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Reliability Centered Modeling of an Automated Waste Sorting Robotic Arm System

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Abstract
In contemporary society, proper waste management is a critical issue that needs to be addressed. Waste management encompasses the collection, transportation, segregation and disposal of waste material to minimize health risks and environment impact. Manual segregation, along with other methods of waste segregation, is often inefficient and time consuming. To address this challenge, numerous industries have embraced automation in waste sorting to expedite the process through advanced technological interventions. Therefore, the optimal functioning of the components of these automated systems is crucial for rapid and accurate segregation of waste. This work is oriented toward the study of a system which is used to segregation of waste and known as waste sorting robotic arm system (WSRAS). Author used Markov decision process, along with mathematical modeling, to evaluate the different performance measure of WSRAS. Sensitivity analysis have also been done to understand the contribution of different components failure in overall performance of WSRAS.

Key Words- WSRAS, Markov decision process, Sensitivity analysis, Expected profit function.

1. Introduction

As a futuristic society, it's very much needed to pay attention towards the sustainable and reliable arrangement of the waste that come from day-to-day life as well as from industrial outputs. Waste sorting robotic arm system (WSRAs) is one of the dedicated systems which deals with sorting the waste with the help of a robotic arm. In literature the different way of systematic waste disposal was discussed (Geethamani et al., 2021; Karajvandani et al., 2024). It can be easily seen at different places that garbage is dumbed without proper segregation which is not good for the society as well as the individuals who are involved in this. This can affect the fertility of the sounding land as well as a big challenge for local authorities. So, it becomes very important to manage this waste effectively through different methodology.

Some authors have proposed the concept of the 4 R's: Refuse, Reuse, Recycle and Recover, as a strategy for sustainable waste management (Ravindra et al., 2016; Thakur et al., 2018; Gautam and Bhadra, 2023).

- Refuse- Say NO to items that are not needed.
- Reuse- Use each as many times as possible.

- Recycle- It means the items which are thrown in the garbage should be properly segregated so that they can be easily recycled.
- Recover- It means to get those items from the garbage which can be sold or which can be useful to someone else.

India's expanding population, rapid urbanization, and industrial growth have made waste management a significant environmental and social concern. According to estimates, India produces 62 million tons of trash annually, and this amount is projected to rise sharply in the years to come. Effective waste management is essential for addressing problems such as resource depletion, public health threats, and environmental degradation. It has been determined that about 96 per cent of the total waste originates from residential zones, while the remaining four per cent is generated from mixed zones which include industries, construction activities, agriculture operations, deceased animals, hospitals, and other miscellaneous sources.

As a result of population growth, the incremental change in the amount of garbage put scientists and researchers to think on some technology-based solution e.g. sensor-based system, systems inculcated with robotic arms, smart trash bins installations, AI based systems and many more, to manage and recycle the waste (Kumar et al., 2017; Chakraborty et al., 2021). These AI-IOT based system can monitor and provide the real time information related to garbage levels at different location and which can be latter managed by the authorities. Also based on the information the garbage can be classified in different category and disposed/used accordingly. In literature, it is suggested to use sensor-based system to differentiate the waste into different category as well to send the alter (Jain and Bagherwal, 2017; Sharanya et al., 2017; Sathish et al., 2017; Vamsi et al., 2021). Shyam et al. (2017) developed an IoT approach using sensors to provide a comprehensive garbage collection management solution by equipping trash cans with AI. It is capable of gathering, sending, and reading a sizable amount of data through the Internet. A smart alert ultrasonic sensor-based system was developed by Dugdhe et al. (2016). The system sent the alert about the garbage level to the server for further action. GSM technology through IR sensor is used by Bhor et al. (2015), to monitor the garbage level of smart dustbins. If garbage is not collected within the stipulated time period, a record of this is automatically sent to the authorities, who can take action.

Some authors have emphasized the use of robots for waste sorting over IoT-based sorting because robots can handle huge volume of waste with great accuracy and speed (Bansal et al., 2019; Sunil et al., 2020; Gupta and Tripathi, 2021). Rokade et al. (2018) developed a smart garbage separator robot capable of sorting waste into degradable and non-degradable categories. When the obstacle sensor is triggered, the camera is actuated and through image processing the type of waste is detected. The robot can receive and react to signals sent via a remotely controlled PC. The trash bin is mounted on the robot and with the use of an arm waste is dumped into the designated space on the robot.

The Importance of deep learning, through YOLOV3, in waste sorting was discussed by Ugale et al. (2021). The system, under study, segregate the waste into five different categories namely plastic, wet, dry, and lethal waste. After segregation the waste will be dumped into different bins as per the category. The efficacy of stochastic processes in modeling of industrial systems such as turnstile and waste data collection sensors were already shown in literature (Kumar and Kumar, 2021; Kumar and Kumar, 2023). Developing a reliability model for an engineering system is crucial to gauge the performance of the system at any time (Forghani-Elahabad and Kagan, 2019, Hao et al., 2020). Also, Forghani-Elahabad (2022) developed and illustrated algorithms for assessing the reliability of sending commodities through minimal and disjoint minimal paths in a multistate flow network, providing complexity results and addressing related optimization problems.

It is clear from the above research studies that no prior work has developed through Markov decision process to determine the reliability indices of a waste sorting robotic arm system. Therefore, the authors propose a Markov process-based modeling to build a model of the waste sorting robotic arm system. The aim is to determine various reliability measures for the same. Additionally, the authors perform a sensitivity analysis to identify the critical components of the system. Furthermore, the authors analyze and present the expected profitability generated by the system, providing insight into its economic viability and potential returns on investment.

The novelty of this paper lies in the development of a mathematical model that determines the reliability measures of a waste segregation system and identifies its weakest components; an aspect not yet addressed by previous research. While many authors have developed various applications and instruments for sorting waste, none have created a model focused on system's reliability. Our model also aids the plant in planning the maintenance activity in a timely manner to prevent system failures.

With this problem in mind, our goal is to determine the several reliability indices of the WSRAS using reliability approach, which has never been explored previously. The robotic arms proposed in this paper efficiently sort two types of objects- plastic and metal- with high accuracy and speed.

2. System Description

Garbage segregation employs various advanced technologies, including robotic arms. Robotic arms are utilized to separate papers, plastics, glass, and metals from the waste materials effectively. The system must operate correctly when in use to ensure effective waste segregation. Any failure in system components such as conveyor belt, vision system, robotic arms, control system or power supply, lowers the systems' performance, impacting revenue generation of the waste treatment plant. Therefore, it is imperative to determine the various reliability measures of the system. By doing so, the overall system performance can be improved through timely repair and replacement of system components. The considered WSRAS consists of conveyor belt, Vision system, Control system and Robotic arm as its major components. By doing so, the overall system performance can be improved through timely repair and replacement of system components. The considered WSRAS consists of conveyor belt, Vision system, Control system and Robotic arm as its major components. The complete/partial failure of theses unit impact the overall performance of WSRAS.

