International Journal of Mathematical, Engineering and Management Sciences
Vol. 10, No. 5, 1539-1562, 2025
https://doi.org/10.33889/IJMEMS.2025.10.5.073

Ram Arti Publishers

Impact of Cogeneration System Failure on the Reliability of Sugar Mill System

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(Received on March 1, 2025; Revised on April 12, 2025 & May 16, 2025; Accepted on May 18, 2025)

Abstract
The cogeneration system, a reliable source for meeting energy needs and ensuring system profitability, is widely utilized in the sugar industry. Its impact on the performance of the sugar manufacturing system (SMS) is significant. Therefore, assessing the reliability of a two-unit sugar mill system, consisting of the SMS and the cogeneration system, is crucial. This paper aims to assess their collaboration from the perspective of reliability, which is vital for the mill's overall economic growth and operational efficiency. Reliability measures concerning the system, such as mean time to system failure (MTSF), availability, busy period of the repair person, expected time for preventive maintenance, and expected down time of the system, are derived by applying the semi-Markov process and the regenerative point technique. The system's profitability is analyzed through graphs, and a sensitivity analysis is conducted to evaluate the impact of variations in system parameters on its overall performance, particularly regarding MTSF and availability.

Keywords- Regenerative point technique, Seasonal, Preventive maintenance, Sensitivity analysis, Reliability measures.

1. Introduction

The sugar industry is one of the largest agricultural industries worldwide. Sugar's transformation from a rare luxury to a common commodity highlights its significant impact on global economies. It is a fundamental ingredient in numerous food products, beverages, and confectioneries, making the sugar industry a cornerstone of the food processing sector. Within this industry, the bagasse-based cogeneration system plays a crucial role, offering an efficient means of utilizing waste to meet the energy demands of the production process. Although the system provides sustainable energy production, it also comes with challenges, including the risk of failure. System failures can have a significant impact on production, often bringing the entire process to a halt. This leads to lost production hours, which in turn directly affect the industry's output and revenue. The cogeneration system is considered the backbone of the sugar industry as it addresses all energy requirements. Consequently, its failures may have a profound impact on the production process. To tackle the potential failures and their consequences, reliability analysis is essential. This paper analyzes the reliability of a two-unit system comprising the SMS and the cogeneration system. The reliability approach outlined in this study identifies key factors, addressing which can help minimize risks, ensure smooth operations, and sustain the plant's overall profitability.

Cogeneration systems are linked to renewable energy-driven multigeneration systems that are designed to produce multiple outputs, such as heat, electricity, and hot water, from a single energy source.

Multigeneration systems have been extensively studied over the years. Among the various methods developed for their analysis, Dincer and Rosen (2012) proposed widely recognized approaches. Al-Sulaiman et al. (2011), Bhatti et al. (2023), Coskun et al. (2012), Esfahani and Ifaei (2024), Eslami et al. (2020), Malico et al. (2009), and Shabbir and Mirzaeian (2016) have also analyzed multigeneration systems based on different configurations. The simplest multigeneration systems are cogeneration systems that produce power and heat simultaneously. Over the decades, cogeneration systems have gained popularity due to their substantial benefits. In the sugar industry, a bagasse-based cogeneration system is a form of combined heat and power technology that utilizes bagasse, the fibrous by-product remaining after extracting juice from sugarcane, as a fuel to produce electricity and useful heat. The sugar plant under consideration has one more advantage of supplying surplus electricity to the grid. Extensive research has been carried out on cogeneration systems in the sugar industry from various perspectives to improve their efficiency and sustainability. While significant studies by Abouelsoud et al. (2025), Birru et al. (2019), Ghani et al. (2020), Sheikh et al. (2024), and Skolpap and Kasemwong (2019) focus on enhancing energy efficiency and promoting environmental sustainability, fewer studies, such as Ramesh and Saravannan (2011), have addressed the reliability assessment of cogeneration systems.

The reliability assessment of industrial systems is an important and growing field because it helps evaluate how well a system performs and identify factors that affect its performance, both positively and negatively. Over the years, various researchers have contributed to this domain using different analytical techniques and system models. Gupta and Sharma (1986) analyzed the reliability behavior of a power plant using the Boolean function technique under an arbitrary failure time distribution. Gopalan and Murulidhar (1991) conducted a cost analysis of a single-unit system incorporating on-line preventive maintenance and repair strategies using the regenerative point technique. Mokaddis and Tawfek (1995) performed a stochastic analysis of a two-dissimilar-unit warm-standby redundant system with multiple repair facilities, while El-Said and El-Sherbeny (2006) carried out a comparative study of the reliability characteristics of different systems. The application of fault tree analysis for power system reliability was explored by Volkanovski et al. (2009), while Shakuntla et al. (2011) employed the supplementary variable technique for analyzing polytube industry reliability. Kumar and Ram (2013) focused on improving reliability measures and conducting sensitivity analysis for coal-handling units in thermal power plants. Further studies have incorporated various strategies, including stochastic biometric system modeling with a rework strategy (Ram and Manglik, 2016), the use of the Gumbel-Hougaard copula for two dissimilar-unit parallel systems (Chopra and Ram, 2019), and the application of correlated lifetime distribution for a parallel-configured two dissimilar-unit system (Singh and Poonia, 2019). Reliability analysis has also been extended to specialized industrial applications, such as building cable manufacturing plants (Taj et al., 2020), milk processing plants (Barak et al., 2021), and food industrial systems with multiple repair facilities (Monika and Chopra, 2022; 2023). Recent studies have continued to refine stochastic modeling of the industrial systems with various concepts, including cold standby systems with refreshment (Kumar et al., 2023), comparative analysis of hot and cold standby redundant systems (Malhotra et al., 2023), and aviation industry reliability modeling (Prawar et al., 2024). Additionally, Kaur and Malhotra (2024) examined a two-unit standby autoclave system with inspection under varying demand, while Pinki et al. (2024) conducted a stochastic analysis of gas turbine systems under different humid conditions. These studies collectively highlight the importance of reliability modeling in optimizing industrial system performance.

Among industrial systems, sugar manufacturing systems have also been studied from the reliability perspective. Kumar et al. (1988) performed the reliability analysis of the sugar mill's crushing system. Aggarwal et al. (2017) employed a fuzzy probabilistic approach to assess the parameters that have a major impact on the availability of the crystallization subsystem in the sugar mill. Kadyan and Kumar (2017) assessed the B-pan crystallization system, which comprises three subsystems, to assess the expected profit.

Kumar and Ram (2018) proposed a mathematical model to evaluate various performance metrics and identified the components critical for system reliability and mean time to failure. Saini and Kumar (2019) examined the most sensitive subsystem in the evaporation system. Dahiya et al. (2019) formulated a mathematical model of an A-pan crystallization system using the fuzzy reliability approach. Saini et al. (2022) analyzed a sugar manufacturing plant with five subsystems in a series configuration. Sharma et al. (2023) examined the evaporation and crystallization unit of a sugar plant to determine maintenance priorities. Their findings offer valuable insights into improving the reliability of various subsystems within the sugar manufacturing process.

While existing research focuses either on the reliability assessment of SMS components or on the sustainability of cogeneration systems in isolation, it does not address their combined operation. This separation overlooks the interdependence between the two systems, where the performance of the cogeneration system directly affects the functioning of the SMS. In our paper, we introduce a novel approach by integrating both systems within a unified framework. Although the methodology builds on existing techniques, the integrated assessment of reliability and performance across both systems constitutes a new and meaningful contribution to the field. To gain a detailed understanding of sugar mill operations, a site visit was conducted to collect real-time data and insights from mill officials. This visit provided valuable information about the mill’s processes, enabling the development of a mathematical model. The proposed model consists of two interconnected units: the SMS and the cogeneration system. By integrating these two systems, the model aims to enhance the overall performance of the sugar mill. The regenerative point technique is used to solve the mathematical model and to obtain various measures of system effectiveness. Sensitivity analysis is also conducted to derive the parameters that affect the system performance measures both positively and negatively.

