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Dynamics Modeling for Key Performance Indicators in Higher Education Through Optimization Methods

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Abstract
Dynamic models of higher education Key Performance Indicators (KPI) help in understanding how internal and external factors affect future KPI achievement. This study aims to construct a dynamic model of university KPI variables and estimate model parameters value. Several used steps to achieve goals are problem definition, variables, model formulation, prerequisite estimation, and conformity analysis. This model involves eight KPIs and three types of funding. Three optimization methods used are Type I constrained optimization, Type II constrained optimization, and unconstrained optimization. The results showed that the percentage of graduates getting decent jobs (KPI 1) in the future year is strongly influenced by two KPIs, namely the work of lecturers (KPI 5) and study programs with international accreditation (KPI 8). The existence of active practitioners (KPI 4) opens opportunities for international cooperation (KPI 6) and collaborative learning (KPI 7). Significant investment in improving the quality of lecturers and the right allocation of funds has proven to impact achieving the goals of higher education positively.

Keywords- Dynamic models, Key performance indicators, Data limitations, Parameter estimation, Optimization methods.

1. Introduction

Higher education performance requires measuring instruments, one of which is Key Performance Indicators (KPIs). KPI is a critical measurement instrument used to assess the performance of universities, covering various aspects such as teaching, research, and community service (Li et al., 2023; Varouchas et al., 2018). KPI assists universities in evaluating the achievement of goals and maintaining consistent improvement in various fields, especially in the fields of teaching, research, and service (Abril-Jiménez et al., 2024; Zheng et al., 2023).

There are a variety of criteria that continue to evolve in the measurement of college KPI. Some research related to KPI measurement criteria has been carried out in various countries, namely Russia, South Korea, and Saudi Arabia. Studies in Russia showed that there are four KPI criteria, namely education, research, international, and financial (Guseva et al., 2022). KPI in South Korea narrowed down to four KPI criteria including finance, customer, learning, and growth as well as internal processes (Kim et al., 2018), while sustainable environmental is included as KPI in Saudi Arabia Universities (Bashir et al., 2023). In addition to KPI measurement criteria, other studies have been conducted related to the design of decision-making models that focus on improving university performance and understanding the socio-economic impact of KPI changes (Abril-Jiménez et al., 2024; Karnitis et al., 2023; Zhang et al., 2020; Zis et al., 2023). The study conducted directs universities to evaluate the extent to which university performance achieves the goals and targets that have been set based on KPI measurement criteria. As done in some of the countries above, Performance measurement based on KPI is also carried out at universities in Indonesia.

The measurement of higher education KPI in Indonesia consists of three criteria, namely graduates, lecturers, and curriculum and learning. The three criteria are translated into eight KPI values. The criteria for graduates are divided into two KPIs, namely the percentage of graduates getting decent jobs and the percentage of students getting off-campus experience. The criteria for lecturers are divided into three KPIs, namely the percentage of lecturers who do activities outside the campus, the percentage of practitioners teaching on campus, and the number of lecturers' work used by the community or received international recognition. Furthermore, curriculum and learning criteria are divided into three, namely the percentage of study programs in collaboration with world-class partners, the percentage of collaborative and participatory classes, and the percentage of undergraduate study programs that have international accreditation or certification recognized by the government. The eight KPIs are measuring tools for evaluating university performance.

Understanding the dynamics of KPI development over time can be done with several approaches, one of which is mathematical modeling. In the context of the KPI problem, in this study, the chosen mathematical modeling is dynamic. Dynamic model is a method of problem analysis to understand system changes over time and describe variables that affect each other in a system (Legaard et al., 2023; Sun et al., 2024; Zhang et al., 2024). Analysing internal and external factors that impact KPI attainment in the future is made easier with the aid of dynamic models. According to Chakrabarty et al. (2022) dynamic models can be widely used in various fields, including higher education KPI studies.

Study-related dynamic modeling has been carried out in recent years. The latest research has been conducted by Son et al. (2024) on dynamic modeling and policy simulation aimed at reducing the risk of heat-related diseases caused by urban heat waves in Seoul, Korea. Aysun and Wright (2024) developed a dynamic factor model to forecast inflation in a small open economy. Legaard et al. (2023) utilize artificial neural network-based models (*neural networks*) in dynamic system simulation. Research by Lin et al. (2023) focuses on developing models of optimal dynamics and control for the dissemination of multi-information in the Social Internet of Things (SIoT) environment. Additionally, KPI-related research was

conducted by Agbali et al. (2018) on KPI problems in *smart cities*. Some studies of these dynamic models emphasize the potential applications of dynamic systems, making them a reference for modeling and analysing KPIs in higher education. However, the research that has been conducted has not examined the construction of dynamic models for higher education KPI in the realm of teaching. The goal of this study is to develop a dynamic model of higher education KPI in Indonesia, as the dynamic model approach has never been implemented in that country.

Research on the dynamics modeling of higher education KPI can be used to estimate the future development of higher education and control related to university performance. Through this approach, it is expected to aid in a better comprehension of the ways in which the different KPI components interact with one another. The application of KPI measurements in universities in Indonesia has only been implemented in recent years. As a result, this research is faced with limited data, so appropriate strategies are needed for the development of appropriate models.

Limited data is often an obstacle in developing the right model. In this context, some researchers have developed iterative algorithms to identify identifiable systems and methods of estimating parameters (Cantó et al., 2017; Ipkovich et al., 2024; Saxton et al., 2024). To get the modeling that best suits the data, several methods are needed to find parameters.

The method chosen to find the estimated value of the KPI parameters in this study is the optimization method. The optimization method is an effective method for finding parameter estimation values in nonlinear models (Chenhao et al., 2024; Milovanović et al., 2021; Ming et al., 2024; Rezk et al., 2024; Xu, 2023). Determining the estimated value of parameters in the KPI case involves the use of optimization methods to find parameter values that minimize the difference between the actual KPI value and the estimated value. Such optimization can be combined with some existing constraints or limitations (Balavignesh et al., 2024; Griva et al., 2009; Khalid, 2024; Nocedal et al., 2014; Wang et al., 2024).

The questions in this study are twofold. First, how to construct a dynamic model that can explain the relationship between model parameters that have been identified in the context of higher education KPI? Second, how to find the estimated value of model parameters with limited data?

