

Sustainable Preservation Strategies with Deterioration Management and Environment Sensitive Demand

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Abstract

Sustainable operations are concerned with the eco-friendly system to postulate intergenerational equity on social, economical, and environmental responsibility. This paper addresses the deterioration with a preservation strategy without compromising the environmental performance of the product. Also, the loss due to deterioration is managed completely with the idea of salvage trade. Furthermore, the upcoming demand is assumed to be “price and environment-sensitive”. This model maximizes the seller’s total profit by optimizing the unit selling price and minimizing the investment to maintain the environmental performance of the product and preservation technology at the same time. A numerical example and related graphs are illustrated to validate the model.

Keywords- Environment sensitive demand, Green preservation investment, Deterioration management, Pricing.

1. Introduction

Deterioration is a phenomenon which leads to monetary loss for the industry, but it is a very natural process which cannot be avoidable. Eventually, it can be controlled with preservation technology investment which ultimately comes out to be an additional cost. The very first article addressing the deterioration was conceded by Ghare and Schrader (1963), followed by Jaggi and Aggarwal (1994), Aggarwal and Jaggi (1995), etc. To maintain the usability of the product, preservation technology is a competent tool that contributes towards the better management of the deterioration phenomena noticeably. Even though preservation technology investment adds to the cost components of the system, but on another hand, it reduces the deterioration cost significantly. Mishra (2015) introduced an EOQ model with preservation technology under the assumption of variable demand. Khanna et al. (2020b), Priyamvada et al. (2021) proposed different models with

deteriorating items and preservation concepts with stock-dependent demand.

Further, it is a well-known fact that the price of the item has an immense influence on demand. Thus, selling price-dependent demand has great significance in practical models. Some of the models considering price-sensitive demand are Rabbani et al. (2016), Jaggi et al. (2015) considered the problem of credit-financing for a deteriorating product under the assumption of price-sensitive demand and shortages. Panda et al. (2017) introduced a volume-flexible inventory policy for deteriorating products with price reliant demand. Jayaswal et al. (2020) studied an inventory model under a fuzzy environment.

As per global concern, the researcher integrates environmental factors i.e., carbon emission control, carbon tax policies, etc., in the production and inventory decision models. Guiffrida et al. (2011) proposed an inventory model for the supply chain with an environmental performance measure. Zanoni et al. (2014) considered the model with price and environment-sensitive demand. Marchi et al. (2018) studied the green supply chain with environmental investment. Khanna et al. (2020a) discussed the two different carbon policies. Gilotra et al. (2020) studied an environmentally responsible inventory model.

1.1 Research Gap and Contribution

The present study bridges the gap toward the existing literature by providing an inventory scenario for the deteriorating items and the deterioration rate is controlled with an investment in preservation techniques. It is observed that the preservation process causes carbon emissions which have an adverse effect on the environment. Further, emissions are a major concern to the atmosphere, therefore an investment in preservation techniques is an innovative approach to control the emissions and intact the environmental performance. Due to the customer's consciousness towards the environment firms needs to invest in eco-friendly products. Moreover, the organic and biodegradable waste can be used for the fertilizer industry as raw material, so the firms use the salvage trade to utilize their deteriorated units. Hence, firms generate revenue by salvage trading and at the same time, they support the environment by managing their waste adequately. The main objective is to capitalize on the total profit, by optimizing the unit selling-price, investment in preservation, and the investment to improve the environmental performance of the item. Some significant managerial insights have been explicated from numerical exercise. To restate, the present study addresses the following research queries:

- (i) What is the ideal preservation technology investment for the deteriorating products to intact the environmental performance?
- (ii) What will be the optimum selling price, investment in preservation, investment to improve environmental performance, and the total profit for a sustainable model?
- (iii) What will be the effect of different model parameters on the optimal ordering policy?

2. Assumptions and Notations

This section includes the notations and assumptions used throughout the paper.

To develop the proposed model, the following assumptions are taken into consideration:

- (i) Demand rate of the inventory system is proportionate to the price and environmental performance i.e., $D(p, G) = a - bp + qG$, $a > 0$, $0 < b, q < 1$, and “environmental

performance measure”, $G = \beta \cdot \ln(I)$ where I is the investment to improve the environmental performance of the products and $0 < \beta < 1$.

