., & Kim, P. (2017). Reliability–redundancy allocation problem considering optimal redundancy strategy using parallel genetic algorithm. *Reliability Engineering & System Safety*, *159*, 153-160.

Kim, J.H., & Yum, B.J. (1993). A heuristic method for solving redundancy optimization problems in complex systems. *IEEE Transactions on Reliability*, *42*(4), 572-578.

Kuo, W., & Prasad, V.R. (2000). An annotated overview of system-reliability optimization. *IEEE Transactions on Reliability*, *49*(2), 176-187.

Kuo, W., Lin, H.H., Xu, Z., & Zhang, W. (1987). Reliability optimization with the Lagrange-multiplier and branch-and-bound technique. *IEEE Transactions on Reliability*, *36*(5), 624-630.

Kuo, W., Prasad, V.R., Tillman, F.A., & Hwang, C.L. (2001). *Optimal reliability design: Fundamentals and applications*. Cambridge university press. UK.

Liu, Y., & Qin, G. (2014a). A hybrid TS-DE algorithm for reliability redundancy optimization Problem. *Journal of Computers*, *9*(9), 2050-2057.

Liu, Y., & Qin, G. (2014b). A modified particle swarm optimization algorithm for reliability redundancy optimization problem. *Journal of Computers*, *9*(9), 2124-2131.

Ouyang, Z., Liu, Y., Ruan, S.J., & Jiang, T. (2019). An improved particle swarm optimization algorithm for reliability-redundancy allocation problem with mixed redundancy strategy and heterogeneous components. *Reliability Engineering & System Safety*, *181*, 62-74.

Raikar, S.L., & Gaonkar, R.S.P. (2022). Jaya algorithm in estimation of P [X> Y] for two parameter Weibull distribution. *AIMS Mathematics*, *7*(2), 2820-2839.

Rao, R. (2016). Jaya: A simple and new optimization algorithm for solving constrained and unconstrained optimization problems. *International Journal of Industrial Engineering Computations*, *7*(1), 19-34.

Rao, R.V., Savsani, V.J., & Vakharia, D.P. (2011). Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems. *Computer-Aided Design*, *43*(3), 303-315.

Sahoo, L., Bhunia, A.K., & Roy, D. (2010). A genetic algorithm based reliability redundancy optimization for interval valued reliabilities of components. *Journal of Applied Quantitative Methods*, *5*(2), 270-287.

Sun, M.X., Li, Y.F., & Zio, E. (2019). On the optimal redundancy allocation for multi-state series–parallel systems under epistemic uncertainty. *Reliability Engineering & System Safety*, *192*, 106019. https://doi.org/10.1016/j.ress.2017.11.025.

Tian, Z., Levitin, G., & Zuo, M.J. (2009). A joint reliability–redundancy optimization approach for multi-state series–parallel systems. *Reliability Engineering & System Safety*, *94*(10), 1568-1576.

Wang, L., & Li, L.P. (2012). A coevolutionary differential evolution with harmony search for reliability–redundancy optimization. *Expert Systems with Applications*, *39*(5), 5271-5278.

Wu, P., Gao, L., Zou, D., & Li, S. (2011). An improved particle swarm optimization algorithm for reliability problems. *ISA Transactions*, *50*(1), 71-81.

Yeh, W.C. (2019). A novel boundary swarm optimization method for reliability redundancy allocation problems. *Reliability Engineering & System Safety*, *192*, 106060. https://doi.org/10.1016/j.ress.2018.02.002.

Yeh, W.C., & Hsieh, T.J. (2011). Solving reliability redundancy allocation problems using an artificial bee colony algorithm. *Computers & Operations Research*, *38*(11), 1465-1473.

Zhang, E., Wu, Y., & Chen, Q. (2014). A practical approach for solving multi-objective reliability redundancy allocation problems using extended bare-bones particle swarm optimization. *Reliability Engineering & System Safety*, *127*, 65-76.

Zou, D., Wu, J., Gao, L., & Wang, X. (2010). A modified particle swarm optimization algorithm for reliability problems. In *2010 IEEE Fifth International Conference on Bio-Inspired Computing: Theories and Applications* (pp. 1098-1105). IEEE. Changsha, China.