The remainder of the paper is structured as follows. Section 2 discusses the notations used throughout the paper. Section 3 focuses on the system description and the various assumptions taken while developing the mathematical model. Section 4 provides the solution to the mathematical model. Section 5 contains a detailed summary of all the relevant rates and costs required to solve the mathematical model. Section 6 includes the graphical interpretations of the results and their respective discussions. Section 7 examines the sensitivity of system effectiveness metrics, specifically MTSF and availability, to various parameters. Section 8 summarizes the research findings and discusses the scope for future work.

2. Notations

This section outlines the various notations used in the paper, with their explanations provided in **Table 1**. The notation table serves as a consistent reference for the symbols used throughout the study, ensuring clarity and readability.

Table 1. Notations.

Notation	Meaning
λ_1	Constant failure rate of SMS.
λ_2	Constant failure rate of cogeneration system.
$g_1(t)/G_1(t)$	pdf/cdf of repair time of SMS.
$g_2(t)/G_2(t)$	pdf/cdf of repair time of cogeneration system.
δ_1	Rate of going from reduced capacity to full capacity period.
δ_2	Rate of going from full capacity to reduced capacity period.
δ_3	Rate of going for preventive maintenance.
δ_4	Rate of transitioning to the operative state (S_0) during the reduced capacity period following preventive maintenance.
η_1	Rate of going to down state due to the unavailability of sugarcane.
η_2	Rate of transitioning to the operative state (S_5) during the full capacity period after sugarcane availability.
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of the time to transit from state ‘S_i’ to state ‘S_j’.

Table 1 Continued…

$q_{ij}^{(k)}/Q_{ij}^{(k)}$	pdf/cdf of the time to transit from state 'S_i'to state 'S_j' via state 'S_k' once.
$\phi_i(t)$	cdf of first passage time from a regenerative state 'S_i' to a failed one.
A_0^s/A_0^d	The steady-state availability of the system in the case when only cogeneration system is operative/both units are operative.
B_0	Busy period of the repair person.
P_0	Expected preventive maintenance time of the system.
D_0	Expected down time of the system.
$M_i(t)$	The probability that a system, initially up in the regenerative state 'S_i', stays operational at time t without moving to another regenerative state.
$W_i(t)$	The probability that the repair person, initially engaged in repairing a system in the regenerative state 'S_i', remains involved in the repair at time t without transitioning to another regenerative state.
$m_{ij}/m_{ij}^{(k)}$	The expected time for the system to reach any regenerative state 'S_j', measured from its entry into state 'S_i' without going to any other state/visiting state 'S_k' only once.
μ_i	The expected time for which the system stays in the regenerative state 'S_i' before transitioning to any other state.
*/©	Notation used for Laplace transform / Laplace convolution.
**/Ⓢ	Notation used for Laplace Stieltjes transform/ Laplace Stieltjes convolution.

3. System Description

This study analyzes a two-unit system of a sugar mill located in Haryana, India. The first unit is the SMS, which is responsible for sugar production, while the second unit is the cogeneration system, which supplies the necessary electricity and heat to support the production process. Additionally, the cogeneration system not only generates energy for its primary use but also transmits surplus electricity to the nearby grid, generating additional profit. Since sugar production is seasonal, the mill usually runs from November to May, a timeframe referred to as the crushing season. During the remaining months, the system undergoes preventive maintenance. Based on the supply of sugarcane to the mill, the season is divided into two periods:

***Full capacity period*:** This is the time during which the supply of sugarcane is abundant, and the mill operates at full capacity to optimize sugar production.

***Reduced capacity period*:** This is the period during which the mill has a limited supply of sugarcane, causing the mill to operate at reduced capacity. This usually happens at the start or the end of the crushing season.

Irrespective of the reduced capacity period, the considered system also remains in a down state during the full capacity period due to various reasons, such as the strike of the farmers, heavy rainfall creating unreachability of the cane to the mill, etc. A mathematical model is developed by considering all aspects. The possible states are given in **Figure 1** and are described in **Table 2**.

Table 2. Description of the states of the system.

State	Description
S_0	Both units are operative during the reduced capacity period.
S_1	The system is undergoing preventive maintenance following the crushing season.
S_2	SMS is in down state, and the cogeneration system is under repair during the reduced capacity period.
S_3	SMS is under repair, and the cogeneration system is working during the reduced capacity period.
S_4	SMS is under repair from the previous state, and the failed cogeneration system is waiting for repair during the reduced capacity period.
S_5	Both units are operative during the full capacity period.
S_6	SMS is in down state, and the cogeneration system is under repair during the full capacity period.
S_7	SMS is under repair, and the cogeneration system is working during the full capacity period.
S_8	SMS is waiting for repair, and the cogeneration system is under repair during the full capacity period.
S_9	The system is in down state due to the unavailability of sugarcane.
●	Regenerative point indicating a regenerative state.

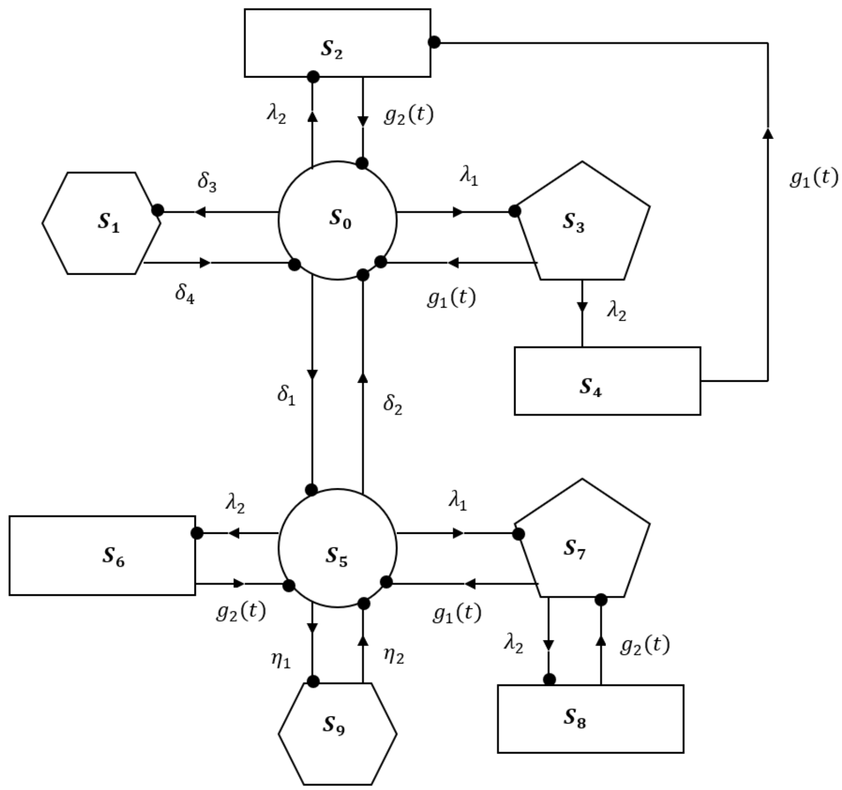

Figure 1. State transition diagram.

The considered mathematical model is based on the assumptions given below:

(i) The two-unit system experiences complete failure if the cogeneration system fails, regardless of the SMS's operational status. In this case, if the SMS is operational, it enters a down state.
(ii) The system operates in a degraded mode when the cogeneration system is functioning, but the SMS has failed.
(iii) During the full-capacity period, if the cogeneration system fails while the SMS is undergoing repair, priority will be given to repairing the cogeneration system.
(iv) There is a single repair facility to handle all system failures.
(v) The system works as new after every repair.
(vi) All failure time and transition time distributions are assumed to be exponential, whereas repair time distributions in this study are treated as general.

