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Estimation of Stress Strength Reliability for Three-Parameter Weibull Distribution Using Jaya Algorithm

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Abstract
Jaya algorithm is a highly effective recent metaheuristic technique. In this article, a study is carried out on estimating reliability considering the stress and strength following Weibull distribution with same shape & location parameter but different scale parameter. The novelty of this study lies in using and analyzing the effectiveness of Jaya algorithm in estimation of reliability via maximum likelihood estimation. The simulation studies are carried out with different sample sizes to validate the proposed methodology. The technique is also applied to real life data to show its implementation and the results are compared with the other methods in the literature. The proposed methodology is simple and gives precise results. The inclusion of location parameter in the study significantly impacts the estimated reliability.

Keywords- Reliability, Stress strength interference, Weibull distribution, Maximum likelihood estimation, Jaya algorithm.

1. Introduction

The technological developments in the field of aircraft, nuclear power plants, infrastructure, transportation, etc. have raised serious concerns with reliability and safety as a small error in the design of applications in these sectors can cause a huge disaster. Novel challenges are posed every day and thus the field of reliability and safety is gaining increasing importance. The designing and assessment of components or operating procedures in the above-mentioned industries based on reliability can be very effective in preventing failures or accidents (Gaonkar et al., 2011; Peng et al., 2021). Properties of a component like stress and strength require precise designing as these are vital in determining the safety of the component. In real life, properties like stress and strength of mechanical components do not take a fixed value due to the various uncertainties in materials, loading conditions, environmental conditions, etc. Thus, they can be considered to follow a particular distribution which can be determined based on the application or prior data. Traditional methods may not be the best approach for designing of mechanical components as it does not take uncertainties into consideration. Hence, reliability-based design will be suitable in such cases which takes into account the probability of failure if the stress and strength takes various values within its range. In the context of stress and strength, reliability can be defined as the probability of strength being greater than stress (Abbas & Tang, 2014; An et al., 2008; Xie et al., 2004). If stress and strength follow some probability distribution, then the interference area of the two distributions give the probability of failure as can be seen in **Figure 1**.

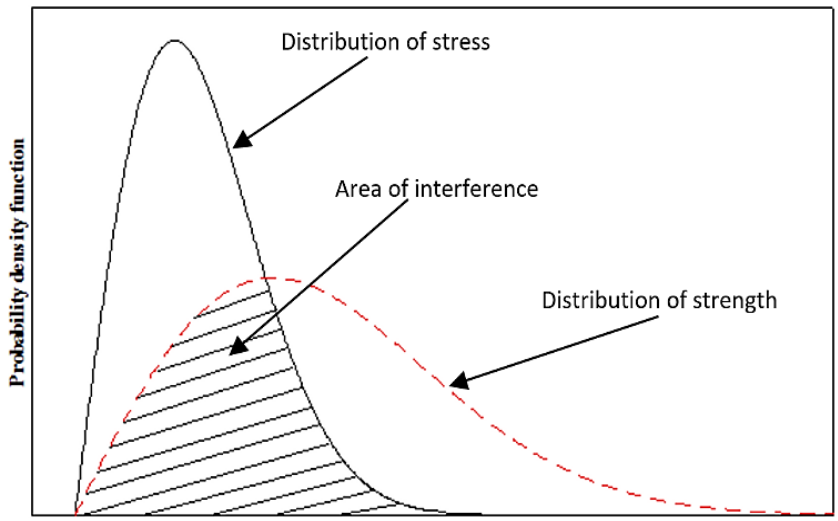

Figure 1. Stress-strength interference.

In order to determine reliability, estimating the parameters of stress strength distribution is crucial. Vast research has been carried in estimation of reliability of components subjected to various distributions of stress and strength. Estimation of stress strength parameters and reliability has been carried out for exponential Frechet distribution using different estimation methods like Maximum likelihood estimation, Bayes estimation and uniformly minimum variance unbiased estimator (Badr et al., 2019). Researchers have worked on evaluating multicomponent stress strength reliability for Chen distribution using classical and Bayes estimation method. Studies have been carried out on stress and strength following Chen-distribution with common shape parameter using classical and Bayesian methods evaluating the methods and carrying out analysis using Monte Carlo simulations and applications on real data sets (Kayal et al., 2020). One of the studies have presented estimation of stress strength reliability for Maxwell-distributed variables with different parameters using maximum likelihood and Bayesian methods based on progressive type-II censored samples (Chaudhary & Tomer, 2018). A research has been carried out on Kumaraswamy distribution in estimating reliability using frequentist and Bayesian methods including Lindley's approximation comparing estimators via Monte Carlo simulations and real data analysis (Kızılaslan & Nadar, 2018). Also research has been presented on Pareto distributed stress and strength using maximum likelihood estimation and comparing estimator performance through Monte Carlo simulations (Rezaei et al., 2015). Similar studies have been conducted by many other researchers in estimation of stress strength reliability analyzing different estimation methods for stress and strength following various distributions (Louzada et al., 2016; Raqab & Kundu, 2005; Rezaei et al., 2015). Recent studies have shown increasing usage of metaheuristics in estimation of parameters (Abbasi et al., 2006; Örkcü et al., 2015a; Örkcü et al., 2015b).