This research has two main objectives. First, a dynamic model that can explain the relationship between identified parameters and the KPIs of higher education in Indonesia, as well as allow long-term analysis and prediction of the impact of parameter changes on the performance of educational institutions. Second, look for parameters that are appropriate to limited data, considering possible changes due to new policies related to KPI. The novelty of this research lies in two main aspects, namely the formation of an innovative KPI model compartment diagram and the application of a model parameter search method designed to overcome data limitations. By combining these two approaches, it is hoped that this research can provide a valuable contribution to an in-depth understanding of higher education performance and the impact of policy changes on KPIs.

The development of the KPI dynamic model provides benefits in the world of education. First, the KPI dynamic model results in the construction of models in decision-making related to education management by university leaders in Indonesia. This model will enable long-term analysis of KPIs and can be used to predict the impact of changes in certain parameters on college performance. Second, the KPI dynamic model can be a reference or guide in dynamic modeling and optimal control applications for policy-making problems in designing more effective strategies to improve higher education KPI achievements.

2. Materials and Methods

This study aims to construct the KPI variable model and find the estimated value of the model parameters. Therefore, to achieve this goal, several steps are needed as follows.

2.1 Definition and Formulations of the KPI Model

At the definition and model formulation stage, the type of model built is a linear discrete dynamic model in the form of a system of simultaneous equations between KPI variables and funding variables. This design begins by dividing eight compartments as KPI variables and three funding variables presented in **Table 1**.

Table 1. Description of KPI variables and funding variables of the KPI model.

Variable KPI	Description
$x_1(k)$	Percentage of graduates who land good employment
$x_2(k)$	Percentage of students that have experience away from campus
$x_3(k)$	Percentage of lecturers engaged in activities away from campus
$x_4(k)$	Percentage of professionals who instruct on campus
$x_5(k)$	The quantity of work done by lecturers is recognized internationally or used by the community.
$x_6(k)$	Percentage of study programs that collaborate with prestigious partners
$x_7(k)$	Percentage of classes that include participation and collaboration
$x_8(k)$	Percentage of undergraduate programs with government-recognized international certification or accreditation
Variable funding	**Description**
$u_1(k)$	Percentage of funds used to raise student standards
$u_2(k)$	Percentage of funds allocated to raise lecturer quality
$u_3(k)$	Percentage of funds allocated to raise the standard of instruction and curriculum

Based on **Table 1**, a flowchart for the KPI model was designed to represent the following four assumptions.
Assumption 1. The quality of lecturers affects the quality of graduates.
Assumption 2. The quality of lecturers affects the quality of curriculum and learning.
Assumption 3. The quality of curriculum & learning affects the quality of graduates.
Assumption 4. Influence between KPI variables.

Based on the assumptions, a linear discrete dynamic model is recommended as follows:

$$
\begin{aligned}
x_1(k+1) &= a_{11}x_1(k) + a_{12}x_2(k) + a_{13}x_3(k) + a_{14}x_4(k) + a_{15}x_5 + a_{16}x_6(k) + a_{17}x_7(k) + a_{18}x_8(k) + b_{11}u_1(k)\\
x_2(k+1) &= a_{21}x_1(k) + a_{22}x_2(k) + a_{23}x_3(k) + a_{24}x_4(k) + a_{25}x_5(k) + a_{26}x_6(k) + a_{27}x_7(k) + a_{28}x_8(k) + b_{12}u_1(k)\\
x_3(k+1) &= a_{33}x_3(k) + a_{34}x_4(k) + a_{35}x_5(k) + a_{36}x_6(k) + a_{38}x_8(k) + b_{23}u_2(k)\\
x_4(k+1) &= a_{42}x_2(k) + a_{44}x_4(k) + a_{46}x_6(k) + a_{47}x_7(k) + a_{48}x_8(k) + b_{24}u_2(k)\\
x_5(k+1) &= a_{53}x_3(k) + a_{55}x_5(k) + a_{56}x_6(k) + a_{58}x_8(k) + b_{25}u_2(k)\\
x_6(k+1) &= a_{61}x_1(k) + a_{63}x_3(k) + a_{64}x_4(k) + a_{65}x_5(k) + a_{66}x_6(k) + a_{68}x_8(k) + b_{36}u_3(k)\\
x_7(k+1) &= a_{73}x_3(k) + a_{74}x_4(k) + a_{75}x_5(k) + a_{76}x_6(k) + a_{77}x_7(k) + a_{78}x_8(k) + b_{37}u_3(k)\\
x_8(k+1) &= a_{81}x_1(k) + a_{82}x_2(k) + a_{83}x_3(k) + a_{84}x_4(k) + a_{85}x_5(k) + a_{86}x_6(k) + a_{87}x_7(k) + a_{88}x_8(k) + b_{38}u_3(k)
\end{aligned}
\tag{1}
$$

From Equation (1) eight simultaneous equations are obtained where, k represents the current year and $k+1$ represents the next year. Based on Equation (1), 58 parameters were found that showed the influence between parameters namely a_{ij} and b_{ij}.

2.2 Model Parameter Estimation

Parameter estimation aims to bring the results obtained close to the data so that it is necessary to minimize the difference between variable estimates and data. Therefore, several optimization methods are needed to obtain the parameters in the KPI model equation. This study used three different ideas to look for model parameters. These three different ideas are categorized into three methods, namely optimization with Type I constraints, optimization with Type II constraints, and optimization without constraints. The parameters that have been found are then validated with a chi-squared test for goodness of fit to assess the extent to which the model fits the existing data. The data used at this stage is the KPI data of Universitas Syiah Kuala for 2019-2022 which is presented in **Table 2**.

Table 2. KPI data & funding data for 2019-2022.