4. Mathematical Formulation and Solution of the Proposed Model

The specified mathematical model is solved using the regenerative point technique, which incorporates the concept of transition probabilities. To define these probabilities, let (X_n, T_n) represent a sequence of random variables, where X_n represents the state space and T_n denotes the jump times of the states with $\tau_n = T_n - T_{n-1}$ as the inter-arrival times between them. If $P[X_{n+1} = j, \tau_{n+1} \leq t \,|(X_0, T_0), (X_1, T_1), \dots, (X_n = i, T_n)] = P[X_{n+1} = j, \tau_{n+1} \leq t | X_n = i]$ for $n \geq 0$, $t \geq 0$; $i, j \in \{0, 1, 2, \dots, 9\}$, then the sequence (X_n, T_n) is called the Markov renewal process. The stochastic process with continuous parameter $\{X(t), t \geq 0\}$, where $X(t) = X_n, T_n \leq t \leq T_{n+1}$, defines a semi-Markov process, and correspondingly $Q_{ij}(t) = P[X_{n+1} = j, \tau_{n+1} \leq t | X_n = i]$ is the semi-Markov kernel that defines the transition probabilities.

In the considered model, there are ten possible states. Out of these, states S_0, S_1, S_2, S_3, S_5, S_6, S_7, S_8 and S_9 are regenerative states, while S_4 is a non-regenerative state. Additionally, the states S_2, S_4, S_6, and S_8 are the failed states.

The transition probabilities $q_{ij}(t)$, where, $q_{ij}(t) = \lim_{\Delta t \to 0} \frac{P[X_{n+1}=j, t<\tau_{n+1}<t+\Delta t | X_n = i\,]}{\Delta t}$ are determined to be as follows:

$$q_{01}(t) = \delta_3 e^{-(\delta_3+\lambda_1+\lambda_2+\delta_1)t} \quad (1)$$

$$q_{02}(t) = \lambda_2 e^{-(\delta_3+\lambda_1+\lambda_2+\delta_1)t} \quad (2)$$

$$q_{03}(t) = \lambda_1 e^{-(\delta_3+\lambda_1+\lambda_2+\delta_1)t} \quad (3)$$

$$q_{05}(t) = \delta_1 e^{-(\delta_3+\lambda_1+\lambda_2+\delta_1)t} \quad (4)$$

$$q_{10}(t) = \delta_4 e^{-\delta_4 t} \quad (5)$$

$$q_{20}(t) = g_2(t) \quad (6)$$

$$q_{30}(t) = e^{-\lambda_2 t}.g_1(t) \quad (7)$$

$$q_{34}(t) = \lambda_2 e^{-\lambda_2 t}.\overline{G_1(t)} \quad (8)$$

$$q_{32}^{(4)}(t) = \left(\lambda_2 e^{-\lambda_2 t} © 1\right) g_1(t) \quad (9)$$

$$q_{50}(t) = \delta_2 e^{-(\delta_2+\lambda_1+\lambda_2+\eta_1)t} \quad (10)$$

$$q_{56}(t) = \lambda_2 e^{-(\delta_2+\lambda_1+\lambda_2+\eta_1)t} \quad (11)$$

$$q_{57}(t) = \lambda_1 e^{-(\delta_2+\lambda_1+\lambda_2+\eta_1)t} \quad (12)$$

$$q_{59}(t) = \eta_1 e^{-(\delta_2+\lambda_1+\lambda_2+\eta_1)t} \quad (13)$$

$$q_{65}(t) = g_2(t) \quad (14)$$

$$q_{75}(t) = e^{-\lambda_2 t}.g_1(t) \quad (15)$$

$$q_{78}(t) = \lambda_2 e^{-\lambda_2 t}.\overline{G_1(t)} \quad (16)$$

$$q_{87}(t) = g_2(t) \quad (17)$$

$$q_{95}(t) = \eta_2 e^{-\eta_2 t} \quad (18)$$

Now the corresponding p_{ij}'s, where, $p_{ij} = \lim_{s \to 0} q_{ij}{}^*(s)$, are found to be as follows:

$$p_{01} = \frac{\delta_3}{\delta_3+\lambda_1+\lambda_2+\delta_1} \quad (19)$$

$$p_{02} = \frac{\lambda_2}{\delta_3+\lambda_1+\lambda_2+\delta_1} \quad (20)$$

$$p_{03} = \frac{\lambda_1}{\delta_3+\lambda_1+\lambda_2+\delta_1} \quad (21)$$

$$p_{05} = \frac{\delta_1}{\delta_3+\lambda_1+\lambda_2+\delta_1} \quad (22)$$

$$p_{10} = 1 \quad (23)$$

$p_{20} = 1$ (24)

$p_{30} = g_1^*(\lambda_2)$ (25)

$p_{32}^{(4)} = 1 - g_1^*(\lambda_2)$ (26)

$p_{50} = \frac{\delta_2}{\delta_2+\lambda_1+\lambda_2+\eta_1}$ (27)

$p_{56} = \frac{\lambda_2}{\delta_2+\lambda_1+\lambda_2+\eta_1}$ (28)

$p_{57} = \frac{\lambda_1}{\delta_2+\lambda_1+\lambda_2+\eta_1}$ (29)

$p_{59} = \frac{\eta_1}{\delta_2+\lambda_1+\lambda_2+\eta_1}$ (30)

$p_{65} = 1$ (31)

$p_{75} = g_1^*(\lambda_2)$ (32)

$p_{78} = 1 - g_1^*(\lambda_2)$ (33)

$p_{87} = 1$ (34)

$p_{95} = 1$ (35)

From these, we can conclude that

$p_{01} + p_{02} + p_{03} + p_{05} = 1$ (36)

$p_{50} + p_{56} + p_{57} + p_{59} = 1$ (37)

$p_{30} + p_{32}^{(4)} = 1$ (38)

$p_{75} + p_{78} = 1$ (39)

The unconditional mean times m_{ij} and $m_{ij}^{(k)}$, where $m_{ij} = \int_0^\infty t\, dQ_{ij}(t) = -q_{ij}^{*'}(0)$ and $m_{ij}^{(k)} = \int_0^\infty t\, dQ_{ij}^{(k)}(t) = -q_{ij}^{(k)*'}(0)$, are calculated as:

$m_{01} = \frac{\delta_3}{(\delta_3+\lambda_1+\lambda_2+\delta_1)^2}$ (40)

$m_{02} = \frac{\lambda_2}{(\delta_3+\lambda_1+\lambda_2+\delta_1)^2}$ (41)

$m_{03} = \frac{\lambda_1}{(\delta_3+\lambda_1+\lambda_2+\delta_1)^2}$ (42)

$m_{05} = \frac{\delta_1}{(\delta_3+\lambda_1+\lambda_2+\delta_1)^2}$ (43)

$m_{10} = \frac{1}{\delta_4}$ (44)

$m_{20} = -g_2^{*'}(0)$ (45)

$m_{30} = -g_1^{*'}(\lambda_2)$ (46)