Weibull distribution is widely used in reliability studies because of its flexibility and ability to fit a wide range of data (Elmahdy, 2015; Kumar & Ram, 2018; Marković et al., 2009; Raikar & Gaonkar, 2023). Extensive research has been carried out in estimating the reliability when stress and strength follow Weibull distribution. (Kundu & Raqab, 2009; Nadarajah & Jia, 2017; Valiollahi et al., 2013). If X and Y denote the Weibull random variables for the strength and stress respectively having common shape (p) & location parameter (μ) and different scale parameters (σ_1 & σ_2), then their probability density function (pdf) is given by

$$f(x;\mu,\sigma_1,p)=\frac{p}{\sigma_1}(x-\mu)^{p-1}\,exp\left\{-\frac{1}{\sigma_1}(x-\mu)^p\right\}, x>\mu,\sigma_1>0,p>0 \tag{1}$$

and

$$f(y;\mu,\sigma_2,p)=\frac{p}{\sigma_2}(y-\mu)^{p-1}\,exp\left\{-\frac{1}{\sigma_2}(y-\mu)^p\right\}, y>\mu,\sigma_2>0,p>0 \tag{2}$$

respectively. The corresponding cumulative distribution function (CDF) for strength and stress is given by

$$F(x;\mu,\sigma_1,p)=1-exp\left\{-\frac{1}{\sigma_1}(x-\mu)^p\right\} \tag{3}$$

and

$$F(y;\mu,\sigma_2,p)=1-exp\left\{-\frac{1}{\sigma_2}(y-\mu)^p\right\} \tag{4}$$

respectively, where $x>\mu$, $y>\mu$, $\sigma_1>0$, $\sigma_2>0$ and $p>0$.

2. Maximum Likelihood Estimation (MLE)

One of the classical and efficient method used in estimation of parameters is maximum likelihood estimation (Ramos et al., 2020; Rodrigues et al., 2018). MLE provides precise estimates with smallest variance. The method is considered to be simple, effective, consistent and the estimation accuracy increases with increase in sample size (Newey & McFadden, 1994). MLE can get accurate results if assisted with proper computational techniques. Let x be a random sample of size n drawn from W(μ, σ_1, p) and y be the random sample of size m from W(μ, σ_2, p). Then the likelihood function can be given as:

$$L=\prod_{i=1}^{n}f(x_i)\prod_{j=1}^{m}f(y_j) \tag{5}$$

$$L=\prod_{i=1}^{n}\frac{p}{\sigma_1}(x_i-\mu)^{p-1}\,exp\left\{-\frac{1}{\sigma_1}(x-\mu)^p\right\}.\prod_{j=1}^{m}\frac{p}{\sigma_2}(y_j-\mu)^{p-1}\,exp\left\{-\frac{1}{\sigma_2}(y-\mu)^p\right\} \tag{6}$$

$$ln\,L=(m+n)\,ln\,p-\;n\,ln\,\sigma_1-m\,ln\,\sigma_2+\;(p-1)\left[\sum_{i=1}^{n}ln\,(x_i-\mu)+\sum_{j=1}^{m}ln\big(y_j-\mu\big)\right]-$$
$$\frac{1}{\sigma_1}\sum_{i=1}^{n}(x_i-\mu)^p-\frac{1}{\sigma_2}\sum_{j=1}^{m}\big(y_j-\mu\big)^p \tag{7}$$

The purpose is to maximize the log-likelihood Equation (7) i.e. the parameter values at which the log-likelihood function attains it's maximum. Solving likelihood equations involving nonlinear functions using numerical methods can be difficult because of the problems associated with it like non convergence, slower convergence and convergence to wrong values. Hence using a heuristic technique can be a good choice in solving the likelihood equations. In this paper, Jaya algorithm is used to maximize the above likelihood equation.

3. Jaya Algorithm

Metaheuristics have been used for a long time for solving optimization problems and are found to be effective in converging to optimal solutions (Kumar et al., 2017a; Kumar et al., 2017b; Pant et al., 2017; Ram & Davim, 2017). Particle Swarm Optimization has been used to solve a reliability optimization problem with conflicting objectives of minimizing system cost and maximizing reliability demonstrating its effectiveness in generating a well-distributed Pareto optimal set for decision-makers (Kumar et al., 2017b). Gray Wolf Optimizer, a nature-inspired metaheuristic has been applied to complex reliability optimization problems (Kumar et al., 2017a). Studies have been carried out in reliability optimization using the universal moment generating function approach combined with Ant Colony optimization algorithm to maximize system reliability (Meziane et al., 2005). Jaya algorithm is one such recently developed metaheuristic for solving optimization problems effectively. The specialty of the algorithm is that it constantly tries to move towards success and away from failure with each iteration. The algorithm has been used by many researchers in solving optimization problems (Ding et al., 2022; Du et al., 2018; Rao et al., 2017; Rao & Waghmare, 2017). Jaya algorithm has been used for parameter identification of an aerofoil demonstrating superior identification accuracy and robustness compared to other approaches, even under quasi-periodic oscillations (Ding et al., 2022). The algorithm has also been used for solving an optimization-based damage identification problem in engineering structures by formulating a hybrid objective function that combines the multiple damage location assurance criterion and modal flexibility change, demonstrating its robustness and efficiency in accurately detecting and quantifying structural damage even under high noise levels (Du et al., 2018). In machining processes, the algorithm has been implemented in optimization demonstrating its effectiveness in achieving optimal process parameters (Rao et al., 2017). In Jaya algorithm, the initial population is randomly generated using upper and lower bounds. Then, each candidate is updated based on the equation:

$$A'_{j,k,i} = A_{j,k,i} + r_{1,j,i} \left(A_{j,best,i} - \left|A_{j,k,i}\right|\right) - r_{2,j,i} \left(A_{j,worst,i} - \left|A_{j,k,i}\right|\right) \quad (8)$$

where, $A'_{j,k,i}$ is the new value of the k^{th} variable for j^{th} candidate solution, $A_{j,k,i}$ is the old value of the k^{th} variable for j^{th} candidate solution and i is the iteration number. $r_{1,j,i}$ and $r_{2,j,i}$ are the random variables. The term $(A_{j,best,i} - |A_{j,k,i}|)$ takes the candidate solution towards the best solution and the term $(A_{j,worst,i} - |A_{j,k,i}|)$ takes the candidate solution away from the worst solution. r_1 and r_2 are random variables between 0 and 1.

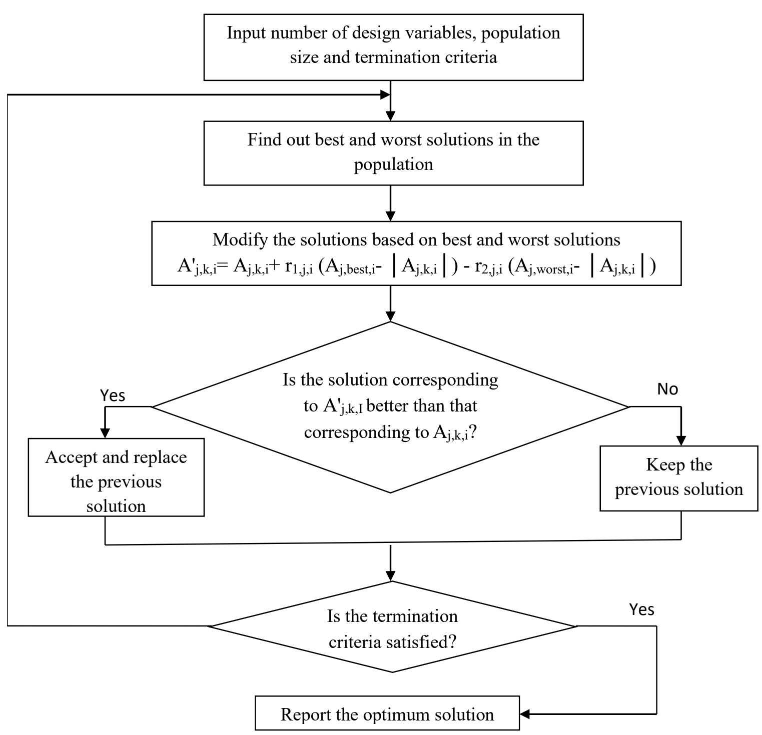

Figure 2. Flowchart of Jaya algorithm.

The candidate solutions are updated among new and old values based on the best function value (maximum or minimum). **Figure 2** shows the flowchart of Jaya algorithm which outlines the iterative process, starting from the initialization of design variables, population size and termination criteria followed by identifying the best and worst solutions within the population. The algorithm then updates solutions based on their proximity to the best and worst candidates. The decision-making process evaluates whether the new solutions outperform the previous ones. The process iterates until the termination criteria are met and the optimal solution is reported.

4. Reliability Estimation using Jaya Algorithm

For the cases involving stress-strength interference, the reliability can be given as P (X > Y) which is equal to

$$\mathrm{R} = \mathrm{P}\,(\mathrm{X} > \mathrm{Y}) = \int_0^{\infty}\left(f(x;\mu,\sigma_1,p)\int_0^{x} f(y;\mu,\sigma_2,p)dy\right)dx \tag{9}$$

$$\mathrm{R} = \mathrm{P}\,(\mathrm{X} > \mathrm{Y}) = \int_0^{\infty}\begin{pmatrix} \frac{p}{\sigma_1}(x-\mu)^{p-1}\, exp\left\{-\frac{1}{\sigma_1}(x-\mu)^{p}\right\} \\ \int_0^{x}\frac{p}{\sigma_2}(y-\mu)^{p-1}\, exp\left\{-\frac{1}{\sigma_2}(y-\mu)^{p}\right\}dy \end{pmatrix}dx \tag{10}$$