3. State Description and Notations

3.1 State Description

The details of all the possible states, in which the system can be present during the operation, of the waste sorting robotic arm systems are given in **Table 1**.

Table 1. States of the system.

States	Description
S_0	PS with all the components of the system working perfectly
S_1	FS due to the failure of the Conveyor belt
S_2	FS due to the failure of the Control System
S_3	FS due to the failure of the Vision System
S_4	DS due to the failure of first robotic arm system
S_5	FS due to the failure of a conveyor belt after the failure of the first robotic arm system
S_6	FS due to the failure of a control system after the failure of the first robotic arm system
S_7	FS due to the failure of a vision system after the failure of the first robotic arm system
S_8	FS due to the failure of the second robotic arm system
S_9	FS due to the failure of a conveyor belt after the failure of the second robotic arm
S_{10}	FS due to the failure of a control system after the failure of the second robotic arm system
S_{11}	FS due to the failure of a vision system after the failure of the second robotic arm system
S_{12}	FS due to the failure of both the robotic arms systems

3.2 Notations

The notation used in the paper are given in **Table 2** below.

Table 2. Notations of the model.

Symbols	Description
t	Time variable
s	Laplace transformation variable
$P_i(t)$	Probability of the system being in the state S_i , i=0, 1, …. ,12
$\overline{P_i}(s)$	Laplace transformation of $P_i(t)$
λ_{cyb}	Represents the failure rate of the conveyor belt.
λ_{cs}	Represents the failure rate of the control system
λ_{vs}	Represents the failure rate of the vision system
λ_{R1}	Represents the failure rate of the first robotic arm system
λ_{R2}	Represents the failure rate of the second robotic arm system
μ_{cyb}	Represents the failure rate of the conveyor belt.
μ_{cs}	Represents the failure rate of the control system
μ_{vs}	Represents the failure rate of the vision system
μ_{R1}	Represents the failure rate of the first robotic arm system
μ_{R2}	Represents the failure rate of the second robotic arm system
PS/FS/FS	Perfect state/Degraded state/Failed state

4. Transition State Diagram of WSRAS

Based on various potential failures and repair that can occur during operation, a transition state diagram of the waste segregation system has been developed (**Figure 2**). This stochastic modeling approach is used in many applications to develop state-based model of the system. In stochastic modeling, any engineering system can be found in one of the following states at any given time: "***perfect working***", "***degraded***" or "***failed***". For the proposed waste segregation system, the authors define the following states of the system. The initial state is the S_0 state which is the perfect working state of the system. The state S_4 and S_8 denote the degraded state of the system. In these states, due to the failure of one of the robotic arms, the system functions at a reduced efficiency. The remaining states of the system ($S_1, S_2, S_3, S_5, S_6, S_7, S_9, S_{10}, S_{11}, S_{12}$) represent the failed states. In these states, the system is completely inoperable. One example is presented in the following **Table 3** and **Figure** 1. Components of the system have only two states "working" or "failed". But system has three possible states namely perfectly working, degraded states, and failed state. The perfectly working state is represented by the numerical value 1. In this state, all the components of the system are functioning correctly. A degraded state occurs when some components of the system have failed but the system continues to operate at reduced efficiency, indicated by table values between 0 and 1. The failed state is represented by a numerical value of 0, indicating complete failure.

Table 3. Possible combinations of components and system states.

Time (days)	Component 1	Component 2	Component 3	Component 4	Component 5	System's performance
0	1	1	1	1	1	1.0
50	1	1	1	1	0	0.8
100	1	1	1	0	0	0.5
150	1	1	1	0	0	0.5
200	1	1	0	0	0	0.4
250	1	1	0	0	0	0.4
300	1	0	0	0	0	0.2
350	1	0	0	0	0	0.2
400	1	0	0	0	0	0.2
450	0	0	0	0	0	0.0
500	0	0	0	0	0	0.0

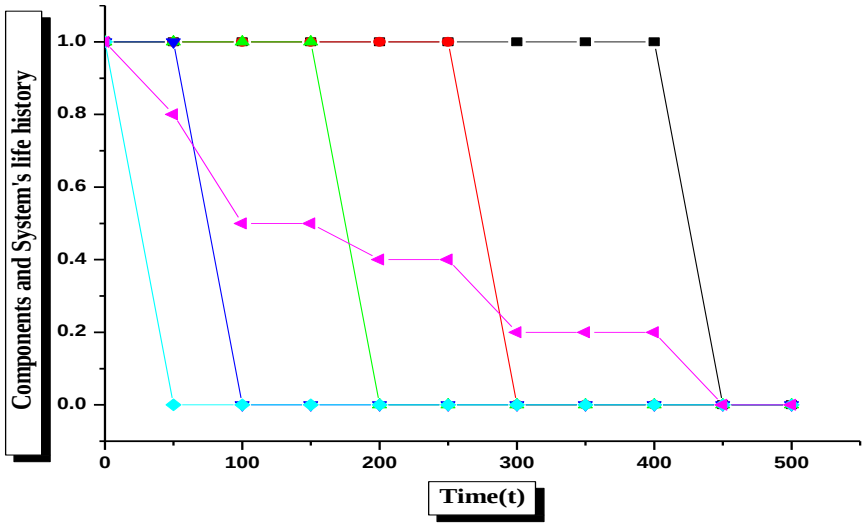

Figure 1. Possible combinations of components and system states.