$m_{34} = \frac{1-g_1^*(\lambda_2)}{\lambda_2} + g_1^{*'}(\lambda_2)$ (47)

$$m_{32}^{(4)} = -g_1^{*'}(0) + g_1^{*'}(\lambda_2) \tag{48}$$

$$m_{50} = \frac{\delta_2}{(\delta_2+\lambda_1+\lambda_2+\eta_1)^2} \tag{49}$$

$$m_{56} = \frac{\lambda_2}{(\delta_2+\lambda_1+\lambda_2+\eta_1)^2} \tag{50}$$

$$m_{57} = \frac{\lambda_1}{(\delta_2+\lambda_1+\lambda_2+\eta_1)^2} \tag{51}$$

$$m_{59} = \frac{\eta_1}{(\delta_2+\lambda_1+\lambda_2+\eta_1)^2} \tag{52}$$

$$m_{65} = -g_2^{*'}(0) \tag{53}$$

$$m_{75} = -g_1^{*'}(\lambda_2) \tag{54}$$

$$m_{78} = \frac{1-g_1^*(\lambda_2)}{\lambda_2} + g_1^{*'}(\lambda_2) \tag{55}$$

$$m_{87} = -g_2^{*'}(0) \tag{56}$$

$$m_{95} = \frac{1}{\eta_2} \tag{57}$$

It can be verified that

$$m_{01} + m_{02} + m_{03} + m_{05} = \mu_0 \tag{58}$$

$$m_{10} = \mu_1 \tag{59}$$

$$m_{20} = \mu_2 \tag{60}$$

$$m_{30} + m_{32}^{(4)} = -g_1^{*'}(0) = K_1(say) \tag{61}$$

$$m_{30} + m_{34} = \mu_3 \tag{62}$$

$$m_{50} + m_{56} + m_{57} + m_{59} = \mu_5 \tag{63}$$

$$m_{65} = \mu_6 \tag{64}$$

$$m_{75} + m_{78} = \mu_7 \tag{65}$$

$$m_{87} = \mu_8 \tag{66}$$

$$m_{95} = \mu_9 \tag{67}$$

The mean sojourn times μ_i, where $\mu_i = E(t) = \int_0^\infty P(T > t)dt = \sum_j (m_{ij} + \sum_k m_{ij}^{(k)})$, are calculated as follows:

$$\mu_0 = \frac{1}{\delta_3+\lambda_1+\lambda_2+\delta_1} \tag{68}$$

$$\mu_1 = \frac{1}{\delta_4} \tag{69}$$

$$\mu_2 = -g_2^{*'}(0) \tag{70}$$

$$\mu_3 = \frac{1-g_1^*(\lambda_2)}{\lambda_2} \tag{71}$$

$$\mu_5 = \frac{1}{\delta_2+\lambda_1+\lambda_2+\eta_1} \tag{72}$$

$$\mu_6 = -g_2^{*'}(0) \tag{73}$$

$$\mu_7 = \frac{1-g_1^*(\lambda_2)}{\lambda_2} \tag{74}$$

$$\mu_8 = -g_2^{*'}(0) \tag{75}$$

$$\mu_9 = \frac{1}{\eta_2} \tag{76}$$

The different metrics for assessing system effectiveness are as follows:

4.1 Mean Time to System Failure (MTSF)

The MTSF is the system effectiveness measure representing the average time it takes for a system to fail. To calculate the MTSF, the following recurrence relations for $\phi_i(t)$ are used, where failed states are considered absorbing states:

$$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t) + Q_{03}(t)Ⓢ\phi_3(t) + Q_{05}Ⓢ\phi_5(t) + Q_{02}(t) \tag{77}$$

$$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) \tag{78}$$

$$\phi_3(t) = Q_{30}(t)Ⓢ\phi_0(t) + Q_{34}(t) \tag{79}$$

$$\phi_5(t) = Q_{50}(t)Ⓢ\phi_0(t) + Q_{57}(t)Ⓢ\phi_7(t) + Q_{59}Ⓢ\phi_9(t) + Q_{56}(t) \tag{80}$$

$$\phi_7(t) = Q_{75}(t)Ⓢ\phi_5(t) + Q_{78}(t) \tag{81}$$

$$\phi_9(t) = Q_{95}(t)Ⓢ\phi_5(t) \tag{82}$$

Taking Laplace Stieltjes transform of these Equations (77) to (82), we have

$$\phi_0^{**}(s) = \frac{N_1(s)}{D_1(s)} \tag{83}$$

where,

$$N_1(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ q_{34}^*(s) & 0 & 1 & 0 & 0 & 0 \\ q_{56}^*(s) & 0 & 0 & 1 & -q_{57}^*(s) & -q_{59}^*(s) \\ q_{78}^*(s) & 0 & 0 & -q_{75}^*(s) & 1 & 0 \\ 0 & 0 & 0 & -q_{95}^*(s) & 0 & 1 \end{vmatrix} \text{ and}$$

$$D_1(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 \\ -q_{30}^*(s) & 0 & 1 & 0 & 0 & 0 \\ -q_{50}^*(s) & 0 & 0 & 1 & -q_{57}^*(s) & -q_{59}^*(s) \\ 0 & 0 & 0 & -q_{75}^*(s) & 1 & 0 \\ 0 & 0 & 0 & -q_{95}^*(s) & 0 & 1 \end{vmatrix}.$$

Thus, when the system starts from the state 'S_0', the MTSF is given by:

$$\text{MTSF} = \lim_{s\to 0}\frac{(1-\phi_0^{**}(s))}{s} = \frac{N_1}{D_1} \tag{84}$$

where,

$$N_1 = \big((1-p_{59}-p_{57}.p_{75})(\mu_0+p_{01}.\mu_1+p_{03}.\mu_3)+p_{05}(\mu_5+p_{57}.\mu_7+p_{59}.\mu_9)\big) \tag{85}$$

and

$$D_1 = (1-p_{01}-p_{03}.p_{30})(1-p_{59}-p_{57}.p_{75})-p_{05}.p_{50} \tag{86}$$

4.2 Availability

Availability generally refers to a system's ability to remain operational and accessible whenever required. A system can be fully available when all components function without failure or partially available (degraded state) when some components experience malfunctions. In this context, the availability of the given two-unit system is examined under two scenarios:

Single-unit availability: The system is operational but in a degraded state (states S_3 and S_7). This happens when only the cogeneration system is working, and the mill profits exclusively from supplying electricity to the grid.

Dual-unit availability: The system is operational due to the functioning of both units (states S_0 and S_5). This leads to both sugar being produced and electricity being supplied to the grid.

4.2.1 Dual-Unit Availability of the System

To derive, let $A_i^d(t)$ corresponds to the probability that the system is in an operational state at the time 't' pertaining to the dual-unit availability of the system, provided it moves into the regenerative state 'S_i' at $t=0$. The following iterative relations are derived:

$$A_0^d(t) = M_0(t)+q_{01}(t)©A_1^d(t)+q_{02}(t)©A_2^d(t)+q_{03}(t)©A_3^d(t)+q_{05}(t)©A_5^d(t) \tag{87}$$

$$A_1^d(t) = q_{10}(t)©A_0^d(t) \tag{88}$$

$$A_2^d(t) = q_{20}(t)©A_0^d(t) \tag{89}$$

$$A_3^d(t) = q_{30}(t)©A_0^d(t)+q_{32}^{(4)}(t)©A_2^d(t) \tag{90}$$

$$A_5^d(t) = M_5(t)+q_{50}(t)©A_0^d(t)+q_{56}(t)©A_6^d(t)+q_{57}(t)©A_7^d(t)+q_{59}(t)©A_9^d(t) \tag{91}$$

$$A_6^d(t) = q_{65}(t)©A_5^d(t) \tag{92}$$

$$A_7^d(t) = q_{75}(t)©A_5^d(t)+q_{78}(t)©A_8^d(t) \tag{93}$$

$$A_8^d(t) = q_{87}(t)©A_7^d(t) \tag{94}$$

$$A_9^d(t) = q_{95}(t)©A_5^d(t) \tag{95}$$

where,
$M_0(t) = e^{-(\delta_3+\lambda_1+\lambda_2+\delta_1)t}$ and $M_5(t) = e^{-(\delta_2+\lambda_1+\lambda_2+\eta_1)t}$.