- (ii) Time horizon will be infinite and negligible lead time is considered.
- (iii) Shortages are not allowed.
- (iv) Deterioration rate is constant throughout the cycle time.
- (v) Investment in preservation technology decreases the deterioration rate with the help of an expression, which holds the subsequent conditions $\partial y(\tau)/\partial \tau < 0$ and $\partial^2 y(\tau)/\partial \tau^2 > 0$. Consequently, this model assumes that $y(\tau) = y_0 e^{-\alpha \tau}$ and $0 < \alpha < 1$ where α is sensitivity parameter of investment.

The notations used in this research are given in Table 1.

Table 1. Notations.

<i>Decision parameters</i>	
p	Unit selling price
τ	Preservation technology investment per unit time
I	Investment to increase the “environmental performance measure” of the product per unit time
<i>Constant parameters</i>	
C_o	Cost of ordering (per order)
$D(p, G)$	Demand rate
h	Holding cost per unit per cycle time
C_p	Purchase cost per unit
G	Product environmental performance measure
q	Demand sensitivity parameter related to the environmental performance of the product
T	Cycle time
a	Demand function intercept
b	Demand sensitivity parameter related to the selling price of the product
q	Demand sensitivity parameter related to the environmental performance of the product
y_0	Deterioration rate without investment in preservation technology
α	Sensitivity parameter of investment where $0 < \alpha < 1$
p_1	Unit selling price for deteriorated units
$y(\tau)$	Deterioration rate with investment in preservation technology
Q	Order quantity
TP	Total profit per unit time
D_t	Deteriorated units

3. Mathematical Modeling

In daily life, consumers are more attentive towards the price and “environmental performance” of the product. Therefore, the environmental performance of the products influences the purchasing

decision of consumers. The demand pattern can be considered as: $D(p, G) = a - bp + qG$, $a, b, q > 0, 0 < b < 1$, and environment performance measure $G = \beta \cdot \ln(I)$. I is the investment to improve the environmental performance of the products and $0 < \beta < 1$.

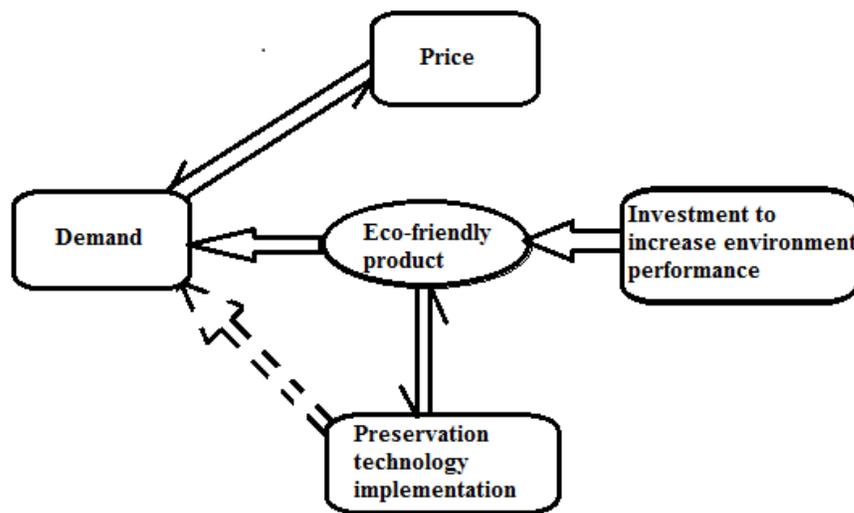


Figure 1. Relationship of demand and different factors which affect the demand.

Figure 1 represents the factors affecting the demand. Price affects the demand inversely and the investment use to improve the “environmental performance” of the item accelerates the demand positively. Preservation techniques influence the demand indirectly because it minimizes the rate of deterioration and hence the quality remains intact for a longer period. Also, the preservation technology affects the environmental performance of products inversely as the techniques used for preservation is not environment friendly but at the same time preservation reduces the loss due to deterioration significantly. Hence, it is a win-win scenario to implement the preservation techniques as well as investment to increase environmental performance. This paper fulfills the research gap by improving the performance of the products and also considers some investments I is a more sophisticated way to reduce the preservation effect.

Consider an inventory scenario where the inventory level depletes with deterioration and demand. The performance of the inventory-level at any time t is presented in Figure 2.