Variable	Year			
	2019	2020	2021	2022
Variable KPI				
$x_1(k)$	0.4835	0.6671	0.9681	0.7794
$x_2(k)$	1.0700	0.4860	0.5523	0.5130
$x_3(k)$	0.7268	0.8496	1.3300	1.5328
$x_4(k)$	0.8051	0.9622	1.5291	1.6918
$x_5(k)$	0.5833	1.1000	0.9833	0.8167
$x_6(k)$	1.0000	1.1461	1.1654	1.4286
$x_7(k)$	0.4687	0.9800	1.3767	1.1095
$x_8(k)$	0.9271	0.0000	0.0000	0.3714
Variable funding				
$u_1(k)$	1.4541	1.0000	1.0393	2.7376
$u_2(k)$	0.4309	1.0000	1.0533	0.8564
$u_3(k)$	0.6347	1.0000	1.1165	1.6450

Source: *Syiah Kuala University Data (Normalized Data)* 2019-2022.

2.2.1 Optimization with Type I Constraints

This first method is a nonlinear optimization method used to optimize model parameters by considering the constraints of equations and inequalities (Griva et al., 2009; Nocedal and Wright, 2000). This method aims to minimize parameter values in the model. a_{ij} is the parameter that links x_i and x_j. Meanwhile, b_{ij} is the parameter that links u_i and x_j. In this method, the constraints are in the form of inequalities and equations as follows.

$$\min_{a_{ij},b_{ij}} f(a_{ij},b_{ij}) = \sum_{i,j\epsilon\mathcal{S}} a_{ij}^2 + \sum_{i,j\epsilon\mathcal{T}} b_{ij}^2$$

Subject to $\left(x_j(k+1) - f_j(x(k),u(k))\right)_r = 0$

$$0 < a_{ij} < 1, \forall(i,j) \in \mathcal{S} \quad (2)$$

$$0 < b_{ij} < 1, \forall(i,j) \in \mathcal{T}$$

where, $\mathcal{S}$ and $\mathcal{T}$ is a set of pairs (i,j) are relevant.

To solve optimization problems with Type I constraints, the numerical computational approach used in this study is the method of interior point (Nocedal and Wright, 2000). In this method, the constraints are combined into the Lagrange function. Next, iterative steps are used to find the optimal point that satisfies the stationary conditions of the Lagrange function.

2.2.2 Optimization with Type II Constraints

This second method is a nonlinear optimization used to find model parameters by minimizing the square of the difference between the estimated value and the actual data. The constraints used in optimization are inequality constraints. The numerical computational approach used is the interior point method. The form of the formulation is as follows.

$$\min_{a_{ij},b_{ij}} f\left(a_{ij},b_{ij},x(k),u(k)\right) = \sum_{r=1}^{3}\sum_{j=1}^{8}\left(x_j(k+1) - f_j\left(x(k),u(k)\right)\right)_r^2$$

Subject to $0 < a_{ij} < 1, \forall(i,j) \in \mathcal{S}$ (3)

$0 < b_{ij} < 1, \forall(i,j) \in \mathcal{T}$

where, $\mathcal{S}$ and $\mathcal{T}$ is a set of pairs (i,j) are relevant.

2.2.3 Unconstrained Optimization

This third method is a nonlinear optimization method used to find model parameters without considering certain constraints (Burden and Faires, 2011; Nocedal and Wright, 2000). In Unconstrained optimization, there are no special constraints that must be adhered to other than finding parameter values that optimize the function of the goal. This method is simpler because there are no obstacles to be met, so the search for optimal solutions becomes easier.

This method aims to minimize quadratic differences but without inequality constraints. This method takes an objective function on the second idea but does not use inequality constraints. In this study, the numerical computational approach used was the Quasi-Newtonian method. This method is a seamless optimization, minimizing the following functions:

$$\min_{a_{ij},b_{ij}} f\left(a_{ij},b_{ij},x(k),u(k)\right) = \sum_{r=1}^{3}\sum_{j=1}^{8}\left(x_j(k+1) - f_j\left(x(k),u(k)\right)\right)_r^2 \quad (4)$$

These three methods are used because data related to KPI is still limited. To obtain appropriate parameters, these three methods are used. The results of the three methods will select an appropriate method based on the results of the chi-squared test for goodness of fit.

3. Results and Discussion

3.1 Parameter Estimation Results

In this section, the results of parameter estimation obtained by three optimization methods that have been described in the methodology will be presented. The results obtained were 58 parameters consisting of a_{ij} and b_{ij}.

3.1.1 Optimization with Type I Constraints

The solution procedure for this method uses function minimum constrained (fmincon) with the interior point algorithm (Byrd et al., 2000; Fazlyab et al., 2016; Nocedal et al., 2014; Waltz et al., 2006) as follows.

Step 1. Initialization
Choose an initial barrier parameter $\mu > 0$. The initial guess chosen for the optimized parameters (a_{ij}, b_{ij}), for example $a_{ij} = 0.1$ dan $b_{ij} = 0.1$.

The problem is:
$\min_{a_{ij},b_{ij}} f(a_{ij}, b_{ij}) = \sum_{(i,j)\epsilon\mathcal{S}} a_{ij}^2 + \sum_{(i,j)\epsilon\mathcal{T}} b_{ij}^2$

Subject to $\left(x_j(k+1) - f_j(x(k), u(k))\right)_r = 0$

$0 < a_{ij} < 1, \forall (i,j) \in \mathcal{S}$ (5)

$0 < b_{ij} < 1, \forall (i,j) \in \mathcal{T}$

where, $\mathcal{S}$ and $\mathcal{T}$ is a set of pairs (i,j) are relevant.

To solve the approximate problem in Equation (5), the algorithm uses one of two main types of steps at each iteration, namely the conjugate gradient step, using a trust region.