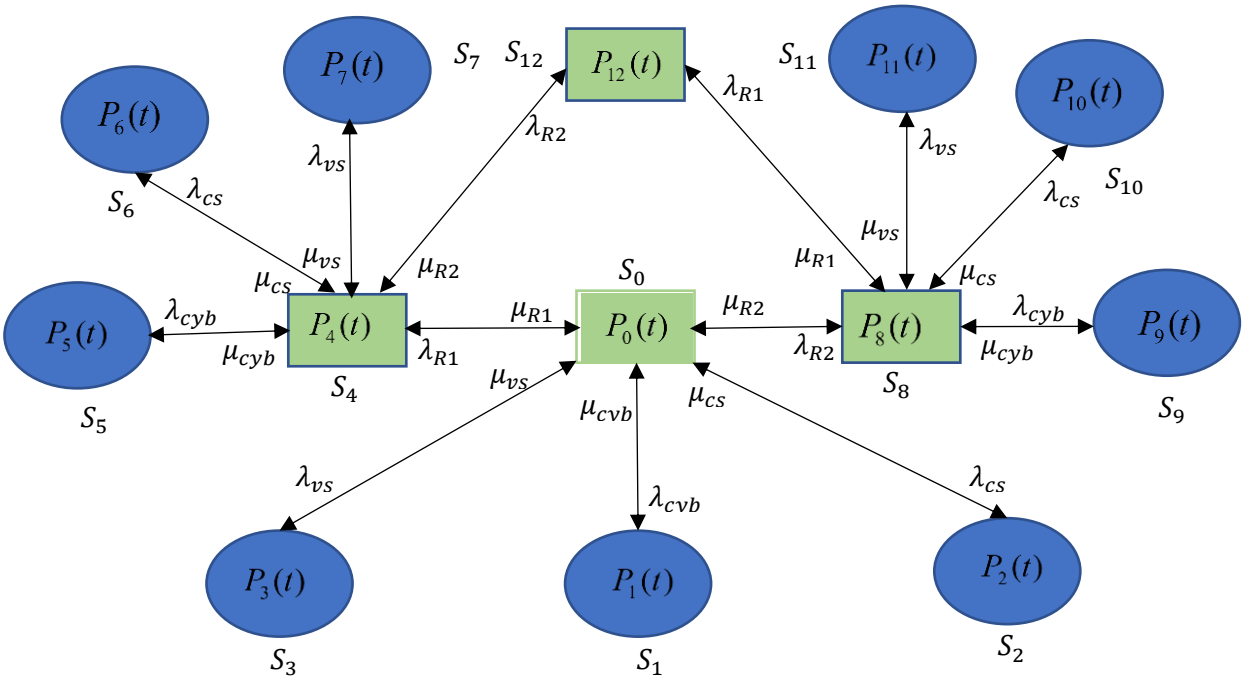

Figure 2. Transition state diagram of WSRAS.

5. Mathematical Formulations and Solution

Based on the transition state diagram (**Figure 2**), the following sets of Chapman-Kolmogorov differential equations are generated to govern the present mathematical model.

$$\left[\frac{d}{dt}+\lambda_{cyb}+\lambda_{cs}+\lambda_{vs}+\lambda_{R_1}+\lambda_{R_2}\right]P_0(t)=\mu_{cyb}P_1(t)+\mu_{cs}P_2(t)+\mu_{vs}P_3(t)+\mu_{R_1}P_4(t)+\mu_{R_2}P_8(t) \quad (1)$$

$$\left[\frac{d}{dt}+\mu_{cyb}\right]P_1(t)=\lambda_{cyb}P_0(t) \quad (2)$$

$$\left[\frac{d}{dt}+\mu_{cs}\right]P_2(t)=\lambda_{cs}P_0(t) \quad (3)$$

$$\left[\frac{d}{dt}+\mu_{vs}\right]P_3(t)=\lambda_{vs}P_0(t) \quad (4)$$

$$\left[\frac{d}{dt}+\lambda_{cyb}+\lambda_{cs}+\lambda_{vs}+\lambda_{R_2}+\mu_{R_1}\right]P_4(t)=\mu_{cyb}P_5(t)+\mu_{cs}P_6(t)+\mu_{vs}P_7(t)+\mu_{R_2}P_{12}(t)+\lambda_{R_1}P_0(t) \quad (5)$$

$$\left[\frac{d}{dt}+\mu_{cyb}\right]P_5(t)=\lambda_{cyb}P_4(t) \quad (6)$$

$$\left[\frac{d}{dt}+\mu_{cs}\right]P_6(t)=\lambda_{cs}P_4(t) \quad (7)$$

$$\left[\frac{d}{dt}+\mu_{vs}\right]P_7(t)=\lambda_{vs}\,P_4(t) \quad (8)$$

$$\left[\frac{d}{dt}+\lambda_{R_2}+\lambda_{vs}+\lambda_{cs}+\lambda_{cyb}+\mu_{R_2}\right]P_8(t)=\mu_{R_2}P_{12}(t)+\mu_{vs}P_{11}(t)+\mu_{cs}P_{10}(t)+\mu_{cyb}P_9(t)+\lambda_{R_2}P_0(t) \quad (9)$$

$$\left[\frac{d}{dt}+\mu_{cyb}\right]P_9(t)=\lambda_{cyb}P_8(t) \quad (10)$$

$$\left[\frac{d}{dt}+\mu_{cs}\right]P_{10}(t)=\lambda_{cs}P_8(t) \quad (11)$$

$$\left[\frac{d}{dt}+\mu_{vs}\right]P_{11}(t)=\lambda_{vs}P_8(t) \quad (12)$$

$$\left[\frac{d}{dt}+\mu_{R_2}\right]P_{12}(t)=\lambda_{R_2}P_8(t) \quad (13)$$

with initial condition

$$P_i(0)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases} \quad (14)$$

On taking Laplace inverse of Equations (1)-(13), (Kumar and Kumar, 2021), we obtain the following set of equations,

$$[s+\lambda_{cyb}+\lambda_{cs}+\lambda_{vs}+\lambda_{R_1}+\lambda_2]\bar{P}_0(s)=1+\mu_{cyb}\bar{P}_1(s)+\mu_{cs}\bar{P}_2(s)+\mu_{vs}\bar{P}_3(s)+\mu_{R_1}\bar{P}_4(s)+\mu_{R_2}\bar{P}_8(s) \quad (15)$$

$$[s+\mu_{cyb}]\bar{P}_1(s)=\lambda_{cyb}\bar{P}_0(s) \quad (16)$$

$$[s+\mu_{cs}]\bar{P}_2(s)=\lambda_{cs}\bar{P}_0(s) \quad (17)$$

$$[s+\mu_{vs}]\bar{P}_3(s)=\lambda_{vs}\bar{P}_0(s) \quad (18)$$

$$[s+\lambda_{cyb}+\lambda_{cs}+\lambda_{vs}+\lambda_{R_2}+\mu_{R_1}]\bar{P}_4(s)=\mu_{cyb}\bar{P}_5(s)+\mu_{cs}\bar{P}_6(s)+\mu_{vs}\bar{P}_7(s)+\mu_{R_2}\bar{P}_{12}(s)+\lambda_{R_1}\bar{P}_0(s) \quad (19)$$

$$[s+\mu_{cyb}]\bar{P}_5(s)=\lambda_{cyb}\bar{P}_4(s) \quad (20)$$

$$[s+\mu_{cs}]\bar{P}_6(s)=\lambda_{cs}\bar{P}_4(s) \quad (21)$$

$$[s+\mu_{vs}]\bar{P}_7(s)=\lambda_{vs}\bar{P}_4(s) \quad (22)$$

$$[s+\lambda_{R_2}+\lambda_{vs}+\lambda_{cs}+\lambda_{cyb}+\mu_{R_2}]\bar{P}_8(s)=\mu_{R_2}\bar{P}_{12}(s)+\mu_{vs}\bar{P}_{11}(s)+\mu_{cs}\bar{P}_{10}(s)+\mu_{cyb}\bar{P}_9(s)+\lambda_{R_2}\bar{P}_0(s) \quad (23)$$