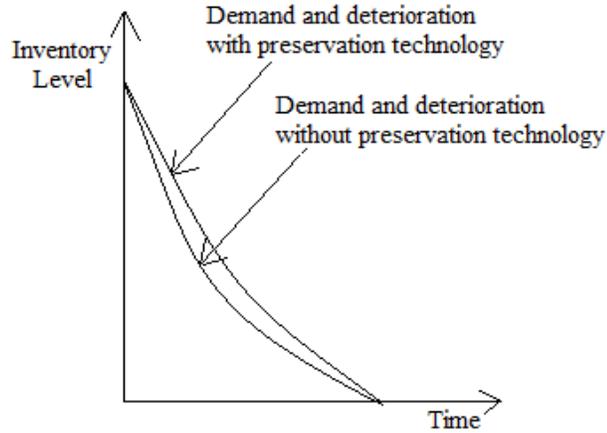


Figure 2. Graphical representation of inventory depletion.

Equation overriding the inventory scenario is assumed as:

$$\frac{dI_1(t)}{dt} + y(\tau)I(t) = -D(p, G), \quad 0 \leq t \leq T \quad (1)$$

After solving the above expression (1) with the condition $I_1(T) = 0$, the inventory level is specified as:

$$I_1(t) = \frac{D(p)}{y(\tau)} \left[e^{y(\tau)(T-t)} - 1 \right] \quad (2)$$

Thus, the initial inventory is:

$$Q = I_1(0) = \frac{D(p, G)}{y(\tau)} \left[e^{y(\tau)T} - 1 \right] \quad (3)$$

and the deteriorated units are:

$$D_t = \frac{D(p, G)}{y(\tau)} \left[e^{y(\tau)T} - 1 - Ty(\tau) \right] \quad (4)$$

The total cost of the system is summing up of the given components:

- Ordering cost = C_o (5)

- Purchase cost = $C_p Q$ (6)

- Holding cost = $h \int_0^T I_1(t) dt = \frac{hD(p, G)}{(y(\tau))^2} [e^{y(\tau)T} - T(y(\tau)) - 1]$ (7)

- Preservation technology investment with investment used for better environmental performance
= $\tau T + \frac{I}{D(p, G)}$ (8)

Summing equations (5)-(8), the total cost of the system will be:

$$= \frac{C_o}{T} + \frac{hD(p, G)}{T(y(\tau))^2} [e^{y(\tau)T} - T(y(\tau)) - 1] + \tau + \frac{I}{TD(p, G)} + \frac{C_p Q}{T} \quad (9)$$

Here revenue can also be generated from the deteriorated units. The present paper deals with the food items, so the deteriorated items can be used for the production of bio-degradable organic fertilizer. Reselling the deteriorated units to such industries is much more profitable since the model considers the items which have improved environmental performance. Hence the total revenue is,

$$\text{Total revenue per unit time (TR)} = p D(p, G) + \frac{p_1 D_t}{T} \quad (10)$$

Total profit of the system will be (TP)

$$= p D(p, G) + \frac{p_1 D_t}{T} - \frac{C_o}{T} - \frac{hD(p, G)}{T(y(\tau))^2} [e^{y(\tau)T} - T(y(\tau)) - 1] - \tau - \frac{I}{TD(p, G)} - \frac{C_p Q}{T} \quad (11)$$

Substitute the value of Q from equation (3) and Since $e^{y(\tau)T} < 1$, after the approximation of exponential function up to second order, eq. (11) reduces,

$$TP = p D(p, G) + \frac{p_1 D(p, G) y(\tau) T}{2} - \frac{C_o}{T} - \frac{hD(p, G) T}{2} - \tau - \frac{I}{TD(p, G)} - \frac{C_p}{T} \left(\frac{D(p, G) T}{2} [2 + y(\tau) T] \right) \quad (12)$$

4. Optimality and Solution Procedure

The main objective of this article is to capitalize on the profit of the present model by mutually optimizing the investment (I), the total investment in preservation technology (τ), and unit selling price (p). To prove optimality, the necessary and sufficient conditions are following.