Step 2. Interior point step with variable slack

(i) Variable Slack
Define the slack variable $s_{a_{ij}}$ and $s_{b_{ij}}$ to ensure that constraints are met. Slack variables $s_{a_{ij}}$ and $s_{b_{ij}}$ are used to ensure that each value a_{ij} and b_{ij} is in the range $0 < a_{ij} < 1$ and $0 < b_{ij} < 1$. Slack variable $s_{a_{ij}}$ and $s_{b_{ij}}$ allows the solution to stay within the given limits:

(ii) Calculate and estimate the solution for the barrier problem (Equation (6)). For each $\mu > 0$ and s is the vector of slack variables is implicitly assumed to have a positive value. The approximate problem is
$\min_{a_{ij},b_{ij},s_{a_{ij}},s_{b_{ij}}} f_\mu\left(a_{ij}, b_{ij}, s_{a_{ij}}, s_{b_{ij}}\right) = \min_{a_{ij},b_{ij},s_{a_{ij}},s_{b_{ij}}} f(a_{ij}, b_{ij})$

Subject to $(x_j(k+1) - f_j(x(k), u(k)))_r = 0$

$a_{ij} + s_{a_{ij}} = 1, \forall (i,j) \in \mathcal{S}$ (6)

$b_{ij} + s_{b_{ij}} = 1, \forall (i,j) \in \mathcal{T}$

$s_{a_{ij}} > 0, \forall (i,j) \in \mathcal{S}$

$s_{b_{ij}} > 0, \forall (i,j) \in \mathcal{T}$

Step 3. Conjugate gradient step using a trust region
Select an initial trust region radius $\Delta > 0$, contraction parameters a_{ij} dan b_{ij} within the interval (0, 1), and a penalty parameter ν > 0 for the merit function in Equation (7).

$$f_\mu\left(a_{ij}, b_{ij}, s_{a_{ij}}, s_{b_{ij}}\right) + v \left\| \begin{matrix} (x_j(k+1) - f_j(x(k), u(k)))_r \\ a_{ij} + s_{a_{ij}} - 1 \\ b_{ij} + s_{b_{ij}} - 1 \end{matrix} \right\| \quad (7)$$

Step 4. Update parameters and slack variables

(i) Reduce the barrier parameter μ and return to Step 1. The interior point algorithm will stop if the solution is feasible or exceeds the maximum iteration limit.

(ii) The output of the interior point algorithm is a parameter that produces the minimum value of the objective function.

3.1.2 Optimization with Type II Constraints

The solution procedure for this method also uses function minimum constrained (fmincon) with the interior point algorithm.

Step 1. Initialization

Choose an initial barrier parameter $\mu_0 > 0$. The initial guess chosen for the optimized parameters is 0.1. The problem is

$$\min_{a_{ij},b_{ij}} f\left(a_{ij}, b_{ij}, x(k), u(k)\right) = \sum_{r=1}^{3}\sum_{j=1}^{8}\left(x_j(k+1) - f_j\left(x(k), u(k)\right)\right)_r^2$$

$$\text{Subject to } 0 < a_{ij} < 1, \forall (i,j) \in \mathcal{S} \tag{8}$$

$$0 < b_{ij} < 1, \forall (i,j) \in \mathcal{T}$$

where, $\mathcal{S}$ and $\mathcal{T}$ is a set of pairs (i, j) are relevant.

To solve the approximate problem Equation (8), the algorithm uses one of two main types of steps at each iteration, namely the conjugate gradient step, using a trust region.

Step 2. Interior point step with variable slack

(i) Variable Slack

Define the slack variable $s_{a_{ij}}$ and $s_{b_{ij}}$ to ensure that constraints are met. Slack variables $s_{a_{ij}}$ and $s_{b_{ij}}$ are used to ensure that each value a_{ij} and b_{ij} is in the range $0 < a_{ij} < 1$ and $0 < b_{ij} < 1$. Slack variable $s_{a_{ij}}$ and $s_{b_{ij}}$ allows the solution to stay within the given limits.

(ii) Calculate and estimate the solution for the barrier problem (Equation (9)). For each $\mu > 0$ and s is the vector of slack variables is implicitly assumed to have a positive value. The approximate problem is

$$\min_{a_{ij},b_{ij},s_{a_{ij}},s_{b_{ij}}} f_\mu\left(a_{ij}, b_{ij}, s_{a_{ij}}, s_{b_{ij}}\right) = \min_{a_{ij},b_{ij},s_{a_{ij}},s_{b_{ij}}} f\left(a_{ij}, b_{ij}\right)$$

$$-\mu\left(\sum_{(i,j)\in\mathcal{S}} \ln\left(s_{a_{ij}}\right) + \sum_{(i,j)\in\mathcal{T}} \ln\left(s_{b_{ij}}\right)\right)$$

$$\text{Subject to } a_{ij} + s_{a_{ij}} = 1, \forall (i,j) \in \mathcal{S} \tag{9}$$

$$b_{ij} + s_{b_{ij}} = 1, \forall (i,j) \in \mathcal{T}$$

$$s_{a_{ij}} > 0, \forall (i,j) \in \mathcal{S}$$

$$s_{b_{ij}} > 0, \forall (i,j) \in \mathcal{T}$$

Step 3. Conjugate gradient step using a trust region

Select an initial trust region radius $\Delta > 0$, contraction parameters a_{ij} dan b_{ij} within the interval (0, 1), and a penalty parameter $\nu > 0$ for the merit function in Equation (10).

$$f_{\mu}\left(a_{ij}, b_{ij}, s_{a_{ij}}, s_{b_{ij}}\right) + v\left\|\begin{matrix} a_{ij} + s_{a_{ij}} - 1 \\ b_{ij} + s_{b_{ij}} - 1 \end{matrix}\right\| \quad (10)$$

Step 4. Update parameters and slack variables
(i) Reduce the barrier parameter μ and return to Step 1. The interior point algorithm will stop if the solution is feasible or exceeds the maximum iteration limit.
(ii) The output of the interior point algorithm is a parameter that produces the minimum value of the objective function.

Here is the flowchart for the two optimization methods, namely optimization with Type I constraints and optimization with Type II constraints, as presented in **Figure 1**.