$$[s+\mu_{cyb}]\bar{P}_9(s)=\lambda_{cyb}\bar{P}_8(s) \quad (24)$$

$$[s+\mu_{cs}]\bar{P}_{10}(s)=\lambda_{cs}\bar{P}_8(s) \quad (25)$$

$$[s+\mu_{vs}]\bar{P}_{11}(s)=\lambda_{vs}\bar{P}_8(s) \quad (26)$$

$$\left[s+\mu_{R_2}\right]\bar{P}_{12}(s)=\lambda_{R_2}\bar{P}_8(s) \tag{27}$$

$$\bar{P}_{12}(s)=\frac{\lambda_{R_2}}{S+\mu_{R_2}}\ \bar{P}_8(s) \tag{28}$$

$$\bar{P}_{11}(s)=\frac{\lambda_{vs}}{S+\mu_{vs}}\ \bar{P}_8(s) \tag{29}$$

$$\bar{P}_{10}(s)=\frac{\lambda_{cs}}{S+\mu_{cs}}\ \bar{P}_8(s) \tag{30}$$

$$\bar{P}_9(s)=\frac{\lambda_{cyb}}{S+\mu_{cyb}}\ \bar{P}_8(s) \tag{31}$$

$$\bar{P}_8(s)=\frac{\lambda_{R_2}}{\mathrm{Y}_0}\ \bar{P}_0(s) \tag{32}$$

$$\bar{P}_7(s)=\frac{\lambda_{vs}}{S+\mu_{vs}}\ \bar{P}_4(s) \tag{33}$$

$$\bar{P}_6(s)=\frac{\lambda_{cs}}{S+\mu_{cs}}\ \bar{P}_4(s) \tag{34}$$

$$\bar{P}_5(s)=\frac{\lambda_{cyb}}{S+\mu_{cyb}}\ \bar{P}_4(s) \tag{35}$$

$$\bar{P}_4(s)=\frac{Y_2}{Y_1}\ \bar{P}_0(s) \tag{36}$$

$$\bar{P}_3(s)=\frac{\lambda_{vs}}{S+\mu_{vs}}\ \bar{P}_0(s) \tag{37}$$

$$\bar{P}_2(s)=\frac{\lambda_{cs}}{S+\mu_{cs}}\ \bar{P}_0(s) \tag{38}$$

$$\bar{P}_1(s)=\frac{\lambda_{cyb}}{S+\mu_{cyb}}\ \bar{P}_0(s) \tag{39}$$

$$\bar{P}_0(s)=\frac{1}{Y_3} \tag{40}$$

where,

$$Y_0=\left[s+\lambda_{R_2}+\lambda_{vs}+\lambda_{cs}+\lambda_{cyb}+\mu_{R_2}-\frac{\mu_{R_2}\lambda_{R_2}}{s+\mu_{R_2}}-\frac{\mu_{vs}\,\lambda_{vs}}{s+\mu_{vs}}-\frac{\mu_{cs}\,\lambda_{cs}}{s+\mu_{cs}}-\frac{\mu_{cyb}\,\lambda_{cyb}}{s+\mu_{cyb}}\right] \tag{41}$$

$$Y_1=\left[s+\lambda_{cyb}+\lambda_{cs}+\lambda_{vs}+\lambda_{R_2}+\mu_{R_1}-\frac{\mu_{cyb}\lambda_{cyb}}{s+\mu_{cyb}}-\frac{\mu_{cs}\,\lambda_{cs}}{s+\mu_{cs}}-\frac{\mu_{vs}\,\lambda_{vs}}{s+\mu_{vs}}\right] \tag{42}$$

$$Y_2=\left[\frac{\mu_{R_2}\lambda_{R_2}}{s+\mu_{R_2}}\ \frac{\lambda_{R_2}}{Y_0}+\lambda_{R_1}\right] \tag{43}$$

$$Y_3=s+\lambda_{cyb}+\lambda_{cs}+\lambda_{vs}+\lambda_{R_1}+\lambda_{R_2}-\frac{\mu_{cyb}\lambda_{cyb}}{s+\mu_{cyb}}-\frac{\mu_{cs}\,\lambda_{cs}}{s+\mu_{cs}}-\frac{\mu_{vs}\,\lambda_{vs}}{s+\mu_{vs}}-\frac{\mu_{R_1}\,Y_2}{Y_1}-\frac{\mu_{R_2}\,\lambda_{R_2}}{Y_0} \tag{44}$$

Also from **Figure 2**, WSRAS up state and doen state probabilities can be obtained as given in Equation (45) and Equation (46) respectively.

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_4(s)+\bar{P}_8(s) \tag{45}$$

$$\bar{P}_{down}(s)=\sum_{i=1}^{12}\bar{P}_i(s)-(\bar{P}_4(s)+\bar{P}_5(s)+\bar{P}_8(s)) \tag{46}$$

$$\bar{P}_{up}(s)+\bar{P}_{down}(s)=\frac{1}{S} \tag{47}$$

6. Performance Measures Analysis

In this section, the authors present various reliability parameters of the system. For model validation, data has been collected from the industry to calculate the various reliability measures. Additionally, experts opinions have been gathered to gain a thorough understanding of the system's operation.

6.1 Availability of the WSRAS

The availability of the waste segregation robotic arm system is a measure of the system's preparedness and dependability to perform trash sorting and segregation activities. It is the percentage of time that the robotic arm system is operational and actively involved in the sorting and segregation process compared to the entire time it is projected to be available for this purpose. This metric considers uptime, downtime due to maintenance or unforeseen failures, and any scheduled or unscheduled interruptions in the waste segregation process. To calculate availability of the system, set the failure rates as $\lambda_{cyb} = 0.004$, $\lambda_{vs} = 0.01$, $\lambda_{cs} = 0.002$, $\lambda_{R_1} = 0.003$, $\lambda_{R_2} = 0.0025$ and repair rate as $\mu_{cyb} = 1, \mu_{vs} = 0.5, \mu_{cs} = 0.5, \mu_{R_1} = 0.5, \mu_{R_2} = 0.33$ in Equation (45).

Now taking inverse laplace transformation of Equation (45), one can easily obtain the expression for the availability $A(t)$ of the system. The availability of the puposed system is given in Equation (48).

$$\begin{aligned} A(t) = {} & -7.10229630\ .(10)^{-6}.e^{-1.008491664t} - 4.241276328\ .(10)^{-6}.e^{-1.008362680t} + 0.00416894\ 1487 \\ & e^{-1.00409358\ t} - 0.0001849184\ 11e^{-0.587852143t} - 4.17780297\ (10)^{-8}e^{-0.578239968\ 5t} + 0.01974 \\ & e^{-0.510049783\ 1t} - 9.5362232\ (10)^{-6}e^{-0.440500366t} + 0.0001157811\ 19e^{-0.431279344\ 3t} + 0.00533 \\ & e^{-0.331883422\ 7t} + 2.1282903\ (10)^{-6}e^{-0.325699704\ 4t} + 0.0000637035\ 1313e^{-0.322056450\ 2t} + 0.97 \\ & e^{-9.112206182(10)^{-6}t} \end{aligned} \tag{48}$$

By varying the time unit, one can obtain the following **Table 4** and **Figure 3**, which show the availability of the system over time.

Table 4. Availability of the WSRAS w.r.t time.