On simplification, the reliability can be obtained as (Asgharzadeh et al., 2011)

$$\mathrm{R} = \frac{\sigma_1}{\sigma_1+\sigma_2} \tag{11}$$

The log likelihood function for estimated values of parameters can be given as:

$$ln\,L = (m+n)\,ln\,\hat{p} - n\,ln\,\widehat{\sigma_1} - m\,ln\,\widehat{\sigma_2} + (\hat{p}-1)\left[\sum_{i=1}^{n} ln\,(x_i-\hat{\mu}) + \sum_{j=1}^{m} ln\left(y_j-\hat{\mu}\right)\right] - \frac{1}{\widehat{\sigma_1}}\sum_{i=1}^{n}(x_i-\hat{\mu})^{\hat{p}} - \frac{1}{\widehat{\sigma_2}}\sum_{j=1}^{m}\left(y_j-\hat{\mu}\right)^{\hat{p}} \tag{12}$$

where, $\hat{\mu}$ and $\hat{p}$ are common estimated values of location and shape parameter. $\widehat{\sigma_1}$ is the scale parameter for strength and $\widehat{\sigma_2}$ is the scale parameter for stress. The parameters are to be estimated in such a manner to maximize the likelihood Equation (12). Hence it becomes an optimization problem. In this study, Jaya algorithm has been used to maximize the likelihood Equation (12). **Table 1** shows the detailed steps for evaluating reliability using Jaya algorithm. The number of design variables taken are 4 considering the four parameters to be estimated and the population size is taken as 10. Number of iterations is considered as the termination criteria. The initial population is randomly generated within the specified range and constraints. The best (maximum) and the worst (minimum) solution are calculated based on the log likelihood function. The population is then updated based on Equation (8). If the new population gives a better maximum value for Equation (12) than the previous one then the new population is accepted and the next iteration begins with the updated population. If the new population does not give a better maximum than the previous one the next iteration will be proceeded with the previous population. After the fixed number of iterations are completed and no variation in the convergence is observed, the final variables obtained in the population are the optimum parameter estimates. Run the above steps for a number of times to find the best population which gives the best maximum.

If $\widehat{\sigma_1}$ and $\widehat{\sigma_2}$ are the estimates of scale parameters for strength and stress respectively, then the reliability estimate, bias and mean squared error can be calculated as:

$$\widehat{\mathrm{R}} = \frac{\widehat{\sigma_1}}{\widehat{\sigma_1}+\widehat{\sigma_2}} \tag{13}$$

$$\mathrm{Bias} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}}\left(\widehat{\mathrm{R}}-\mathrm{R}\right) \tag{14}$$

$$\mathrm{MSE} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}}\left(\widehat{\mathrm{R}}-\mathrm{R}\right)^2 \tag{15}$$

Table 1. Steps for reliability estimation using Jaya algorithm.

Step No.	Procedure
1.	Generate a data of 500 samples of Weibull distribution with real parameters.
2.	Input the population size=10 & number of design variables as 4.
3.	Set the maximum number of iterations for each sample.
4.	Specify the boundaries for the variables.
5.	Generate a random population within the constraints.
6.	Compute the maximum likelihood function value from Equation (12).
7.	Update the population based on Equation (8).
8.	If the updated population gives better maximum, then accept the new population else reject. This completes one iteration.
9.	Go for next iteration and similarly run the program till the maximum iteration number is reached.
10.	The final set of variables is the best solution of estimated parameters for the current experiment.
11.	Compute reliability from Equation (13).
12.	Run the program for 500 experiments.
13.	Compute bias and MSE based on Equation (14) & (15) respectively.

5. Simulation Studies

Simulations are performed in order to study the effectiveness of the proposed methodology. The data is randomly generated for strength and stress using Weibull distribution with shape parameter 1.5 and location parameter 2. The study has been carried out by varying the scale parameter and analyzing its effect on reliability. The scale parameter for strength is taken 2, 2.5 and that for stress is considered as 1, 1.5, 2, 2.5 and 3. The sample sizes selected are (25, 25), (50, 50), (100, 100) and (500, 500). Total 500 independent experiments are conducted to check the repeatability of the proposed methodology. Through preliminary experiments it was observed that the optimization process stabilizes well before 200 iterations. Increasing the iteration count beyond this did not significantly improve accuracy but led to higher computational time. Hence 200 iterations were chosen as a reasonable trade-off between precision and efficiency in evaluating the proposed methodology.

Table 2 depicts the estimated values of reliability using the proposed methodology for different sample sizes when $\mu = 2$, $\sigma_1 = 2$, $p = 1.5$ and σ_2 taking values 1, 1.5, 2, 2.5, 3. **Figure 3** illustrates the estimated values of reliability for simulation of 500 experiments. The histograms indicate a normal distribution, with greater spread observed for smaller sample sizes. For instance, in the first histogram, the mean is approximately 0.66667, which corresponds to the true reliability. As the sample size increases, the spread decreases indicating improved estimation accuracy with the histogram values clustering more closely around 0.66667. Similar inferences are observed in the other histograms, which correspond to different values of σ_2. Additionally, the figure demonstrates in accordance with interference theory that as the scale parameter of stress (σ_2) increases, the reliability decreases. The reliability estimates using the proposed methodology are compared with the reliability using Monte Carlo simulations, R (MCS) for estimated parameters. It can be observed that the results of reliability estimates using the proposed methodology are very close to the reliability using Monte Carlo simulations. Comparison is also made by calculating bias, mean standard error and compilation time.