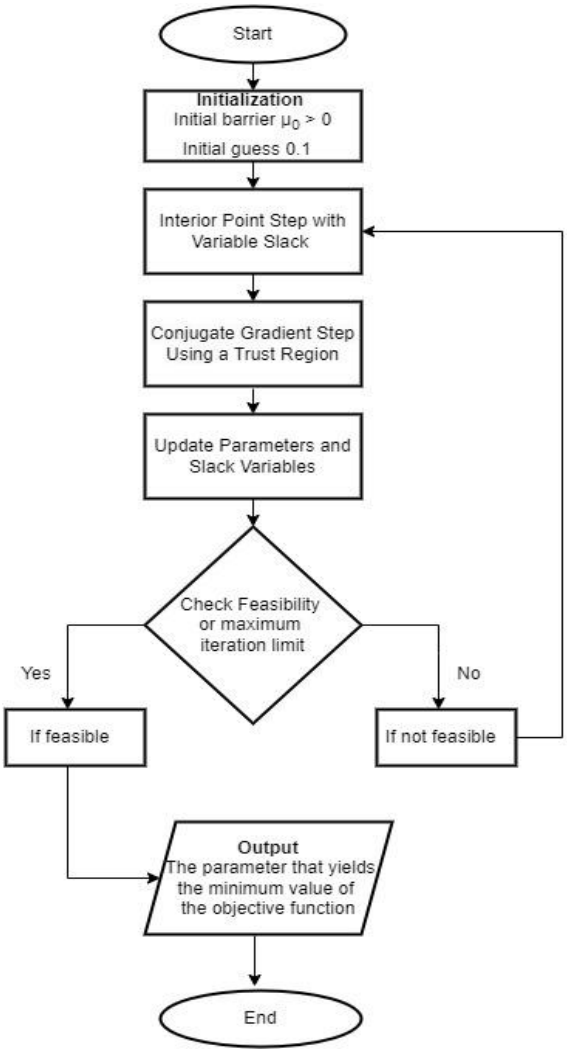

Figure 1. Flowchart of the interior point method.

3.1.3 Unconstrained Optimization

The parameter search process in this method is almost the same as the Type II constrained optimization method. The difference lies only in the method chosen. Type I constrained optimization uses Function Minimum Constrained (fmincon) with the Interior Point method. Meanwhile, the unconstrained optimization method uses Function Minimum Unconstrained (fminunc) with the Quasi-Newton method. Then a general Quasi-Newton method (Griva et al., 2009; Shanno, 1970; Wei et al., 2006) can formally be written as:

Step 1. Initialization

Provide an initial estimate of the solution $(a_{ij}, b_{ij})_0$ and an initial approximation of the Hessian $B_0 = I$, where, I is the scaled identity matrix. The problem is

$$\min_{a_{ij},b_{ij}} f\left(a_{ij}, b_{ij}, x(k), u(k)\right) = \sum_{r=1}^{3}\sum_{j=1}^{8}\left(x_j(k+1) - f_j\left(x(k), u(k)\right)\right)_r^2 \quad (11)$$

Step 2. Iteration

For $k = 0,1, \ldots.1000$

(i) If $(a_{ij}, b_{ij})_k$ is optimal, stop.

(ii) Solve $B_k p = -\nabla(f\left(a_{ij}, b_{ij}\right)_k)$ for p_k, where, p is search direction and $\nabla(f\left(a_{ij}, b_{ij}\right)_k$ is the gradient of the objective function at the current location.

(iii) Use a line search to determine
$(a_{ij}, b_{ij})_{k+1} = (a_{ij}, b_{ij})_k + \alpha_k p_k$.

(iv) Compute
$s_k = (a_{ij}, b_{ij})_{k+1} - (a_{ij}, b_{ij})_k$
$y_k = \nabla f\left((a_{ij}, b_{ij})_{k+1}\right) - \nabla f\left((a_{ij}, b_{ij})_k\right)$.

These methods accumulate curvature information at each iteration to construct a quadratic model problem in Equation (11) of the following form.

$$\underset{p}{\text{minimize}}\, f(p) = f\left(a_{ij}, b_{ij}\right) + \nabla\left(f(a_{ij}, b_{ij})\right)^T p + \frac{1}{2} p^T B_k p \quad (12)$$

where, the Hessian matrix B_k is a symmetric matrix that is positive definite, $(f(a_{ij}, b_{ij}))^T$ is a constant vector, and $f(a_{ij}, b_{ij})$ is a constant value. While p is a search vector indicating the direction of improvement from the current point. Therefore, p is the change in the parameter a_{ij}, b_{ij}. The optimal solution for this problem is achieved when the partial derivatives approach zero (Shanno, 1970; Wei et al., 2006).

(v) Compute

$$B_{k+1} = B_k - \frac{(B_k s_k)(B_k s_k)^T}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k} \quad (13)$$

Step 3. The output of the Quasi-Newton algorithm is a parameter that yields the minimum value of the objective function.

Here is the flowchart for the two optimization methods, namely Optimization with unconstrained optimization, as presented in **Figure 2**.

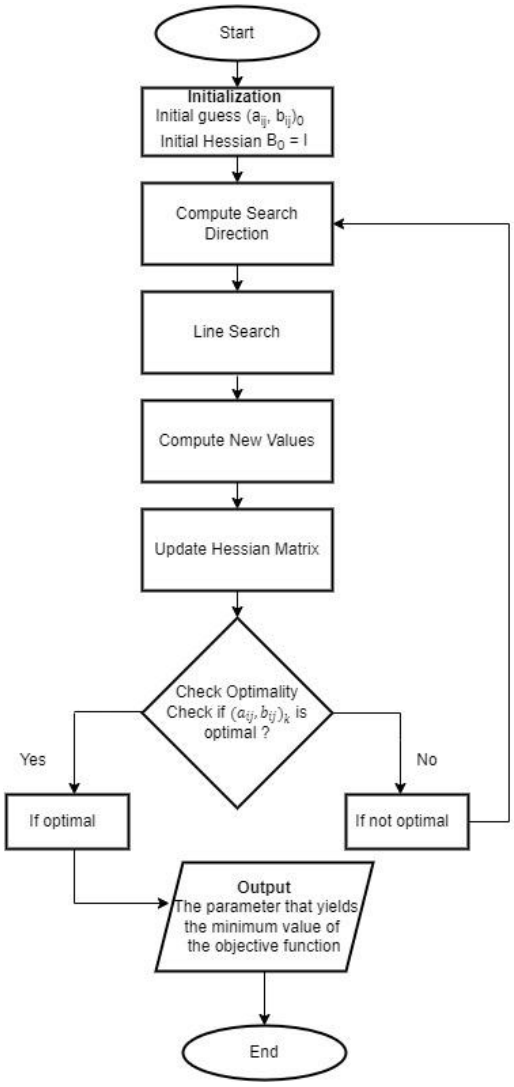

Figure 2. Flowchart of the Quasi-Newton method.