Time (days)	Availability $A(t)$
0	1
50	0.97
100	0.97
150	0.97
200	0.97
250	0.97
300	0.97
350	0.97
400	0.97
450	0.97
500	0.97

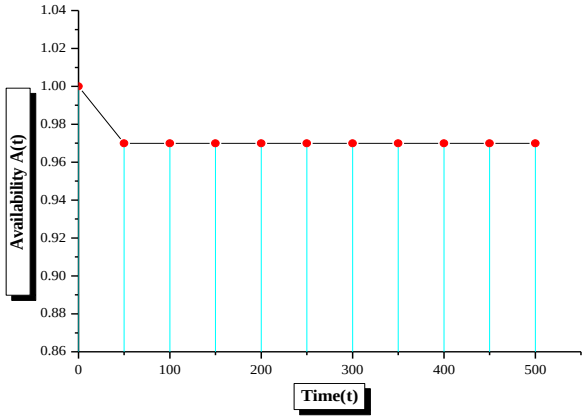

Figure 3. Availability of the WSRAS w.r.t time.

6.2 Reliability of the WSRAS

Reliability of the waste sorting robotic system is the probability that the system will perform its intended task for a specified period of time when operated under prescribed conditions. Mathematically, reliability is defined as the system cannot fail before the time period $'t'$. The same is expressed using the following equation.

$$R(t) = P(T > t) \tag{49}$$

To assess the reliability of the waste sorting robortic arm system, set all the repair rates equal to zero and set all failure rates as $\lambda_{cyb} = 0.004$, $\lambda_{vs} = 0.01$, $\lambda_{cs} = 0.002$, $\lambda_{R_1} = 0.003$, $\lambda_{R_2} = 0.0025$. in Equation (45).

Now taking inverse Laplace transformation of Equation (45), one can easily obtain the expression for the reliability $R(t)$ of the system. The reliability of the puposed system is given in Equation (50).

$$R(t) = e^{-0.02150000000\,t} + 3.666666667e^{-0.02000000000\,t}\sinh(0.001500000000\,t) \tag{50}$$

By varying time unit t from 0 to 100, one can easily obtain the following **Table 5** and **Figure 4** which show the reliability of the system over time.

Table 5. Reliability of the WSRAS w.r.t time.

Time (days)	Reliability $R(t)$
0	1
10	0.85
20	0.72
30	0.61
40	0.52
50	0.44
60	0.37
70	0.31
80	0.26
90	0.22
100	0.19

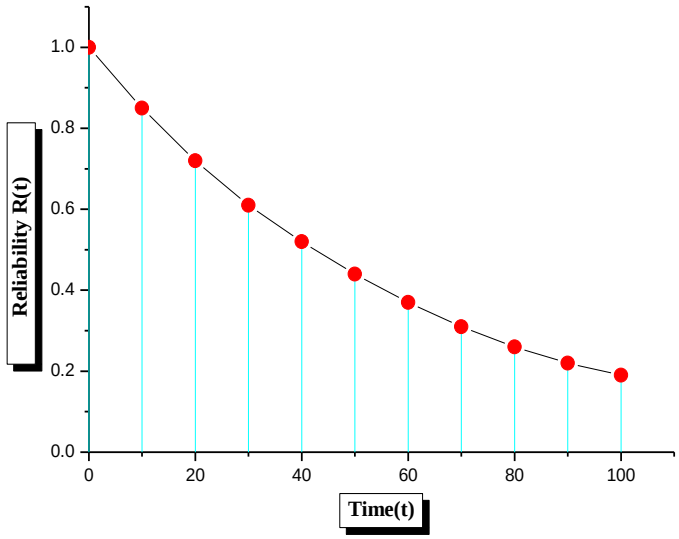

Figure 4. Reliability of the WSRAS w.r.t time.

6.3 Mean Time to Failure (MTTF)

The mean time to failure (MTTF) of the system is the average expected time until the system fails. To calculate the MTTF of the system, one need to have the informartion about the failure rates of all the components of the system. To obtain the expression for the MTTF of the system, set all the repair rate equl to zero in Equation (45) and letting $s \to 0$ in Equation (45), one can obtain the expression of the MTTF of the system as given in Equation (51) (Kumar and Kumar, 2021),

$$\text{MTTF}=\lim_{s \to 0} \bar{P}_{up}(s) \tag{51}$$

$$\text{MTTF}=\left[\frac{1}{(\lambda_{cyb}+\lambda_{vs}+\lambda_{cs}+\lambda_{R_1}+\lambda_{R_2})}+\frac{\lambda_{R_1}}{(\lambda_{cyb}+\lambda_{vs}+\lambda_{cs}+\lambda_{R_1}+\lambda_{R_2})(\lambda_{cyb}+\lambda_{vs}+\lambda_{cs}+\lambda_{R_2})}+\frac{\lambda_{R_2}}{(\lambda_{cyb}+\lambda_{vs}+\lambda_{cs}+\lambda_{R_1}+\lambda_{R_2})(\lambda_{cyb}+\lambda_{vs}+\lambda_{cs}+\lambda_{R_2})}\right] \tag{52}$$

Set failure rates values as $\lambda_{cyb} = 0.004$, $\lambda_{vs} = 0.01$, $\lambda_{cs} = 0.002$, $\lambda_{R_1} = 0.003$, $\lambda_{R_2} = 0.0025$.

Now varing each failure rate one by one from 0.001 to 0.009 and keeping all other failure rates fixed in Equation (52), one can obtain the **Table 6** and **Figure 5** of the MTTF of the system as given below w.r.t variation in the failure rates of the system components.

Table 6. MTTF w.r.t variation in the failure rates.

Variation in the failure rate	Variation in the MTTF w.r.t failure rate λ_{cyb}	Variation in the MTTF w.r.t failure rate λ_{vs}	Variation in the MTTF w.r.t failure rate λ_{cs}	Variation in the MTTF w.r.t failure rate λ_{R_1}	Variation in the MTTF w.r.t failure rate λ_{R_2}
0.001	73.23	126.31	64.11	60.98	61.76
0.002	68.37	112.87	60.33	60.64	60.84
0.003	64.11	101.94	56.98	60.33	59.80
0.004	60.33	92.90	53.96	60.06	58.69
0.005	56.98	85.29	51.25	59.80	57.53
0.006	53.97	78.81	48.80	59.56	56.36
0.007	51.26	73.23	46.56	59.35	55.18
0.008	48.80	68.37	44.52	59.15	54.01
0.009	46.56	64.11	42.65	58.97	52.85

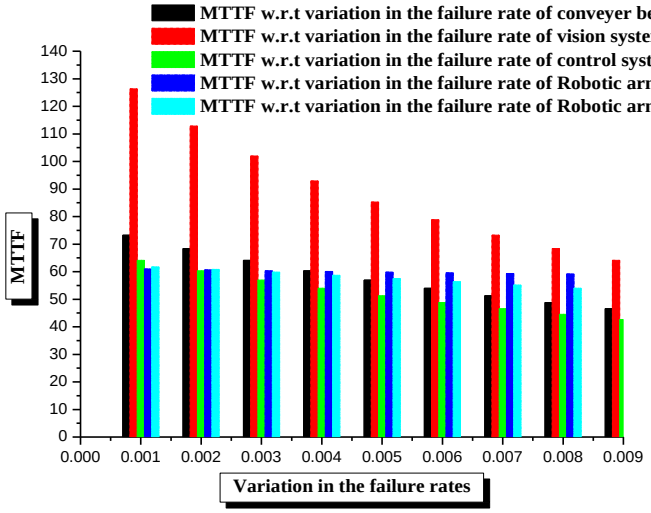

Figure 5. MTTF w.r.t variation in the failure rates.