Taking the Laplace transform of these Equations (87) to (95), the steady-state availability when both units are working is given as

$$A_0^d = \lim_{s\to 0}\left[s.\frac{N_{21}(s)}{D_{21}(s)}\right] = \frac{N_{21}}{D_{21}} \tag{96}$$

where,

$$N_{21}(s)=\begin{vmatrix} M_0^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{32}^{(4)*}(s) & 1 & 0 & 0 & 0 & 0 & 0 \\ M_5^*(s) & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{57}^*(s) & 0 & -q_{59}^*(s) \\ 0 & 0 & 0 & 0 & -q_{65}^*(s) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -q_{75}^*(s) & 0 & 1 & -q_{78}^*(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_{87}^*(s) & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

and

$$D_{21}(s)=\begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -q_{30}^*(s) & 0 & -q_{32}^{(4)*}(s) & 1 & 0 & 0 & 0 & 0 & 0 \\ -q_{50}^*(s) & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{57}^*(s) & 0 & -q_{59}^*(s) \\ 0 & 0 & 0 & 0 & -q_{65}^*(s) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -q_{75}^*(s) & 0 & 1 & -q_{78}^*(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_{87}^*(s) & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

providing

$$N_{21}=p_{75}(p_{50}.\mu_0+p_{05}.\mu_5) \tag{97}$$

and

$$D_{21}=p_{75}\left(p_{50}\left(\mu_0+p_{01}.\mu_1+\left(p_{02}+p_{03}p_{32}^{(4)}\right)\mu_2+p_{03}.K_1\right)+p_{05}(\mu_5+p_{56}.\mu_6+p_{59}.\mu_9)\right)+ p_{05}p_{57}(\mu_7+p_{78}.\mu_8) \tag{98}$$

4.2.2 Single-Unit Availability of the System

To derive, let $A_i^s(t)$ be the probability that the system is in an operational state at the time 't' pertaining to the availability of only the cogeneration system, provided it moves into the regenerative state 'S_i' at $t=0$. The following iterative relations are obtained:

$$A_0^s(t)=q_{01}(t)©A_1^s(t)+q_{02}(t)©A_2^s(t)+q_{03}(t)©A_3^s(t)+q_{05}(t)©A_5^s(t) \tag{99}$$

$$A_1^s(t)=q_{10}(t)©A_0^s(t) \tag{100}$$

$$A_2^s(t)=q_{20}(t)©A_0^s(t) \tag{101}$$

$$A_3^s(t)=M_3(t)+q_{30}(t)©A_0^s(t)+q_{32}^{(4)}(t)©A_2^s(t) \tag{102}$$

$$A_5^s(t)=q_{50}(t)©A_0^s(t)+q_{56}(t)©A_6^s(t)+q_{57}(t)©A_7^s(t)+q_{59}(t)©A_9^s(t) \tag{103}$$

$$A_6^s(t)=q_{65}(t)©A_5^s(t) \tag{104}$$

$$A_7^s(t)=M_7(t)+q_{75}(t)©A_5^s(t)+q_{78}(t)©A_8^s(t) \tag{105}$$

$$A_8^s(t)=q_{87}(t)©A_7^s(t) \tag{106}$$

$$A_9^s(t)=q_{95}(t)©A_5^s(t) \tag{107}$$

where,
$M_3(t) = M_7(t) = e^{-\lambda_2 t}.\overline{G_1(t)}.$

Now, applying the Laplace transform to these Equations (99) to (107) and solving for steady state, the system's availability with only the cogeneration system operative is given as:

$$A_0^S = \lim_{s\to 0}\left[s.\frac{N_{22}(s)}{D_{22}(s)}\right] = \frac{N_{22}}{D_{22}} \tag{108}$$

where,

$$N_{22}(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_3^*(s) & 0 & -q_{32}^{(4)*}(s) & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{57}^*(s) & 0 & -q_{59}^*(s) \\ 0 & 0 & 0 & 0 & -q_{65}^*(s) & 1 & 0 & 0 & 0 \\ M_7^*(s) & 0 & 0 & 0 & -q_{75}^*(s) & 0 & 1 & -q_{78}^*(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_{87}^*(s) & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

and
$D_{22}(s) = D_{21}(s)$

giving

$$N_{22} = p_{03}p_{50}p_{75}\mu_3 + p_{05}p_{57}\mu_7 \tag{109}$$

and

$$D_{22} = D_{21} \tag{110}$$

4.3 Busy Period of the Repair Person

To determine the busy period of the repair person for repairing the failures incurred within the system, let $B_i(t)$ represents the probability that the repair person is fully occupied at the time *'t'*, assuming that the system transitioned to the regenerative state 'S_i' at $t = 0$. Then, the subsequent iterative relations are derived:

$$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t) + q_{03}(t)©B_3(t) + q_{05}(t)©B_5(t) \tag{111}$$

$$B_1(t) = q_{10}(t)©B_0(t) \tag{112}$$

$$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) \tag{113}$$

$$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{32}^{(4)}(t)©B_2(t) \tag{114}$$

$$B_5(t) = q_{50}(t)©B_0(t) + q_{56}(t)©B_6(t) + q_{57}(t)©B_7(t) + q_{59}(t)©B_9(t) \tag{115}$$

$$B_6(t) = W_6(t) + q_{65}(t)©B_5(t) \tag{116}$$

$$B_7(t) = W_7(t) + q_{75}(t)©B_5(t) + q_{78}(t)©B_8(t) \tag{117}$$

$$B_8(t) = W_8(t) + q_{87}(t)©B_7(t) \tag{118}$$

$$B_9(t) = q_{95}(t)©B_5(t) \tag{119}$$

where,

$W_2(t) = \overline{G_2(t)}$, $W_3(t) = e^{-\lambda_2 t}.\overline{G_1(t)} + (\lambda_2 e^{-\lambda_2 t} © 1)\overline{G_1(t)}$, $W_6(t) = \overline{G_2(t)}$, $W_7(t) = e^{-\lambda_2 t}.\overline{G_1(t)}$ and $W_8(t) = \overline{G_2(t)}$.

By applying the Laplace transform to these Equations (111) to (119) and solving them for the steady state, we can determine the repair person's busy period as:

$$B_0 = \lim_{s\to 0}\left[s.\frac{N_3(s)}{D_3(s)}\right] = \frac{N_3}{D_3} \tag{120}$$

where,

$$N_3(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_2^*(s) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_3^*(s) & 0 & -q_{32}^{(4)*}(s) & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{57}^*(s) & 0 & -q_{59}^*(s) \\ W_6^*(s) & 0 & 0 & 0 & -q_{65}^*(s) & 1 & 0 & 0 & 0 \\ W_7^*(s) & 0 & 0 & 0 & -q_{75}^*(s) & 0 & 1 & -q_{78}^*(s) & 0 \\ W_8^*(s) & 0 & 0 & 0 & 0 & 0 & -q_{87}^*(s) & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

and

$D_3(s) = D_{21}(s)$

providing

$$N_3 = p_{50}.\left(p_{75}.\left(\left(p_{02} + p_{03}.p_{32}^{(4)}\right).\mu_2 + p_{03}.\mu_3\right)\right) + p_{05}.(p_{56}.p_{75}.\mu_6 + p_{57}.\mu_7 + p_{57}.p_{78}.\mu_8) \tag{121}$$

and

$$D_3 = D_{21} \tag{122}$$

4.4 Expected Time for Preventive Maintenance

To determine the expected time for the system's preventive maintenance, let $P_i(t)$ represents the probability that the system is undergoing preventive maintenance at an instant 't', given that it entered the regenerative state 'S_i' at $t = 0$. The succeeding iterative relations for $P_i(t)$ are derived:

$$P_0(t) = q_{01}(t)©P_1(t) + q_{02}(t)©P_2(t) + q_{03}(t)©P_3(t) + q_{05}(t)©P_5(t) \tag{123}$$

$$P_1(t) = U_1(t) + q_{10}(t)©P_0(t) \tag{124}$$

$$P_2(t) = q_{20}(t)©P_0(t) \tag{125}$$

$$P_3(t) = q_{30}(t)©P_0(t) + q_{32}^{(4)}(t)©P_2(t) \tag{126}$$

$$P_5(t) = q_{50}(t)©P_0(t) + q_{56}(t)©P_6(t) + q_{57}(t)©P_7(t) + q_{59}(t)©P_9(t) \tag{127}$$

$$P_6(t) = q_{65}(t)©P_5(t) \tag{128}$$

$$P_7(t) = q_{75}(t)©P_5(t) + q_{78}(t)©P_8(t) \tag{129}$$

$$P_8(t) = q_{87}(t)©P_7(t) \tag{130}$$

$$P_9(t) = q_{95}(t)©P_5(t) \tag{131}$$

where,
$U_1(t) = e^{-\delta_4 t}$.