3.2 Optimization Analysis of Parameter Search Methods

In this section, we will discuss the process of finding estimated model parameter values from the three methods. The first is the Type I constrained optimization method. This method uses a fmincon solved through an interior point algorithm (Fazlyab et al., 2016; Nocedal et al., 2014). Method interior point used to solve optimization problems that have constraints, such as mathematical programming problems with inequality constraints (Leyffer, 2005; Mészáros, 1998; Nocedal et al., 2014). The result of the parameter shows a range between 0 and 1 where the initial guess given is 0.1 and the maximum iteration stops at 1000 iterations.

The optimization method is a Type I, providing statistically significant parameter estimation results of 12.04. However, the solution obtained is not feasible i.e. unable to meet all the constraints that have been given in optimization (Nocedal et al., 2014). This shows that after trying various iterations and search steps, the algorithm cannot find a point that meets all the constraints together. This method has impossible

conditions, so no solution meets all the restrictions. Next, try to make initial settings for limits and initial guesses that allow meeting the limits. However, the solution is still not feasible (Baayen and Postek, 2022).

Second, Type II constrained optimization method**.** This method also provides statistically significant parameter estimation results of 0.81. This second method is also solved by an algorithm interior point Like the first method. However, it is very different from the results of the Type I constrained optimization method, which is 12.04 statistically. This shows that the results of parameter estimation with Type II constrained optimization methods are close to the ideal value, which is towards 0. This value shows the difference between the actual data and the modelling data. In the context of optimization, it shows that the optimization algorithm is an interior point that has successfully found the local minimum value and meets all the constraints that have been set. The optimization algorithm has found the local minimum value of the optimized goal function.

The solutions found meet all the limitations set on optimization problems. That is, the value found is a valid solution according to the constraint. Optimization in this method shows that the algorithm has identified the goal function so that it does not decrease in a feasible direction, and this has met the tolerance default for function and constraints. In other words, the algorithm assumes that it has achieved a solution that meets the criteria of the function's predefined goals and constraints (Nocedal and Wright, 2000; Nocedal et al., 2014).

Third, the unconstrained optimization method is employed using the *fminunc* function, which utilizes the Quasi-Newton approach. Specifically, *fminunc* implements the BFGS (Broyden-Fletcher-Goldfarb-Shanno) Quasi-Newton method, incorporating a mixed quadratic and cubic line search procedure. During each iteration, *fminunc* determines the search direction by updating an approximation of the Hessian matrix of the objective function, based on the BFGS formula (Li et al., 2008; Mahdavi-Amiri and Salehi Sadaghiani, 2020; Wei et al., 2006). The BFGS method is a widely recognized Quasi-Newton optimization technique known for its computational efficiency and reliable performance in solving unconstrained optimization problems. It approximates the Hessian matrix using first-order information, making it a practical choice for optimization problems where direct calculation of the Hessian is infeasible (Mahdavi-Amiri and Salehi Sadaghiani, 2020; Pratama et al., 2023; Talbi et al., 2020; Wei et al., 2006; Xue et al., 2024).

This method gives a statistical test result of 0.68. This indicates close to the ideal value of 0. The results of the hypothesis state that the actual frequency does not deviate significantly from the theoretical frequency in the normal distribution. In the context of optimization, Newton's Quasi-algorithm has successfully found the local minimum point in the optimization process (Burden and Faires, 2011). So, the result is a solution that minimizes objective functioning with the initial conditions and constraints provided. Additionally, the optimization process concludes when the gradient of the objective function becomes sufficiently small and reaches a specified tolerance level. This suggests that optimization has reached a state in which the change in objective functions is very small, and the optimization algorithm considers it to be quite close to the minimum. The results of parameter estimation in this method indicate that the optimization has succeeded in finding a local minimum that meets the established tolerance criteria, but not the global minimum (Nocedal and Wright, 2000).

The success of this optimization is based on the size of the gradient that has reached the tolerance level. When viewed statistically its value is significant. However, in the case of KPI, the unconstrained optimization method has a drawback because it does not impose limitations, which can result in negative estimated values for the parameters. This suggests that the constraint on the KPI, which limits the parameter values of a_{ij} and b_{ij} to between 0 and 1, is being violated. This observation aligns with the research by

Cardozo and Aanonsen (2009), which indicates that seamless optimization algorithms may yield unrealistic results. Hence the importance of using a driving optimization algorithm by setting parameter constraints and performing multiple searches from different initial guesses.

The estimated parameter values are assessed using two main criteria based on the outcomes of the three methods. The first criterion is the chi-squared test for goodness of fit, with a focus on values that are close to zero. The second criterion is the reasonableness of the estimated parameter values within the context of the KPI. The values for the parameters of the linear discrete dynamic model (1) are shown in **Table 3**.

Table 3. Parameter values of a linear discrete dynamic model in Type II constrained optimization method.

Equation $x_1(k+1)$		Equation $x_2(k+1)$		Equation $x_3(k+1)$		Equation $x_4(k+1)$	
Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
a_{11}	0.0001	a_{21}	0.0075	a_{33}	0.0009	a_{42}	0.0001
a_{12}	0.0001	a_{22}	0.0212	a_{34}	0.5103	a_{44}	0.6048
a_{13}	0.0001	a_{23}	0.0049	a_{35}	0.7609	a_{46}	0.3342
a_{14}	0.0001	a_{24}	0.0042	a_{36}	0.0002	a_{47}	0.3342
a_{15}	0.8404	a_{25}	0.4546	a_{38}	0.0001	a_{48}	0.0001
a_{16}	0.0001	a_{26}	0.0187	b_{23}	0.0001	b_{24}	0.0001
a_{17}	0.0001	a_{27}	0.0056				
a_{18}	0.1901	a_{28}	0.1775				
b_{11}	0.0001	b_{12}	0.0016				
Equation $x_5(k+1)$		**Equation $x_6(k+1)$**		**Equation $x_7(k+1)$**		**Equation $x_8(k+1)$**	
Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
a_{53}	0.0079	a_{61}	0.2155	a_{73}	0.0001	a_{81}	0.0001
a_{55}	0.0764	a_{63}	0.1576	a_{74}	0.0001	a_{82}	0.0001
a_{56}	0.0229	a_{64}	0.1447	a_{75}	0.6533	a_{83}	0.0001
a_{58}	0.2204	a_{65}	0.3138	a_{76}	0.0001	a_{84}	0.0002
b_{25}	0.8224	a_{66}	0.2175	a_{77}	0.0001	a_{85}	0.0001
		a_{68}	0.2938	a_{78}	0.6193	a_{86}	0.0001
		b_{36}	0.1381	b_{37}	0.0001	a_{87}	0.1655
						a_{88}	0.0001
						a_{38}	0.0001