6.4 Sensitivity Analysis of the WSRAS

Sensitivity analysis is a valuable tool for decision-making and risk management. It helps in identifying the critical components of the system that requires improvements. Sensitivity analysis helps in identifying the impact of variations in the components' performance on the overall system performance. Here, the authors performed the sensitivity analysis of both the MTTF and reliability.

6.4.1 Sensitivity Analysis of the MTTF of the WSRAS

To perform the sensitivity analysis of the MTTF of the system, differentiate Equation (53) one by one w.r.t each failure rates and set the values of the failure rates as $\lambda_{cyb} = 0.004$, $\lambda_{vs} = 0.01$, $\lambda_{cs} = 0.002$, $\lambda_{R_1} = 0.003$, $\lambda_{R_2} = 0.0025$.

Now varing each failure rate one by one from 0.001 to 0.009, keeping other failure rates fixed, one can easily obtain the following **Table 7** and **Figure 6**.

Table 7. MTTF w.r.t variation in the failure rates.

Variation in the failure rate	$\frac{\partial(MTTF)}{\partial\lambda_{cyb}}$	$\frac{\partial(MTTF)}{\partial\lambda_{vs}}$	$\frac{\partial(MTTF)}{\partial\lambda_{cs}}$	$\frac{\partial(MTTF)}{\partial\lambda_{R_1}}$	$\frac{\partial(MTTF)}{\partial\lambda_{R_2}}$
0.001	-5196.07	-14980.60	-4003.44	-355.38	-839.10
0.002	-4542.46	-12056.40	-3553.93	-321.55	-986.81
0.003	-4003.44	-9899.09	-3175.29	-292.34	-1081.70
0.004	-3553.93	-8264.72	-2853.50	-266.93	-1138.94
0.005	-3173.29	-6998.53	-2577.79	-244.69	-1169.21
0.006	-2853.50	-5998.68	-2339.84	-225.13	-1180.16
0.007	-2577.79	-5196.07	-2133.09	-207.82	-1177.27
0.008	-2339.84	-4542.46	-1952.35	-192.43	-1164.55
0.009	-2133.09	-4003.44	-1793.47	-178.69	-1144.89

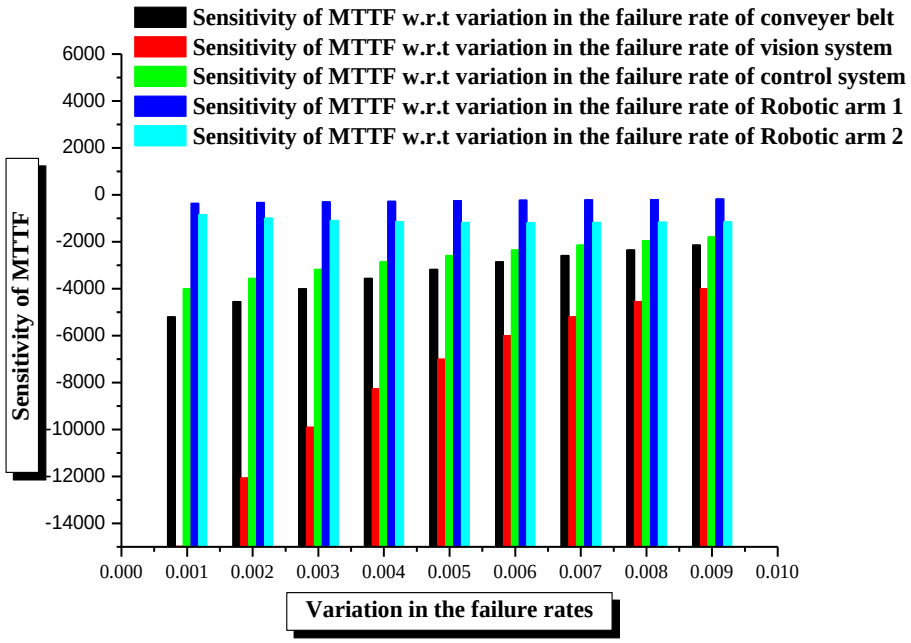

Figure 6. MTTF w.r.t Variation in the failure rates.

6.4.2 Sensitivity Analysis of the Reliability of the WSRAS

To perform the sensitivity analysis of the reliability of the system, set all the failure rates equl to zero in the Eq.(45) and then take laplace inverse of the equation. Differentiate this expression of reliabilit one by one w.r.t each failure rate and set the values of the failure rates as $\lambda_{cyb} = 0.004$, $\lambda_{vs} = 0.01$, $\lambda_{cs} = 0.002$, $\lambda_{R_1} = 0.003$, $\lambda_{R_2} = 0.0025$.

Now varing time unit from 0 to 100, one can easily obtain the **Figure 7** and **Table 8** for the sensitivity of the reliability.

Table 8. Sensitivity of Reliability.

Time (in days)	$\frac{\partial R(t)}{\partial \lambda_{cyb}}$	$\frac{\partial R(t)}{\partial \lambda_{vs}}$	$\frac{\partial R(t)}{\partial \lambda_{cs}}$	$\frac{\partial R(t)}{\partial \lambda_{R_1}}$	$\frac{\partial R(t)}{\partial \lambda_{R_2}}$
0	0	0	0	0	0
10	-7.94	-7.94	-7.94	-0.09	-0.30
20	-12.59	-12.59	-12.59	-0.28	-0.93
30	-14.96	-14.96	-14.96	-0.49	-1.61
40	-15.78	-15.78	-15.78	-0.66	-2.19
50	-15.59	-15.59	-15.59	-0.79	-2.61
60	-14.77	-14.77	-14.77	-0.86	-2.88
70	-13.59	-13.59	-13.59	-0.89	-3.00
80	-12.25	-12.25	-12.25	-0.88	-3.00
90	-10.85	-10.85	-10.85	-0.85	-2.90
100	-9.49	-9.49	-9.49	-0.80	-2.74

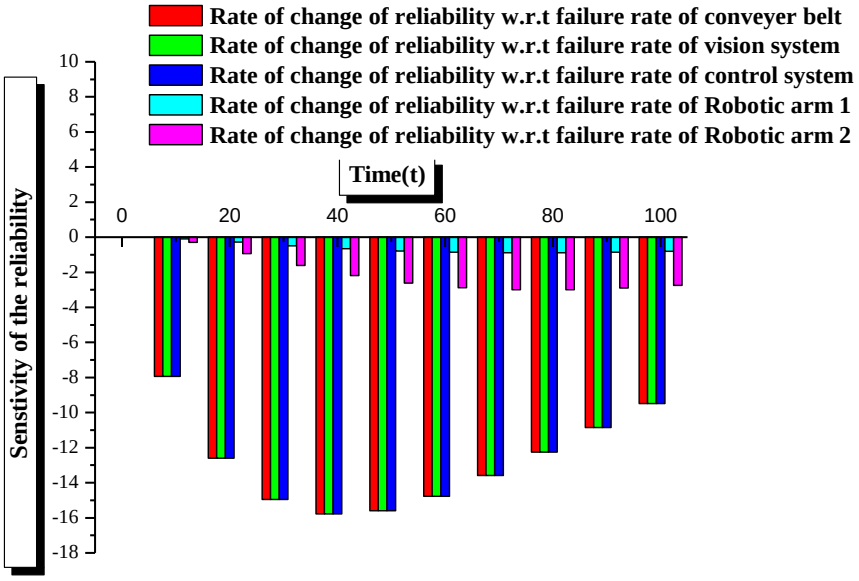

Figure 7. Sensitivity of reliability w.r.t time.