Table 3 and **Figure 4** shows these results for same set of parameters with $\sigma_1 = 2.5$. The trends observed are also similar with the histograms exhibiting lesser variability for larger sample sizes. The expected inverse relationship between the scale parameter of stress (σ_2) and reliability is observed reinforcing the consistency of the findings across different parameter settings. The computational code for Jaya algorithm was compiled using MATLAB 2018 software. The estimated reliability values obtained in all the cases are very close to the real reliability values. Also, as the sample size increases the mean squared error reduces and the estimated reliability moves closer to the real reliability. But, at the same time as the sample size increases the time taken for compilation also increases. So, optimization for reliability is a trade-off between the accuracy of reliability value and the compilation time. **Figure 5** shows the convergence behavior of the proposed algorithm for different sample sizes. It can be seen that the Jaya algorithm values converge towards optimal solutions after at most 50 iterations in all cases.

Table 2. Results of reliability estimate, bias, mean square error (MSE) and compilation time (t) when $\mu = 2$, $\sigma_1 = 2$, $p = 1.5$.

n, m		$\sigma_2 = 1$ (R = 0.66667)	$\sigma_2 = 1.5$ (R = 0.57143)	$\sigma_2 = 2$ (R = 0.5000)	$\sigma_2 = 2.5$ (R = 0.44444)	$\sigma_2 = 3$ (R = 0.4)
(25, 25)	$\hat{R}$	0.67048	0.57646	0.49865	0.44200	0.39395
	Bias	0.00381	0.00503	-0.00135	-0.00245	-0.00605
	MSE	0.00517	0.00248	0.00288	0.00235	0.00232
	t (s)	37.32877	37.31540	37.4313	37.53155	37.40354
	R (MCS)	0.67406	0.57615	0.50143	0.44374	0.39868
(50, 50)	$\hat{R}$	0.67021	0.57283	0.50234	0.44139	0.39831
	Bias	0.00355	0.0014	0.00234	-0.00305	-0.00169
	MSE	0.00094	0.001070	0.00126	0.00109	0.00108
	t (s)	64.15220	65.14447	64.1661	66.30122	65.09571
	R (MCS)	0.66752	0.57394	0.49544	0.44139	0.40095
(100, 100)	$\hat{R}$	0.66876	0.57147	0.49942	0.44363	0.39980
	Bias	0.00209	0.00004	-0.00058	-0.00081	-0.0002
	MSE	0.00047	0.00052	0.00058	0.00057	0.0005
	t (s)	118.114	118.1314	119.1136	118.6877	119.5847
	R (MCS)	0.67021	0.57328	0.49791	0.44162	0.39804
(500, 500)	$\hat{R}$	0.66781	0.57128	0.50026	0.44438	0.40018
	Bias	0.00114	-0.00015	0.00026	-0.00006	0.00018
	MSE	0.00008	0.0001	0.00011	0.00011	0.00009
	t (s)	550.1336	551.3976	551.2057	550.0018	552.2750
	R (MCS)	0.66686	0.57278	0.50246	0.44439	0.40023

Table 3. Results of reliability estimate, bias, mean square error (MSE) and compilation time (t) when $\mu = 2$, $\sigma_1 = 2.5$, $p = 1.5$.

n, m		$\sigma_2 = 1$ (R = 0.714286)	$\sigma_2 = 1.5$ (R = 0.625)	$\sigma_2 = 2$ (R = 0.5556)	$\sigma_2 = 2.5$ (R = 0.5)	$\sigma_2 = 3$ (R = 0.4545)
(25, 25)	$\hat{R}$	0.72190	0.62783	0.55491	0.50477	0.45412
	Bias	0.00761	0.00283	-0.00069	0.00477	-0.00038
	MSE	0.0022	0.00513	0.00373	0.00378	0.00399
	t (s)	37.38062	37.44218	37.30677	37.72256	37.48941
	R (MCS)	0.72272	0.62468	0.55738	0.49725	0.45317
(50, 50)	$\hat{R}$	0.71925	0.62720	0.55715	0.49797	0.45418
	Bias	0.00496	0.00220	0.00155	-0.00203	-0.00032
	MSE	0.00079	0.00101	0.00109	0.00122	0.00106
	t (s)	64.84338	65.1444	65.03510	65.00493	65.6791
	R (MCS)	0.71020	0.62719	0.55680	0.50152	0.45515
(100, 100)	$\hat{R}$	0.71713	0.62689	0.55583	0.49966	0.45510
	Bias	0.00284	0.00189	0.00023	-0.00034	0.00060
	MSE	0.00035	0.00052	0.00058	0.00069	0.00055
	t (s)	119.8956	120.6025	120.1363	120.2996	120.635
	R (MCS)	0.71626	0.62109	0.55707	0.50042	0.00542
(500, 500)	$\hat{R}$	0.71463	0.62610	0.55549	0.50019	0.45443
	Bias	0.00034	0.00110	-0.00011	0.00019	0.00007
	MSE	0.00007	0.00009	0.00010	0.00011	0.00011
	t (s)	637.9747	626.1736	626.3683	631.763	635.9884
	R (MCS)	0.71600	0.62433	0.55234	0.50070	0.45353