By applying the Laplace transform to these Equations (123) to (131) and solving for steady state, the expected time during which the system undergoes preventive maintenance is given by:

$$P_0 = \lim_{s\to 0}\left[s.\frac{N_4(s)}{D_4(s)}\right] = \frac{N_4}{D_4} \tag{132}$$

where,

$$N_4(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 \\ U_1^*(s) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{32}^{(4)*}(s) & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{57}^*(s) & 0 & -q_{59}^*(s) \\ 0 & 0 & 0 & 0 & -q_{65}^*(s) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -q_{75}^*(s) & 0 & 1 & -q_{78}^*(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_{87}^*(s) & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

and
$D_4(s) = D_{21}(s)$

which gives

$$N_4 = p_{01}p_{50}p_{75}\mu_1 \tag{133}$$

and

$$D_4 = D_{21} \tag{134}$$

4.5 Expected Down Time of the System at Full Capacity Period

To determine the expected down time of the system during its full capacity period, let $D_i(t)$ represents the probability that the system is in a down state at the time 't', provided that it entered the regenerative state 'S_i' at $t = 0$. The following iterative relations are derived for $D_i(t)$:

$$D_0(t) = q_{01}(t)©D_1(t) + q_{02}(t)©D_2(t) + q_{03}(t)©D_3(t) + q_{05}(t)©D_5(t) \tag{135}$$

$$D_1(t) = q_{10}(t)©D_0(t) \tag{136}$$

$$D_2(t) = q_{20}(t)©D_0(t) \tag{137}$$

$$D_3(t) = q_{30}(t)©D_0(t) + q_{32}^{(4)}(t)©D_2(t) \tag{138}$$

$$D_5(t) = q_{50}(t)©D_0(t) + q_{56}(t)©D_6(t) + q_{57}(t)©D_7(t) + q_{59}(t)©D_9(t) \tag{139}$$

$$D_6(t) = q_{65}(t)©D_5(t) \tag{140}$$

$$D_7(t) = q_{75}(t)©D_5(t) + q_{78}(t)©D_8(t) \tag{141}$$

$$D_8(t) = q_{87}(t)©D_7(t) \tag{142}$$

$$D_9(t) = U_9(t) + q_{95}(t)©D_5(t) \tag{143}$$

where,
$U_9(t) = e^{-\eta_2 t}$.

By implementing the Laplace transform to these Equations (135) to (143) and solving them for the steady state, the expected down time of the system at full capacity period can be evaluated as:

$$D_0 = \lim_{s\to 0}\left[s.\frac{N_5(s)}{D_5(s)}\right] = \frac{N_5}{D_5} \quad (144)$$

where,

$$N_5(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & -q_{03}^*(s) & -q_{05}^*(s) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{32}^{(4)*}(s) & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -q_{56}^*(s) & -q_{57}^*(s) & 0 & -q_{59}^*(s) \\ 0 & 0 & 0 & 0 & -q_{65}^*(s) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -q_{75}^*(s) & 0 & 1 & -q_{78}^*(s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_{87}^*(s) & 1 & 0 \\ DT_9^*(s) & 0 & 0 & 0 & -q_{95}^*(s) & 0 & 0 & 0 & 1 \end{vmatrix}$$

and
$D_5(s) = D_{21}(s)$

which yields

$$N_5 = p_{05}p_{59}p_{75}\mu_9 \quad (145)$$

and

$$D_5 = D_{21} \quad (146)$$

4.6 Profit Analysis

The profit attained through a system refers to the financial gain achieved when the revenue generated by the system exceeds its total costs. It is a key indicator of the system's efficiency and sustainability, reflecting its ability to create value. The profit generated through the given system can be determined using the following formula:

$$\text{Profit} = C_{01}.A_0^d + C_{02}.A_0^s - C_1.B_0 - C_2.P_0 - C_3.D_0 \quad (147)$$

where,
C_{01} = revenue generated per unit uptime of the system regarding dual-unit availability,
C_{02} = revenue generated per unit uptime of the system regarding single-unit availability,
C_1 = cost per unit time when the system is under repair by the repair person,
C_2 = cost per unit time when the system is under preventive maintenance and
C_3 = loss per unit time when the system is in a down state during the full capacity period.

5. Numerical Evaluation of Reliability Metrics

The current research investigates the influence of various parameters on the system's efficiency. Regarding this, the repair times of the SMS and the cogeneration system are considered to follow an exponential distribution with parameters α and β, respectively. Based on this specification, we have: $p_{30} = \frac{\alpha}{\lambda_2+\alpha}$,

$p_{32}^{(4)} = \frac{\lambda_2}{\lambda_2+\alpha}$, $p_{75} = \frac{\alpha}{\lambda_2+\alpha}$, $p_{78} = \frac{\lambda_2}{\lambda_2+\alpha}$, $\mu_0 = \frac{1}{\delta_3+\lambda_1+\lambda_2+\delta_1}$, $\mu_1 = \frac{1}{\delta_4}$, $\mu_3 = \frac{1}{\lambda_2+\alpha}$, $\mu_5 = \frac{1}{\delta_2+\lambda_1+\lambda_2+\eta_1}$, $\mu_6 = \frac{1}{\beta}$, $\mu_7 = \frac{1}{\lambda_2+\alpha}$, $\mu_8 = \frac{1}{\beta}$, $\mu_9 = \frac{1}{\eta_2}$, and $K_1 = \frac{1}{\alpha}$. Also, as per the data provided by the mill's officials, the parameters associated with the given model are as follows: $\lambda_1 = 0.024$, $\lambda_2 = 0.018$, $\alpha = 0.505$, $\beta = 0.535$, $\delta_1 = 0.00069$, $\delta_2 = 0.00046$, $\delta_3 = 0.00027$, $\delta_4 = 0.0416$, $\eta_1 = 0.0011$, and $\eta_2 = 0.0700$.

The various costs (all in Indian rupees) included in the analysis of the given system are: $C_{01} = 2000$, $C_{02} = 1000$, $C_1 = 2500$, $C_2 = 5000$, and $C_3 = 1000$. Using these rates and costs, the measures of the system's efficiency are found to be as $MTSF = 55.88887$, $A_0^d = 0.91343$, $A_0^s = 0.04281$, $B_0 = 0.07558$, $P_0 = 0.00237$, and $D_0 = 0.00861$.

6. Results and Discussions

Graphical representations are used to visualize key trends in system effectiveness measures, namely MTSF and availability, based on specified parameters.

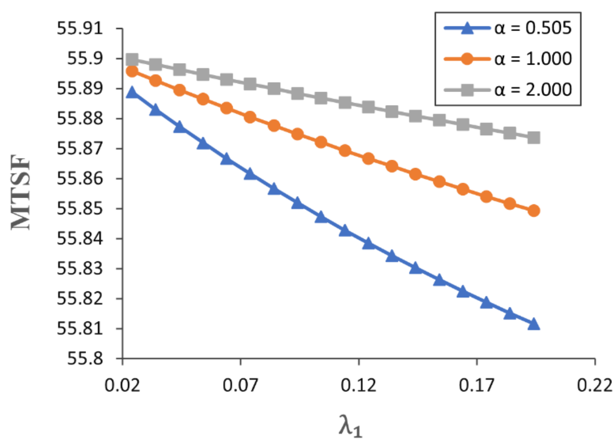

Figure 2. Relation between the MTSF and the failure rate of SMS (λ_1) for its varying repair rates (α).