6.5 Expected Profit of WSRAS

It is always necessary to calculate the expected profit that a system will generate in the future. Therefore, the authors calculated the expected profit of the waste sorting robotic arm industry. This industry sells segregated products/items in the market. The below function can be used to estimate the profit of the system in the time interval $[0, t)$.

$$E_P(t) = K_1 \int_0^t A(t)dt - K_2 t \tag{53}$$

Here the expression $\int_0^t A(t)dt$ gives the total uptime of the system and t here denotes the total time period. The parameter K_1 represents the revenue generated by the system per unit time and K_2 represents the expenditure of the system per unit time. Now set $K_1 = 1000$, and vary the values of K_2 and t, one can easily obtain the following **Table 9** and **Figure 8** of the expected profit that the WSRAS can generate.

Table 9. Expected profit of the system.

Time (in days)	$E_P(t)$ $K_2 = 100$	$E_P(t)$ $K_2 = 200$	$E_P(t)$ $K_2 = 300$	$E_P(t)$ $K_2 = 400$	$E_P(t)$ $K_2 = 500$
0	0	0	0	0	0
20	17475.30	15475.30	13475.30	11475.30	9475.30
40	34895.00	30895.00	26895.00	22895.00	18895.0
60	52318.20	46318.20	40318.20	34318.20	28318.20
80	69744.90	61744.90	53744.90	45744.90	37744.90
100	87175.30	77175.30	67175.30	57175.30	47175.30
120	104609.10	92609.10	80609.10	68609.10	56609.10
140	122046.50	108046.5	94046.50	80046.50	66046.50
160	139487.40	123487.4	107487.40	91487.40	75487.4
180	156931.90	138931.9	120931.90	102931.90	84931.9
200	174379.90	154379.9	134379.90	114379.90	94379.90

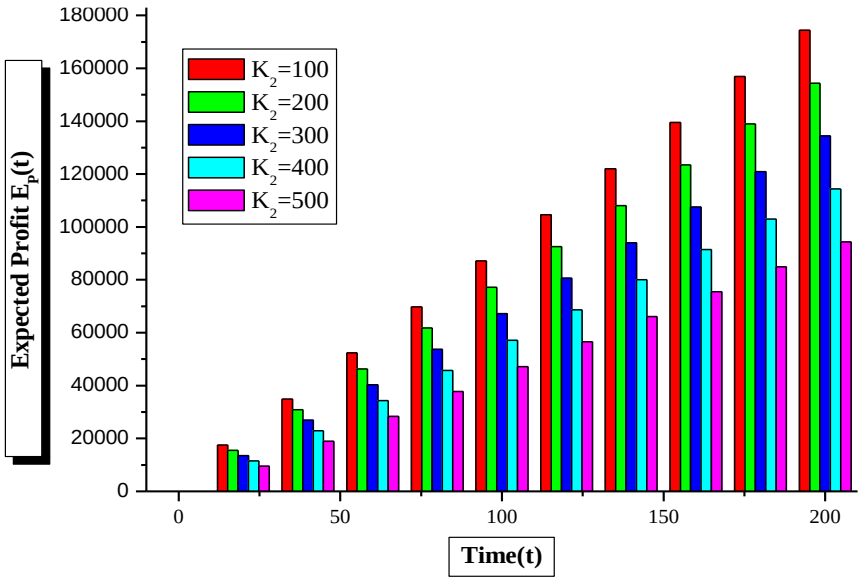

Figure 8. Expected profit of the system.

7. Result and Discussion

In this study, the authors analyzed the performance of the waste sorting robotic arm system using a reliability approach under random operating conditions. They employed a Markov process to construct a stochastic model of proposed system, which includes hardware components such as Conveyer belt, Vision system, control system, and two robotic arms.The authors developed Chapman-Kolmogorov differential

equations based on the system's state transition diagram of the system. These equations were solved by using Laplace transformation and Inverse Laplace transformation. Various reliability metrics such as availability, reliability and MTTF of the system were determined. Additionally, sensitivity analysis was conducted to determine the critical components of the system. Furthermore, the expected profit generated by the system was calculated. The results obtained from the Tables and Figure are summarized below.

- **Table 4** and **Figure 3** depict time dependent availability of the waste sorting robotic arm system. An availability of 1 at time $t = 0$ signifies that system is completely available to perform its intended tasks without any failure. After 50 days of operation, there is minor fluctuation in system's availability and system's availability is 0.97. After 50 days, system maintains a stable availability of 0.97 throughout the remaining observed period. This highly stable availability signifies that system experiences minimal downtime and failures. This suggests that system is well designed and well maintained and is highly available for its usage.
- **Table 5** and **Figure 4** depict the reliability of the waste sorting robotic arm system. At time $t = 0$ reliability of the system is 1. This signifies that initially, the system is highly reliable for the waste sorting opertion. However, the system's reliability consistently decreases over time. After 100 days of its usage, system's reliability is only 0.19, which is very low. It indicates a dimishing level of confidence or trust in the system's ability to perform its intended task without failure over time.
- **Table 6** and **Figure 5** depict MTTF of the system w.r.t vatiation in the failure rates. On careful examination of the Table and Graph of the MTTF of the system, we observe that as the failure rate of Control system's is increasing it is significantly lowering the MTTF of the whole system. The effect of variation in the failure rates of the other components lesser than the variation in the failure rate of the control system's on the system's MTTF. Hence, the control system emerges as the most critical component of the system, expected to fail sooner than the others..
- **Table 7** and **Figure 6** depict sensitivity of the MTTF w.r.t variation rates of the system components. Upon close examination of the Table and Graph of the sensitivity of the MTTF w.r.t variation in the failure rates exhibits that vision system exhibits higher sensitivity, indicating changes in its failure rates have the most significant impact on the overall system's MTTF. The slight change in the failure rate of the vision system changes the value of the system's MTTF significantly. Therefore, it is crucial to ensure high-quality components are always used in the vision system to maintain the system's availability over ectended periods of usage.
- **Table 8** and **Figure 7** depict sensitivity of the reliability w.r.t variation in time. It is evident from the Table and graph that conveyer belt, vision system and control system appear to be more sensitive for system reliability. Ove time, these components exhibit a significant impact on reducing the overall system's reliability.
- **Table 9** and **Figure 8** depict the expected profit of the system for the 200 days. Based on the data it is clear that higher expediture results in lower expected profit. This trend highlights the importance of efficient resource management and cost control measures to maintain profitability in the long run.