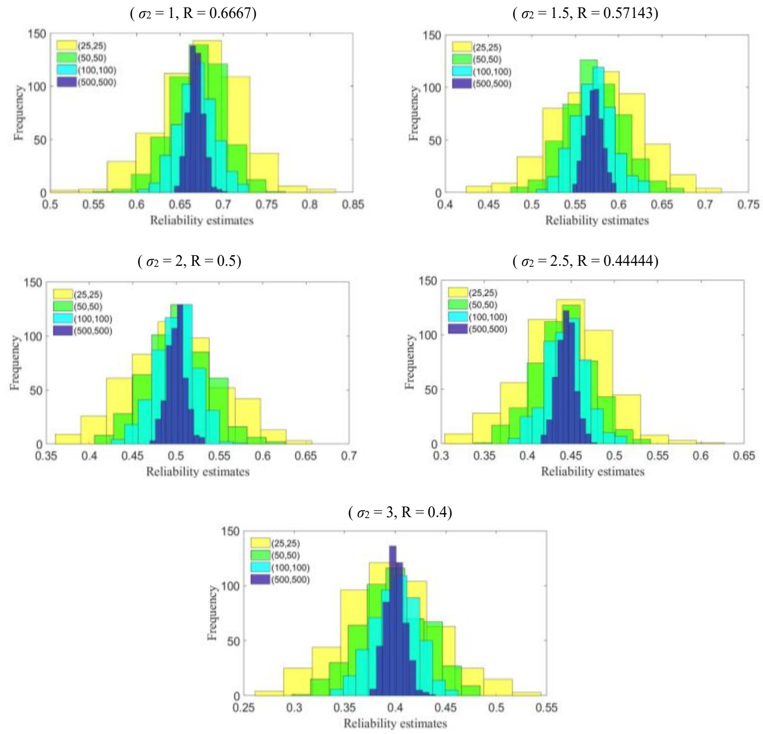

Figure 3. Histogram of 500 experiments for $\mu = 2$, $\sigma_1 = 2$, $p = 1.5$.

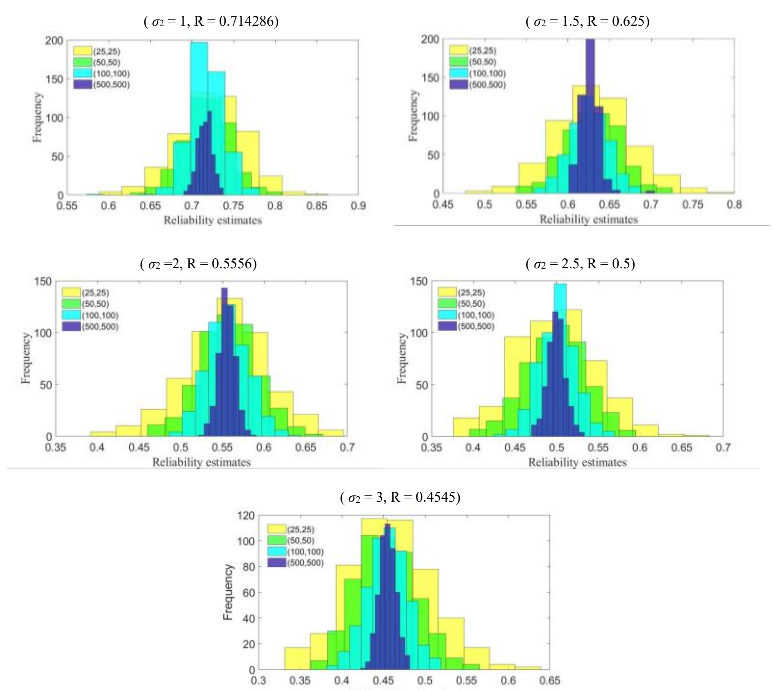

Figure 4. Histogram of 500 experiments for $\mu = 2$, $\sigma_1 = 2.5$, $p = 1.5$.