Furthermore, the KPI model (1) uses parameter estimates obtained from Type II constrained optimization methods. The resulting model, as detailed in Equation (14), is a linear discrete dynamical system represented by a set of eight simultaneous equations.

$$
\begin{aligned}
x_1(k+1) &= 0.0001x_1(k) + 0.0001x_2(k) + 0.0001x_3(k) + 0.0001x_4(k) + 0.8404x_5(k) + \\
&0.0001x_6(k) + 0.0001x_7(k) + 0.1901x_8(k) + 0.0001u_1(k) \\
x_2(k+1) &= 0.0075x_1(k) + 0.0212x_2(k) + 0.0049x_3(k) + 0.0042x_4(k) + 0.4546x_5(k) + \\
&0.0187x_6(k) + 0.0056x_7(k) + 0.1775x_8(k) + 0.0016\,u_1(k) \\
x_3(k+1) &= 0.0009x_3(k) + 0.5103x_4(k) + 0.7609x_5(k) + 0.0002x_6(k) + 0.0001x_8(k) + \\
&0.0001u_2(k) \\
x_4(k+1) &= 0.0001x_2(k) + 0.6048x_4(k) + 0.3342x_6(k) + 0.3342x_7(k) + 0.0001x_8(k) + \\
&0.0001u_2(k) \\
x_5(k+1) &= 0.0079x_3(k) + 0.0764x_5(k) + 0.0229x_6(k) + 0.2204x_8(k) + 0.8224\,u_2(k)
\end{aligned}
\tag{14}
$$

$$x_6(k+1) = 0.2155x_1(k) + 0.1576x_3(k) + 0.1447x_4(k) + 0.3138x_5(k) + 0.2175x_6(k) + 0.2938x_8(k) + 0.1381u_3(k)$$

$$x_7(k+1) = 0.0001x_3(k) + 0.0001x_4(k) + 0.6533x_5(k) + 0.0001x_6(k) + 0.0001x_7(k) + 0.6193x_8(k) + 0.0001u_3(k)$$

$$x_8(k+1) = 0.0001x_1(k) + 0.0001x_2(k) + 0.0001x_3(k) + 0.0002x_4(k) + 0.0001x_5(k) + 0.0001x_6(k) + 0.1655x_7(k) + 0.0001x_8(k) + 0.0001u_3(k)$$

3.3 Analysis of KPI Parameters

The KPI model presented in Equation (14) was then used to analyse the estimated parameter values for each equation. The first equation shows that the variable $x_5(k)$ and $x_8(k)$ has a significant parameter estimation value against $x_1(k+1)$ than any other KPI variable. This indicates that the proportion of lecturers' work recognized by the community or receiving international acclaim influences the percentage of graduates securing suitable employment within the following year by 84.04%. Similarly, the percentage of undergraduate programs with international accreditation or government-recognized certification can impact the rate of graduates obtaining appropriate employment one year later by 19.01%. So, KPI 1 in the next year is influenced by KPI 5 and KPI 8 in the current year. This highlights the importance of lecturers' work in increasing campus credibility and alumni opportunities to obtain appropriate jobs (Hershatter and Epstein, 2010).

The second equation shows that variables $x_5(k)$ and $x_8(k)$ have significant parameter estimation values against $x_2(k+1)$ than other KPI variables. This implies that the extent of lecturers' work that is utilized by the community or gains international recognition influences the percentage of graduates securing suitable employment within the following year by 45.46%. Similarly, the proportion of undergraduate programs with international accreditation or government-recognized certification impacts the rate of graduates obtaining appropriate employment in the subsequent year by 17.75%. So, KPI 2 in the next year is influenced by KPI 5 and KPI 8 in the current year.

The third equation shows that variables $x_4(k)$ and $x_5(k)$ have significant parameter estimation values against $x_3(k+1)$ than other KPI variables. This indicates that the proportion of practitioners teaching on campus influences the percentage of lecturers engaging in off-campus activities over the next year by 51.03%. Similarly, the extent of lecturers' work that is utilized by the community or receives international recognition affects the percentage of graduates obtaining suitable employment in the following year by 76.09%. So, KPI 3 in the next year is influenced by KPI 4 and KPI 5 in the current year.

The fourth equation shows that the variables $x_4(k)$, $x_6(k)$, and $x_7(k)$ has a significant parameter estimation value against $x_4(k+1)$ than any other KPI variable. This means that the percentage of practitioners teaching on campus affects the percentage of lecturers doing off-campus activities 1 year later amounted to 60.48%. Similarly, the percentage of study programs collaborating with world-class partners, along with the proportion of collaborative and participatory classes, influences the percentage of practitioners teaching on campus one year later by 33.42%. So, KPI 4 in the next year is influenced by KPI 4, KPI 6 and KPI 7 in the current year. This indicates that the presence of practitioners who actively teach on campus contributes positively to the achievement of KPI in the coming years (Ingtias et al., 2022). This could mean that practitioners involved in the on-campus teaching process provide valuable insights to students or enhance

the quality of education provided by the institution. These findings can also form the basis for improvement efforts or policies aimed at improving the standard of instruction in higher education.

The fifth equation shows that the variables $x_8(k)$ and $u_2(k)$ have significant parameter estimation values against $x_5(k+1)$ compared to any other KPI variable. This means that the percentage of undergraduate programs with international accreditation or government-recognized certification influences the extent of lecturers' work that is utilized by the community or receives international recognition one year later by 22.04%. Similarly, the proportion of funding allocated to enhancing the quality of lecturers impacts KPI 5 over the next five years by 82.24%. Therefore, KPI 5 in the coming year is affected by both KPI 8 and current-year funding for lecturers. The allocation of funding to improve the quality of lecturers directly has a very significant impact on the achievement of KPI 5 within the next 1 year. This shows that investments made in improving the quality of lecturers directly have great potential to improve the performance and achievement of universities in the long run. Therefore, universities should consider prioritizing and allocating resources to programs and initiatives that enhance the quality of lecturers, with the expectation that this will positively impact the achievement of higher education key performance indicators (Azmat et al., 2017). These findings can also form a foundation for planning funding policies in education.