8. Conclusion and Future Direction for Research

In this research paper, the authors have developed a stochastic model of the waste sorting robotic arm system using Markov modeling. The waste sorting robotic arm system is the fastest technology to sort the garbage with high level of accuracy. This system can easily sort tons of garbage in short time if system and its components function properly. Any component's failure in this system will increase system's downtime and this will reduce system's profitability.

Based on the result presented above, the authors have identified that vision system and control system are the main critical components of the waste sorting robotic arm system. Therefore, it is imperative to

implement proactive maintenance strategies for both these systems. Software updation, components replacement and regular inspection of these two systems can help in increasing the reliability of the overall system. Collabration with experts and imparting training to the employees may further help in increasing the reliability of the system. If these suggestions are properly implemented, system's failure can be effectively mitigated.

The above research work can be extended by incorporating any of the suggestion in the model.

- Introduce more components of the control system and vision system.
- Develop new method which may be capable of handling 40-50 components of the system to determine the reliability measures of the system.
- One may also compare this model with the other model to do the comparative analysis of the studies.
- One can also use fuzzy logic and soft computing techniques to enhance decision-making.

Conflict of Interest

The authors declare that they do not have any conflict of interest related to the presented study.

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References

Bansal, S., Patel, S., Shah, I., Patel, P.A., Makwana, P., & Thakker, D.R. (2019). AGDC: Automatic garbage detection and collection. *arXiv preprint arXiv:1908.05849*.1-12.

Bhor, V., Morajkar, P., Gurav, M., Pandya, D., & Deshpande, A. (2015). Smart garbage management system. *International Journal of Engineering Research & Technology*, *4*(3),1117-1120.

Chakraborty, S., Mehta, A., Sheikh, S., & Jha, A.K. (2021). Smart waste management system. *Journal of Emerging Technologies and Innovative Research*, *8*(5), 250-254.

Dugdhe, S., Shelar, P., Jire, S., & Apte, A. (2016). Efficient waste collection system. In *2016 International Conference on Internet of Things and Applications* (pp. 143-147). IEEE. Pune, India.

Forghani-Elahabad, M. (2022). 3 The disjoint minimal paths reliability problem. In *Operations Research* (pp. 35-66). CRC Press.

Forghani-Elahabad, M., & Kagan, N. (2019). Reliability evaluation of a stochastic-flow network in terms of minimal paths with budget constraint. *IISE Transactions*, *51*(5), 547-558.

Gautam, A., & Bhadra, S. (2023). Rag-pickers and their young children during COVID-19: exploring issues in parenting and feasible social work response. *Journal of Human Rights and Social Work*, *8*(1), 75-90.

Geethamani, R., Rakshana, P., Raveena, P., & Ragavi, R. (2021). Garbage management system. In *2021 7th International Conference on Advanced Computing and Communication Systems* (Vol. 1, pp. 1190-1192). IEEE. Coimbatore, India.

Gupta, M., & Tripathi, M.M. (2021). Smart robot for collection and segregation of garbage. In *2021 International Conference on Innovative Practices in Technology and Management* (pp. 169-173). IEEE. Noida, India.

Hao, Z., Yeh, W.C., Liu, Z., & Forghani-elahabad, M. (2020). General multi-state rework network and reliability algorithm. *Reliability Engineering & System Safety*, *203*, 107048.

Jain, A., & Bagherwal, R. (2017). Design and implementation of a smart solid waste monitoring and collection system based on Internet of Things. In *2017 8th International Conference on Computing, Communication and Networking Technologies* (pp. 1-5). IEEE. Delhi, India.

Karajvandani, S.M.A., Abbasi, G., Amirtaheri, O., & Khishtandar, S. (2024). A hybrid multi-criteria decision-making and allocation model for selection of hospital waste disposal firms. *Journal of Environmental Science and Technology*, *1*, 17-33.

Kumar, P., & Kumar, A. (2021). Tripod turnstile machines performance analysis for the system safety and security without considering simultaneous failures using reliability approach. *International Journal of Mathematical, Engineering and Management Sciences*, *6*(1), 383-395.

Kumar, P., & Kumar, A. (2023). Quantifying reliability indices of garbage data collection IOT-based sensor systems using Markov birth-death process. *International Journal of Mathematical, Engineering and Management Sciences*, *8*(6), 1255-1274.

Kumar, S.V., Kumaran, T.S., Kumar, A.K., & Mathapati, M. (2017). Smart garbage monitoring and clearance system using internet of things. In *2017 IEEE International Conference on Smart Technologies and Management for Computing, Communication, Controls, Energy and Materials* (pp. 184-189). IEEE. Chennai, India.

Ravindra, K., Kaur, K., & Mor, S. (2016). Occupational exposure to the municipal solid waste workers in Chandigarh, India. *Waste Management & Research*, *34*(11), 1192-1195.

Rokade, R., Maurya, A., Khade, V., Tandel, S., & Mali, J. (2018). Smart garbage separation robot with image processing technique. *International Journal of Engineering Research & Technology*, 1-4.

Sathish, A., Prakash, M., Jainulabudeen, S.A.K., & Sathishkumar, R. (2017). Intellectual trash management using Internet of Things. In *2017 International Conference on Computation of Power, Energy Information and Commuincation* (pp. 053-057). IEEE. Melmaruvathur, India.

Sharanya, A., Harika, U., Sriya, N., & Kochuvila, S. (2017). Automatic waste segregator. In *2017 International Conference on Advances in Computing, Communications and Informatics* (pp. 1313-1319). IEEE. Udupi, India.

Shyam, G.K., Manvi, S.S., & Bharti, P. (2017). Smart waste management using Internet-of-Things (IoT). In *2017 2nd International Conference on Computing and Communications Technologies* (pp. 199-203). IEEE. Chennai, India.

Sunil, M.P., Chaand, P.K., Grandhe, B., & Hariprasad, S.A. (2020). Waste segregation robot-A Swachh Bharat Initiation. *International Journal of Innovative Science, Engineering & Technology*, *7*(12), 505-516.

Thakur, P., Ganguly, R., & Dhulia, A. (2018). Occupational health hazard exposure among municipal solid waste workers in Himachal Pradesh, India. *Waste Management*, *78*, 483-489.

Ugale, M., Rane, S., Rajadhyaksha, S., Pednekar, P. (2021). Importance of waste segregation using waste segregating robot. *International Journal of Engineering Research & Technology*, *9*(3), 325-328.

Vamsi, T.M.N., Chakravarthi, G.K., Lanka, P., & Divakar, B. (2021). An IoT based smart garbage monitoring and disposal support system. In *2021 5th International Conference on Computing Methodologies and Communication* (pp. 438-442). IEEE. Erode, India.