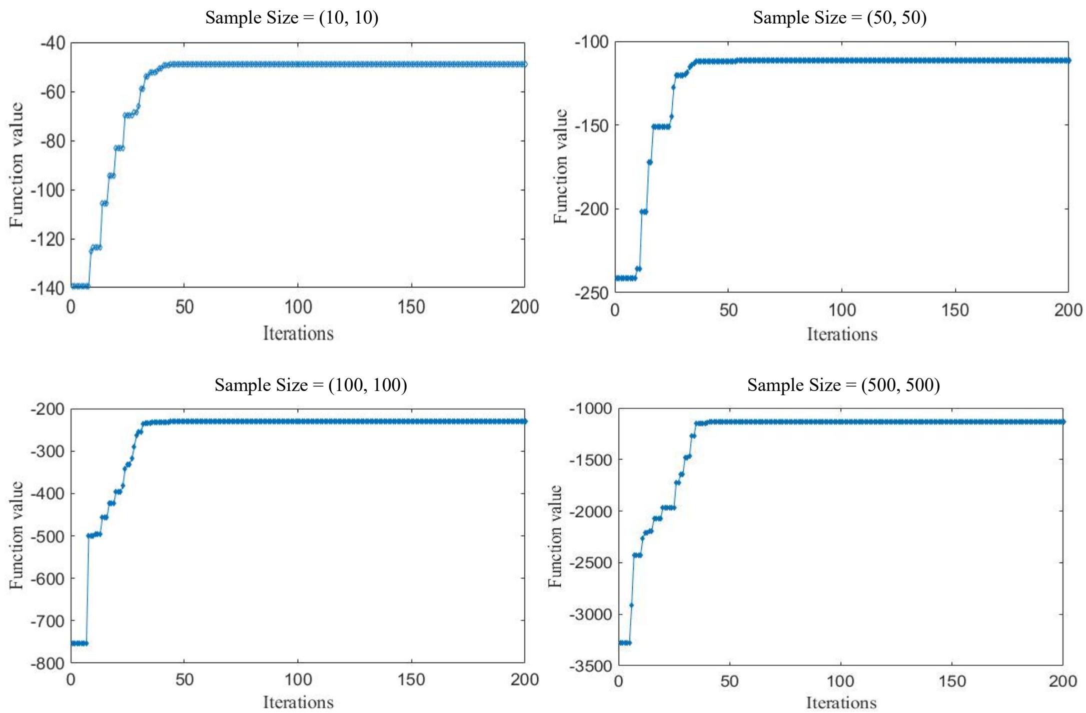

Figure 5. Convergence behavior of Jaya algorithm for different sample sizes.

6. Application to Real Life Data

The proposed methodology has been applied to the strength data shown in **Table 4** and **Table 5** previously used by many researchers in their study (Badar & Priest, 1982; Jia et al., 2017). The data depicts the strength in GPA of single glass fibers for 63 samples of length 10mm and 69 samples of length 20mm. The reliability in this context is evaluated based on the strength of gauge length 10mm being greater than strength of gauge length 20mm using the proposed methodology. The scale parameter for data of gauge length 10 mm is estimated to be 16.1367 and that for gauge length 20 mm is estimated to be 5.1950. The location and shape parameters are considered common for both the data and are estimated to be 0.9980 and 3.3837 respectively. The Kolmogorov-Smirnov test was conducted to check the fitness of estimated parameters with the data. For the data of gauge length 10mm, the Kolmogorov-Smirnov statistic was found to be 0.0410, p value 0.998 and log likelihood function value as -59.4464. The Kolmogorov-Smirnov statistic, p value and log likelihood function value for data of gauge length 20mm are calculated to be 0.0783, 0.8196 and -49.0843 respectively. This shows that the Weibull distribution with estimated parameters gives a good fit for both the data sets. The maximum value of log likelihood function in Equation (12) is obtained as -108.5307. The estimated reliability is 0.7564. The interference of distributions with the estimated parameters can be seen in **Figure 6**. The proposed methodology has diverse applications across multiple domains. In engineering it can be used in reliability analysis ensuring structural safety and material quality. In finance, it can be used to compare investment returns and assess credit risk, while in healthcare, it can be used to evaluate treatment effectiveness. Additionally, it can be implemented in comparing marketing strategies, performance evaluation in sports, model comparison, etc. making it a valuable tool for data-driven decision-making. The economic and commercial impact of this research lies in its potential applications in reliability engineering and risk assessment. By improving the estimation of stress-strength reliability using the Jaya algorithm, industries such as aerospace, automotive, manufacturing and materials science can enhance the accuracy of failure predictions leading to better product design, reduced maintenance costs and improved safety. The methodology can also contribute in optimizing quality control processes, minimizing warranty claims and increasing overall operational reliability making it valuable for businesses aiming to improve product durability and cost-effectiveness.

Table 4. Data of gauge length 10mm.

Strength of glass fibers in GPA for data of gauge length 10mm										
1.901	2.132	2.203	2.228	2.257	2.35	2.361	2.396	2.397	2.445	2.454
2.474	2.518	2.522	2.525	2.532	2.575	2.614	2.616	2.618	2.624	2.659
2.675	2.738	2.74	2.856	2.917	2.928	2.937	2.937	2.977	2.996	3.03
3.125	3.139	3.145	3.22	3.223	3.235	3.243	3.264	3.272	3.294	3.332
3.346	3.377	3.408	3.435	3.493	3.501	3.537	3.554	3.562	3.628	3.852
3.871	3.886	3.971	4.024	4.027	4.225	4.395	5.02	-	-	-

Table 5. Data of gauge length 20mm.