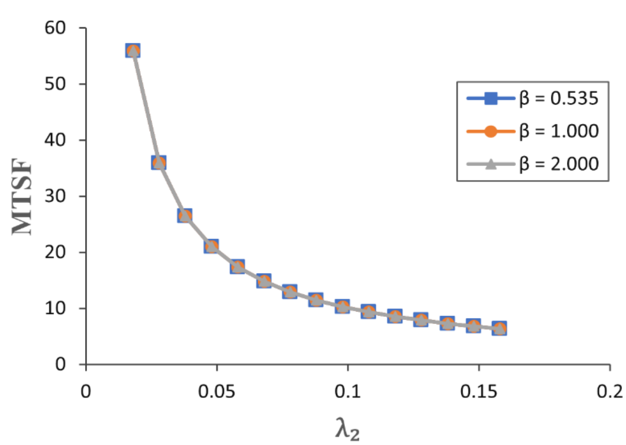

Figure 3. Relation between the MTSF and the failure rate of cogeneration system (λ_2) for its varying repair rates (β).

Figures 2 and **3** illustrate that the MTSF decreases as the failure rate of either the SMS or the cogeneration system increases. However, it increases with a rise in the SMS's repair rate while remaining unaffected by changes in the cogeneration system's repair rate. Moreover, **Figures 2** and **3** illustrate that the MTSF declines more rapidly as the cogeneration system's failure rate increases compared to the SMS failure rate. As a result, prioritizing the reduction of the cogeneration system's failure rate can significantly improve the system's operational efficiency.

The availability perspectives regarding single-unit and dual-unit availability corresponding to the failure rate of the SMS and the cogeneration system are shown in **Figures 4, 5, 6,** and **7**. **Figure 4** shows that the dual-unit availability of the system decreases with an increase in the SMS's failure rate, while **Figure 5** reveals that the single-unit availability of the system increases with an increase in the SMS's failure rate. On the other hand, in the context of the failure rate of the cogeneration system, **Figures 6** and **7** illustrate that both the dual-unit availability and single-unit availability decrease with the rising failure rate.

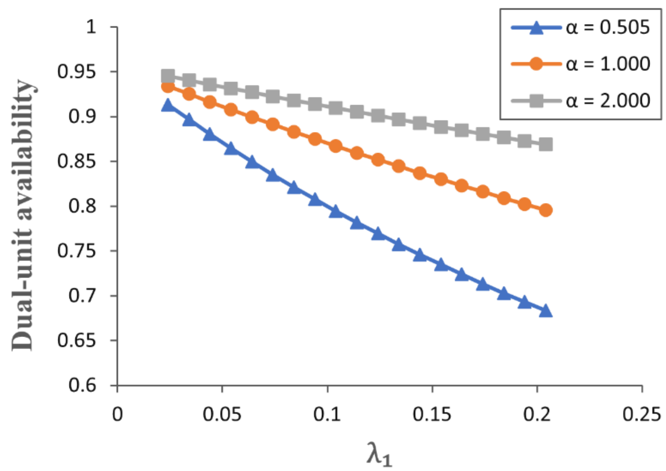

Figure 4. Impact of SMS failure rate (λ_1) on the system's dual-unit availability with its varying repair rates (α).

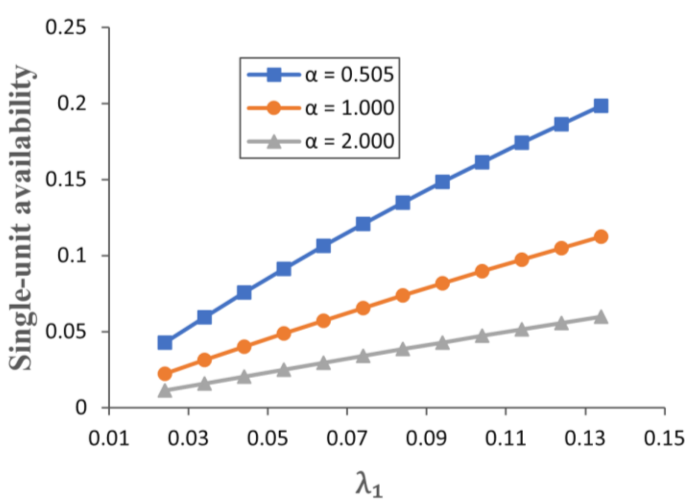

Figure 5. Impact of SMS failure rate (λ_1) on the system's single-unit availability with its varying repair rates (α).

The availability pattern (**Figure 5**), indicating that availability can rise with a growing failure rate, is also evident in the studies of Pinki et al. (2024) and Taneja et al. (2011). Although their system configuration and definitions were different, their findings still validate the possibility of this trend occurring under certain scenarios.

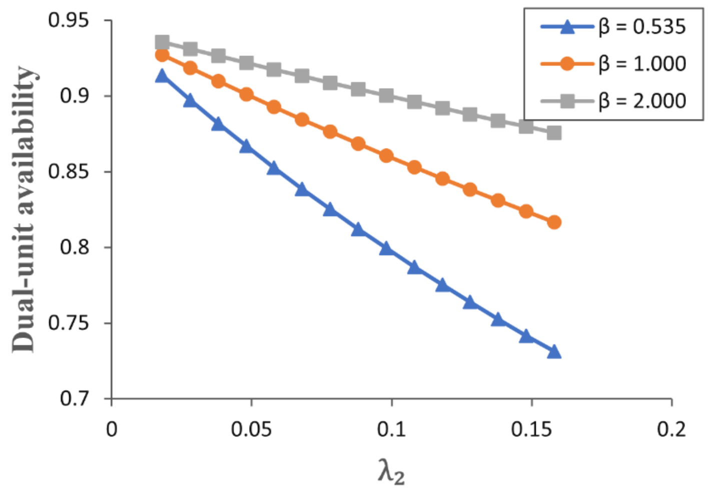

Figure 6. Impact of cogeneration system failure rate (λ_2) on the system's dual-unit availability with its varying repair rates (β).

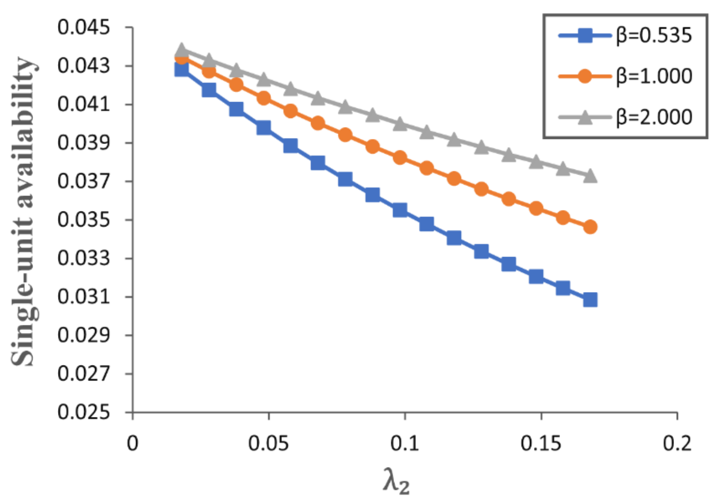

Figure 7. Impact of cogeneration system failure rate (λ_2) on the system's single-unit availability with its varying repair rates (β).

Profitability is the primary and ultimate concern of every industry. Evaluating a system's profitability requires a thorough analysis of key influencing factors. In this study, we focus on two critical aspects: the cost associated with the repair person's busy period for rectifying the failures within the system and the failure rate of the cogeneration system. By examining these parameters, we aim to explore various operational scenarios and their impact on overall system performance.

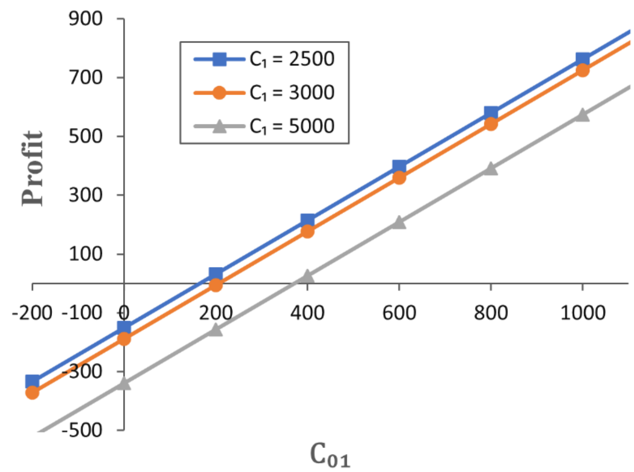

Figure 8. Profit v/s revenue associated with dual-unit availability (C_{01}) across varying repair costs (C_1).