Necessary conditions are:

$$\frac{\partial TP}{\partial \tau} = 0, \frac{\partial TP}{\partial I} = 0 \text{ and } \frac{\partial TP}{\partial p} = 0 \quad (13)$$

$$\frac{\partial TP}{\partial \tau} = \frac{\alpha D(p, G)y(\tau)(C_p + T^2)}{2} - 1 \quad (14)$$

$$\frac{\partial TP}{\partial p} = -\frac{b}{T(D(p, G))^2} - 2bp + \frac{bC_p(y(\tau)T + 2)}{2T} - \frac{by(\tau)T^2}{2} + \frac{bhT}{2} + a + qG \quad (15)$$

$$\frac{\partial TP}{\partial I} = \frac{\beta q}{IT(D(p, G))^2} - \frac{\beta q C_p(y(\tau)T + 2)}{2IT} - \frac{h\beta q T}{2I} + \frac{\beta pq}{I} \quad (16)$$

After solving equations (14), (15), and (16) the optimal values of the investment I^* , investment in preservation technology τ^* , and unit selling price p^* can be calculated. Furthermore, for the optimality and specially sufficiency conditions, the given lemmas are being used,

Lemma 1. “For a fixed investment to improve the environmental performance I^* and preservation technology investment τ^* , there exists a unique unit selling price p^* which maximizes the total profit”.

Proof: Since,
$$\frac{\partial^2 TP}{\partial p^2} = -\frac{2b^2}{T(D(p, G))^3} - 2b = \frac{-2b(b + T(D(p, G))^3)}{T(D(p, G))^3} < 0 \quad (17)$$

This is negative because b can not be negative and hence prove the optimality of the total profit.

Lemma 2. “For a fixed investment to improve the environmental performance I^* and unit selling price p^* there exists a unique preservation technology investment τ^* , which maximizes the total profit”.

Proof: Since,
$$\frac{\partial^2 TP}{\partial \tau^2} = -\frac{\alpha^2 D(p, G)y(\tau)(C_p + T^2)}{2} < 0 \quad (18)$$

which is negative and hence proves the concavity of the total profit expression.

Lemma 3. “For a fixed unit selling price p^* and preservation technology investment τ^* , there exists a unique investment to improve the environmental performance I^* which maximizes the total profit”.

Proof:

Since
$$\frac{\partial^2 TP}{\partial I^2} = -\frac{\beta q}{TI^2(D(p, G))^2} - \frac{2\beta^2 q^2}{TI^2(D(p, G))^3} - \frac{T\beta q(2p - hT) - \beta q C_p(Ty(\tau) + 2)}{2TI^2} < 0, \quad (19)$$

which is negative and hence establishes the optimality of the total profit expression.

5. Numerical Example

The parametric values are given as follows:

$$T = 3 \text{ months}, C_0 = \$80/\text{order}, a = 100, b = 0.5, C_p = \$50/\text{unit}, h = \$1/\text{unit}/\text{time}, \\ y_0 = 0.08, \beta = 0.5, \alpha = 0.05, q = 0.05$$

Using equations (14), (15) and (16) the optimal values of the investment to improve the environmental performance I^* , investment in preservation technology τ^* , and unit selling price p^* are calculated. Further, substituting these values in equation (12), the optimal total profit TP^* is obtained. The result is recorded as follows:

Investment to improve environmental performance (I^*) = \$198.083, Selling Price (p^*) = \$127.585, Investment in preservation technology (τ^*) = \$28.384, Total Profit (TP^*) = \$2572.985.

6. Sensitivity Analysis

In this section, sensitivity analysis has been performed to study the effect of key parameters such as demand parameters (a) and (b), demand sensitivity parameter related to the environmental performance of the product (q) and holding cost (h) on the optimal solution. Using the numerical data given in section 5, results have been recorded in Table 2.

Table 2. Impact of various parameters on the optimal solution.