The sixth equation shows that the variable $x_5(k)$ has a more significant parameter estimation value against $x_6(k+1)$ than any other KPI variable. This indicates that the extent of lecturers' work utilized by the community or recognized internationally influences the percentage of study programs in collaboration with world-class partners one year later, amounting to 31.38%. Therefore, KPI 6 in the following year is impacted by KPI 5 in the current year. This shows that the achievements and contributions of lecturers who receive recognition or play an active role in various community or international initiatives can help increase the reputation and attractiveness College at national and international levels. As a result, study programs can more easily collaborate with international partners for academic program development and education quality improvement (Highman et al., 2023). The findings could also serve as a foundation for colleges to formulate a strategy that emphasizes leveraging lecturers' work to enhance international collaboration.

The seventh equation shows that the variables $x_5(k)$ and $x_8(k)$ had a significant parameter estimation value against $x_7(k+1)$ than any other KPI variable. This indicates that the extent of lecturers' work utilized by the community or recognized internationally impacts the percentage of collaborative and participatory classes one year later, reaching 65.33%. Consequently, KPI 7 for the following year is influenced by KPI 5 from the current year. This happens because the work of lecturers can be applied in the learning process, which then provides great benefits for students. The more faculty work available, the better the opportunities to engage students in productive, collaborative learning. The learning process can also be directed using a learning model approach such as project-based learning or case study, which supports active, participatory and collaborative learning among students (Chu et al., 2011; Sormunen et al., 2020). These findings provide a basis for universities to better utilize the work of lecturers as a valuable resource in improving the quality of learning.

The eighth equation shows that the variable $x_7(k)$ has a significant parameter estimation value against $x_8(k+1)$ than other KPI variables. This implies that the percentage of collaborative and participatory classes impacts the proportion of undergraduate study programs with international accreditation or certification recognized by the government in the following year, by 16.55%. Therefore, KPI 8 for the next year is influenced by KPI 7 from the current year.

4. Conclusion

This study explains the process of constructing dynamic models at higher education KPI. Dynamic model construction is generated with several stages starting with the definition of KPI variables, assumptions, compartment design, models, model parameter estimation, and model conformity analysis. The result of the model construction is a linear discrete dynamic model consisting of eight simultaneous equations. This model can explain the relationship of as many as 58 model parameters that have been identified with the KPI of universities. Next, this study looks for parameter estimation values that correspond to limited data. Parameter estimation aims to bring the results obtained close to the data so that it is necessary to minimize the difference between variable estimates and data. Therefore, several optimization methods are needed to obtain the parameters in the KPI model equation. This study used three optimization methods to find model parameters. Of the three optimization methods, one suitable method is obtained, namely the Type II constrained optimization method. The parameter estimation value of the Type II constrained optimization method is taken based on two considerations. First, the chi-squared test for goodness of fit is assessed by examining values that are close to zero. Second, the rationality of the estimated parameter values in relation to the KPI is evaluated. Model validation is performed by comparing the actual data with the model's predictions. The results for several KPIs, specifically KPI 1, KPI 2, KPI 3, and KPI 6, closely align with the actual data for 2021 and 2022. However, there are notable discrepancies between the model predictions and actual data for KPIs 4, KPI 5, KPI 7, and KPI 8 during the same period.

Analysis of the equation of the estimated parameter value confirms that the performance of universities on the percentage of getting decent jobs for graduates (KPI 1) in the coming year is strongly influenced by two KPIs, namely the work of lecturers (KPI 5) and study programs that have international accreditation (KPI 8). The existence of active practitioners (KPI 4) also opens opportunities for international cooperation (KPI 6) and collaborative learning (KPI 7). Significant investment in enhancing the quality of lecturers, combined with proper allocation of funds, has demonstrably positively impacted the achievement of higher education goals. Therefore, efforts to improve educational quality should concentrate on developing lecturers, boosting international reputation, and managing resources effectively to support the sustainable advancement of higher education.

Overall, the findings of this study result in a discrete linear dynamic model for Key Performance Indicators (KPIs) along with its parameters. Future research will focus on optimal control calculations and simulating various policy scenarios in the field of education through KPIs.

The Interior Point algorithm, while effective for convex optimization, faces significant limitations, particularly in large-scale problems where sensitivity to initial points and parameter selection can hinder computational efficiency and convergence rates (Chalekh and Djeffal, 2023; Charif and Djeffal, 2024; Tao et al., 2024; Shin et al., 2024). These challenges are exacerbated in complex Key Performance Indicator (KPI) optimization scenarios, where the algorithm may struggle with ill-conditioned problems. Meanwhile, the Quasi-Newton algorithm, although widely used, encounters its own set of issues, particularly in estimating the Hessian matrix. This can lead to poor convergence in non-convex problems, especially when dealing with complex nonlinear relationships among KPI parameters (Chalekh and Djeffal, 2023; Du and You, 2024; Huang et al., 2023). The difficulty in accurately estimating the Hessian can result in suboptimal solutions or slow convergence, particularly in landscapes with multiple local minima (Chalekh and Djeffal, 2023). Thus, both algorithms exhibit critical limitations that can impact their effectiveness in KPI optimization tasks.

The Interior Point algorithm, particularly with parallelization techniques, can be applied to large-scale optimization problems, such as network planning, portfolio optimization, and energy system design

(Balavignesh et al., 2024; Premkumar et al., 2024). This technique enables the resolution of large problems by leveraging parallel computing, thereby enhancing efficiency in analyses that require handling large systems and complex constraints, as well as applications in problems with various types of constraints and large data sizes. Meanwhile, the Quasi-Newton algorithm has wide-ranging applications beyond KPI optimization, such as in optimal control problems, optimal design, and inverse problems across various engineering and scientific fields (Mao and Ghahfarokhi, 2024). For instance, this algorithm is used in the design of complex engineering systems and the development of control strategies for industrial processes, where efficient Hessian estimation can facilitate the achievement of optimal solutions.

Conflict of Interest
The authors confirm that there is no conflict of interest to declare for this publication.

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