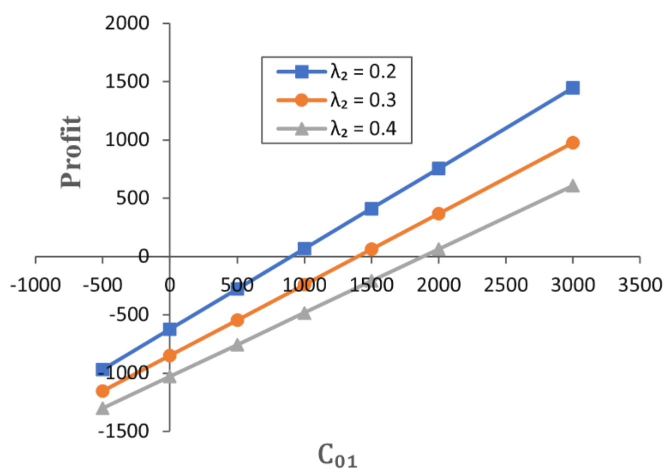

Figure 9. Profit v/s revenue associated with dual-unit availability (C_{01}) across varying failure rates of the cogeneration system (λ_2).

Figure 8 illustrates that for the values of C_1 as 2500, 3000, and 5000, the profit is positive when C_{01} exceeds 166.268607, 207.314801, or 371.4993750, respectively; neutral when it equals these values; and negative when it falls below them. Similarly, **Figure 9** shows that for the values of λ_2 as 0.2, 0.3, and 0.4, the profit remains positive if C_{01} is above 904.51678, 1397.580994, or 1887.89365, respectively; zero if it matches these values; and negative if it is lower than these values.

In general, a positive profit means the system is profitable, covering costs and generating surplus revenue. A neutral profit indicates a break-even point, where there is neither a profit nor a loss, while a negative profit signifies a loss, where costs exceed revenue, highlighting the need for better maintenance or pricing strategies. To sustain profitability, revenue (C_{01}) must remain above the identified thresholds in the given scenarios**.** From the above illustrations, the sugar mill operator can also ensure profitability by prioritizing repairs when revenue approaches the cutoff points, thereby preventing losses. Additionally, maintenance intervals can be adjusted in response to observed failure rates, ensuring timely interventions and minimizing costly downtimes. Furthermore, **Figures 8** and **9** suggest that a higher failure rate of the cogeneration system requires a higher minimum revenue (C_{01}) to maintain profitability, indicating that frequent breakdowns of the cogeneration system impose significant financial pressure. Similarly, as repair cost (C_1) increases, the system requires a higher revenue (C_{01}) to remain profitable, emphasizing the importance of cost management. Based on this analysis of potential variations in repair costs and the failure rate of the cogeneration system, decision-makers can develop the optimal strategy to maximize profit. This can be achieved by strategically pricing the sugar or electricity supplied to the grid.

7. Sensitivity Analysis

Sensitivity analysis is a technique used to assess the influence of input parameters on the system's functionality. It supports better decision-making by identifying the most influential parameters, allowing resources to be prioritized accordingly. Previous studies, including those by Kumar and Ram (2018), Monika and Chopra (2024), Monika et al. (2024), Poonia (2022), Saini and Kumar (2019), Sachdeva et al. (2022), and Tyagi et al. (2024), conducted sensitivity analyses to identify the most effective parameters for improving the performance of their respective systems. In the present paper, sensitivity analyses for MTSF and availability have been performed. Specifically, the sensitivity functions are expressed as partial derivatives of these performance metrics with respect to the parameters of interest 's'. These are:

$$\eta_m = \frac{\partial(MTSF)}{\partial s}, \eta_d = \frac{\partial(A_0^d)}{\partial s}, \eta_s = \frac{\partial(A_0^s)}{\partial s} \tag{148}$$

where, 's' represents the parameters $\lambda_1, \lambda_2, \alpha, \beta, \delta_1, \delta_2, \delta_3, \delta_4, \eta_1, \eta_2$. Here, η_m is the MTSF sensitivity function, η_d is the sensitivity function of dual-unit availability, and η_s is the sensitivity function of the single-unit availability of the system. **Table 3** provides the respective values of the sensitivity functions.

Table 3. Sensitivity of MTSF and availabilities with respect to given parameters.

Parameter 's'	MTSF sensitivity (η_m)	Availability sensitivity	
		Single-unit (η_s)	Dual-unit (η_d)
λ_1	-0.69036	1.70387	-1.70702
λ_2	-3107.37000	-0.10853	-1.63166
α	0.03168	-0.07983	0.08109
β	0	0.00257	0.05493
δ_1	23.68000	0.39345	-2.69286
δ_2	-0.84720	-0.59018	4.03929
δ_3	1232.77000	-0.37602	-8.02269
δ_4	-8.00115	0.00244	0.05207
η_1	26.21100	-0.33520	-7.15165
η_2	-0.41188	0.00526	0.11238

The positive and negative values indicate that an increase in these parameters can either enhance or diminish the system's performance metrics, respectively. From **Table 3**, it can be observed that the failure rate of the cogeneration system has the greatest impact on the MTSF, indicating that even a minor change in its value may significantly impact the MTSF. Regarding availability, single-unit availability is highly sensitive to the failure rate of the SMS, while dual-unit availability is almost equally sensitive to the failure rates of both the SMS and the cogeneration system.

8. Conclusion

This paper provides an analysis of the profitability and sensitivity of a two-unit system in a sugar mill. Various graphical illustrations are made depicting the behaviour of two types of availabilities in the considered system. The following conclusions are drawn:

(i) As the failure rate of SMS increases, both the MTSF and dual-unit availability of the system decrease. However, the single-unit availability increases with an increase in the failure rate of SMS, indicating that the probability of the system operating in a degraded state becomes higher as the failure rate of SMS increases.
(ii) The parametric sensitivity order for MTSF is as: $\lambda_2 > \delta_3 > \eta_1 > \delta_1 > \delta_4 > \delta_2 > \lambda_1 > \eta_2 > \alpha$, for single-unit availability as: $\lambda_1 > \delta_2 > \delta_1 > \delta_3 > \eta_1 > \lambda_2 > \alpha > \eta_2 > \beta > \delta_4$ and for dual-unit availability as: $\delta_3 > \eta_1 > \delta_2 > \delta_1 > \lambda_1 > \lambda_2 > \eta_2 > \alpha > \beta > \delta_4$. These parametric sensitivity orders indicate the influence of the various parameters on the respective reliability metrics, ranked from most to least influential.

The profitability of the considered sugar mill system can be enhanced by monitoring the parameters having a significant effect on the production process, particularly in terms of MTSF or system availability. Furthermore, the present model can be enhanced by adding a standby configuration for the cogeneration system, as sensitivity analysis reveals that MTSF is most significantly affected by the failure rate of the cogeneration system. While this study provides valuable insights into improving system reliability, it has certain limitations. First, the findings are primarily applicable to systems with similar operational characteristics, limiting their generalizability. Second, the study assumes specific failure and repair rate distributions, which may not fully capture real-world variability. Additionally, external factors such as environmental conditions, human intervention, and unforeseen disruptions were not explicitly considered, which may impact actual system performance. To ensure mathematical simplicity and maintain focus on reliability analysis, the profit function considers only fixed costs. While this offers a basic profitability estimate, future research can integrate variable costs for a more detailed financial evaluation. Additionally, validation of the proposed standby configurations and optimization approaches can further enhance reliability. Future studies may also explore advanced algorithms, including machine learning-based predictive maintenance, to improve failure mitigation strategies for specific system designs.

Conflict of Interest
The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments
This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The authors would like to thank the editor and anonymous reviewers for their comments that helped improve the quality of this work.

AI Disclosure
During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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