Parameter	Values	I	τ	p	TP	Q
a	80	101.496	01.498	108.205	1325.242	88.298
	90	146.035	14.897	117.844	1897.500	103.987
	100	198.083	28.384	127.585	2572.985	119.489
	110	257.640	40.178	137.389	3350.867	134.872
	120	324.711	50.673	147.235	4230.608	150.176
b	0.3	439.127	41.189	194.137	5766.484	136.278
	0.4	287.224	35.025	152.506	3751.394	127.927
	0.5	198.083	28.384	127.585	2572.985	119.489
	0.6	140.318	21.171	111.031	1813.262	110.938
	0.7	100.477	13.256	99.273	1293.197	102.240
h	0.25	200.246	27.270	127.006	2616.632	120.568
	0.50	200.189	27.645	127.200	2602.041	120.205
	1.00	198.083	28.384	127.585	2572.985	119.489
	1.25	196.942	28.747	127.777	2558.519	119.132
	1.50	195.987	29.105	127.970	2544.095	118.775
q	0.03	118.614	28.296	127.536	2569.313	119.38
	0.04	158.309	28.339	127.56	2571.102	119.433
	0.05	198.083	28.384	127.585	2572.985	119.489
	0.06	237.947	28.431	127.611	2574.941	119.546
	0.07	277.904	28.478	127.638	2576.96	119.604

From Table 2 the following observations and managerial insights are made:

- (i) When the constant component of the demand function, i.e., (a) increases the profit levels are sure to grow significantly. Also, with higher demand, unit selling price (p), investment in preservation technology (τ) and (I) will increase significantly.

- (ii) With an amplified demand parameter (b) the total demand of the system will decrease which results in lower profit. To overcome this situation the retailer has to reduce the selling price and also the investment in preservation and the investment related to the environmental factor.
- (iii) When the holding cost (h) increases, the investment required in preservation (τ) and the total profit decreases. To combat the effect of increasing cost associated with holding the inventory, the retailer lowers the investment in preservation, and the investment (I) increases the environmental performance.
- (iv) Increase in the parameter related to the environmental performance (q) leads to an increase in investment related to the environment performance (I) which results into higher demand and greater profit. It gives the insight to increase the order size in such a scenario.

When the base demand has reached maturity and is on increasing trend, the retailers can take it positively, and to maintain the quality of the product, investment in preservation technology will increase significantly. Moreover, with higher demand, one has to maintain the environmental performance of the product to match up the standards of customers. This allows an increase in the selling price. Again, to combat the effect of increasing cost associated with holding the inventory, the retailer lowers the investment in preservation technology and hence a decline in total profit. Since the overall increase in total cost needs to be balanced with other significant investments, it becomes viable to reduce a certain amount of investment in environmental performance.

7. Conclusions

In today's global decision-making, consumers are very concerned about the environment and prefer eco-friendly products. Thus, the firms make eco-efficiency an important priority for them to generate more revenue while reducing emissions. Hence, this study summarizes the EOQ model with sustainable preservation techniques with an additional investment to increase the environmental performance of the item. An inventory scenario for the deteriorating items has been considered to study the result of investment in environmental concern and preservation under the assumptions of environment and price-sensitive demand. The customers prioritize environmental factors in addition to the price while purchasing any product and this puts an extra financial burden on the seller to invest in different practices and techniques to maintain the sustainable system. Preservation techniques help to reduce the deterioration, but can't stop it completely. Thus, the idea to trade the deteriorated units for their further utilization in the production of organic fertilizer is a step towards sustainability, and also helps in fetching extra revenue. This facilitates the retailer to improvise their profit instead of investing an additional cost to dispose that deteriorated units. Thus, this paper altogether provides new dimensions to the implementation of preservation technology along with environmental performance for deteriorating items. The key findings are concluded as:

- (i) The environmental aspects are taken into consideration by incorporation of its impact on demand and improvisation of preservation techniques with the help of additional investment.
- (ii) For higher holding costs, it is suggested to increase the investment in preservation technology and prefers to order the small lots to manage the inventory effectively.
- (iii) Investment in preservation technology helps in reducing deterioration which supports the environment significantly.
- (iv) The deteriorated units are used to make some fertilizers which is a great initiative towards sustainability and at the same time, it fetches extra revenue which motivates the retailers to utilize the deteriorated products in the right way.

For future studies, this model can be extended in various ways. For instance, the model can allow shortages for cases of complete and partial backlogging. Further, the demand can be considered as stock-dependent, advertisement-dependent, or stochastic. Also, the occurrence of defective items, credit-linked demand, and inflation on economic policies can be considered. Finally, changes in optimal policies owing to carbon emission consideration can be studied. The focus of the study is limited to organic and biodegradable products.

Conflict of Interest

On behalf of all the authors, the corresponding author declares that there is no conflict of interest.

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