Strength of glass fibers in GPA for data of gauge length 20mm										
1.312	1.314	1.479	1.552	1.7	1.803	1.861	1.865	1.944	1.958	1.966
1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.14	2.179	2.224	2.24
2.253	2.27	2.272	2.274	2.301	2.301	2.359	2.382	2.382	2.426	2.434
2.435	2.478	2.49	2.511	2.514	2.535	2.554	2.566	2.57	2.586	2.629
2.633	2.642	2.648	2.684	2.697	2.726	2.77	2.773	2.8	2.809	2.818
2.821	2.848	2.88	2.809	2.818	2.821	2.848	2.88	2.954	3.012	3.067
3.084	3.09	3.096	3.128	3.233	3.433	3.585	3.585	-	-	-

The proposed methodology applied to the data set has been compared with other studies in the literature to evaluate its effectiveness and accuracy in estimating reliability. **Table 6** shows the results of proposed methodology to that of the existing literature. The proposed methodology gives a better log likelihood value is achieved compared to those obtained using other methodologies in the literature for the considered data. The reliability obtained using proposed methodology is close to the results obtained by Kundu & Gupta (2006) and Kundu & Raqab (2009). The results for reliability estimate is higher compared to the estimate by Valiollahi et al. (2013) and slightly on the lower side compared to the results of Nadarajah & Jia (2017).

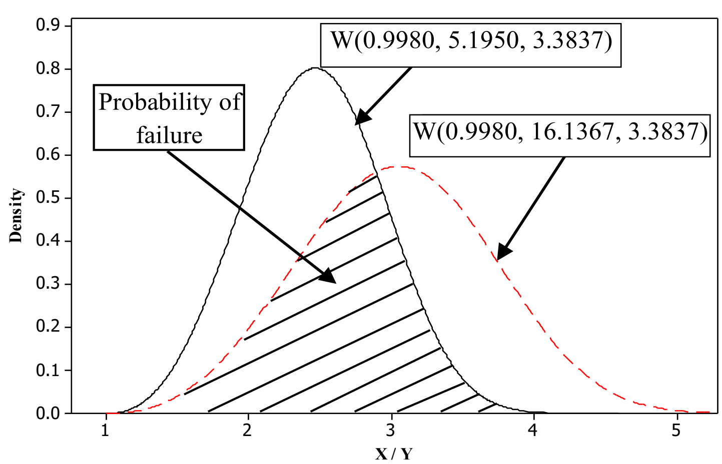

Figure 6. Weibull plots for gauge length data with estimated parameters.

Table 6. Comparison of results for proposed methodology.

Method/Reference	Data set	Shape parameter	Scale parameter	Location parameter	K-S	p-value	Log likelihood value	Reliability
Proposed Methodology	I	3.3837	16.1367	0.9980	0.0410	0.998	-59.4464	0.7564
	II	3.3837	5.1950	0.9980	0.0783	0.8196	-49.0843	
Kundu & Gupta (2006)	I	3.8770	37.2333	-	0.0800	0.8154	-60.1524	0.7624
	II	3.8770	11.6064	-	0.0461	0.9985	-48.8703	
Kundu & Raqab (2009)	I	4.6344	86.9579	1.312	0.0767	0.8525	-	0.7406
	II	4.6344	248.3652	1.312	0.0464	0.9984	-	
Nadarajah & Jia (2017)	I	5.049	0.12547	-	0.088	0.719	-39.438	0.8000
	II	6.725	0.01046	-	0.050	0.997	-68.149	
Valiollahi et al. (2013)	I	5.049	424.574	-	0.0867	0.7197	-	0.5002
	II	5.505	214.131	-	0.0578	0.9658	-	

7. Conclusion

In this study, the estimation of reliability for stress strength interference was carried out using Jaya algorithm via maximum likelihood estimation. It was considered that the stress and strength follow Weibull distribution with common location and shape parameter but different scale parameter. The methodology was applied to simulated data sets of different sample sizes and different scale parameters. The simulation studies assessed reliability using the proposed methodology, the real parameters and Monte Carlo simulation. The estimated reliability from the proposed methodology was compared to real reliability using bias and mean squared error. The results show that the estimated reliability values using the proposed methodology are very close to the real reliability values and the reliability using Monte Carlo simulations for estimated parameters. Additionally, the bias and mean squared error values were observed to be low, further validating the accuracy of the proposed methodology. Also, as the sample size increases, the estimated reliability moves closer to the real reliability and mean squared error also decreases supporting the general trend of estimated values approaching closer to real values. The proposed method using Jaya algorithm efficiently converges to optimal solutions after at the most 50 iterations for all the considered sample sizes. An application to real life data is also shown along with the estimated reliability and interference graph. The inclusion of location parameter in the study significantly affects the reliability results. The comparison of the proposed methodology with other methods for the considered dataset demonstrates that it achieves a higher maximum log-likelihood value, highlighting its effectiveness over existing approaches in the literature. The above methodology can also be applied for other methods of estimation like least squares, weighted least squares, etc. in which optimization is involved. Further research can be carried by varying the location and shape parameters and analyzing its influence on reliability. Also, interference of various other distributions like gamma, Laplace, etc. including cases of systems with multiple components can be considered in evaluation of reliability. Additionally, a comparative analysis of the Jaya algorithm with other metaheuristic and standard numerical optimization methods can be explored under progressively censored data to evaluate its performance in reliability estimation.

Conflict of Interest

The authors confirm that there is no conflict of interest for this publication.

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