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Optimal Profit Analysis of Machine Repair Problem with Repair in Phases and Organizational Delay

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Abstract
In this article, we study machine repair problems (MRP) consisting of the finite number of operating machines with the provisioning of the finite number of warm standby machines under the care of a single unreliable server. For the machining system's uninterrupted functioning, an operating machine is immediately replaced with the available warm standby machine in negligible switchover time whenever it fails. The concept of threshold vacation policy: N-policy is also considered. Under this vacation policy, the server starts to serve the failed machines on the accumulation of a pre-specified number of failed machines in the system. The server continues until the system is empty from the failed machines; after that, the server goes for vacation. The notion of an organizational delay, server breakdown, and repair in multiple phases is also conceptualized to build the studied model more realistic. The recursive matrix method is used to find steady-state queue size distribution, and subsequently, various system performance measures are also developed to validate the studied model. The optimal analysis has been performed to identify the critical design parameters for the governing model. The state-of-the-art of the present study is its mathematical modeling of the multi-machine stochastic problem with varied limitations and strategies. The methodology to obtain queue size distribution, optimal design parameters, is beneficial for dealing with other complex and sophisticated real-time machining problems in the service

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system, computer and communication system, manufacturing and production system, etc. The present problem is limited to fewer machines, which can be extended to more machines with different topologies with high computational facilities.

Keywords - Warm standby, Threshold vacation policy, Unreliable server, Organizational delay, Repair in phases.

1. Introduction

The machine is an integral part of all industries, modernizations, and techno-socio advances, and it is also subject to random failure. Machine repair problem (MRP) is the most worrying issue in manufacturing industries, computer networks, communication systems, production systems, transportation systems, flexible manufacturing systems, etc. or where ever machine is required. Due to the unpredictable machine breakdown, there is a loss in production and a loss in time, resources, and cause inconvenience. When a machine fails, it is immediately sent to an in-house unreliable repair facility to avoid prolonged interruption.

To achieve optimal efficiency and reduce the production capacity's loss, the system usually maintains standby machines in spare. At the time of a failure of any operating machine, the available standby machine automatically replaces perfectly in negligible switchover time. So, the standby machine support helps obtain the high availability of the system. In this article, we study a machine repair problem having a finite number of operating and warm standby machines. The failure rate of the warm standby machine in the spare is less than that of the operating machine.

When the system does not have a failed machine to be repaired, i.e., the system is empty, the server goes for a vacation of random length to diminish expected service cost. Whenever the server returns from its vacation, if the server does not find waiting failed machine(s) in the system, it takes another vacation; otherwise takes a general setup time to repair. Then after, the server immediately starts serving the failed machines. The unreliable server is also prone to random breakdowns in the busy period. The repairman, who restores the server in k phases, requires some setup time, referred to as organizational delay.

The in-depth literature survey prompts the research gap on how vacation, repair in phases, and organization delay; the contradictory and correlated repair facility behaviors affect the multi-unit machining system's performance in a probabilistic environment. There is a need for scientific study to present a mathematical model of such a stochastic problem and obtain optimal strategies to deal effectively. This paper differs from earlier published articles on machine repair problem in: (i) The present study considers the standby provisioning MRP with repair in phases and organizational delay; (ii) the varied reliability characteristics are observed and parametric analysis has been done; (iii) the expected total cost function has been formulated, and Newton-quasi method has been employed to get optimal parameters value. The present study gives state-of-the-art mathematical models of multi-unit stochastic problems with many new machining terminologies and optimal strategies for varied incurred costs for different states.

In this article, we investigate a machine repair problem (MRP) with the unreliable server having M operating machines and S warm standby machines with threshold vacation policy, multi-phase repair, and organizational delay. The remaining of the article is structured as follows: In section 2, we give a brief literature review for the present problem. We describe the model using underlying assumptions and notations in section 3. In section 4, we develop a matrix representation of a governing forward Chapman-Kolmogorov system of equations and employ the matrix approach

for the computation of steady-state probabilities. In section 5, besides the expected total profit function, some performance measures are also presented. In section 6, using the direct-search method and Newton-quasi optimization technique, we determine the decision parameters' optimal values. Numerical results for comparative and optimal analysis are provided in section 7. Finally, in the last section 8, the conclusion is drawn, and future scope is discussed.

2. Literature Review

We consider a machine repair problem (MRP) where a finite number of operating machines are under the supervision of a single unreliable server in the repair facility. These operating machines are assumed unreliable and may fail randomly at any time. This incidence may lead to loss of production because the failed machine must wait in the repair facility for some time (Sztrik and Bunday, 1993; Haque and Armstrong, 2007; Shekhar et al., 2017e; Chopra et al., 2020; Shekhar et al., 2020b). The present study analyzes the machine repair problem (MRP) with the provisioning of warm standby machines whose failure rates are non-zero but lower than those of operating machines. The provisioning of warm standby machines is usually adopted to increase system reliability and availability (Shekhar et al., 2017a; Shekhar et al., 2020a). The server may be a machine in nature and is also prone to fail randomly. Machine repair problems (MRP) having an unreliable server are very commonly found in real life. It is a matter of great concern in a just-in-time (JIT) service system since it directly affects its efficiency. From the inception of the concept of unreliable server in the queueing system by White and Christle (1958), a considerable amount of research work has been done on machine repair problems with the unreliable server during the past decades (Wang, 1997; Grey et al., 2002; Haridass and Arumuganathan, 2008; Jain et al., 2012; Shekhar et al., 2017b; Kumar et al., 2019).

The server's continuous availability is necessary for efficient services, but in view of economic constraints, to avoid overheat or wear/tear, to diminish the server's idle time, vacationing of the server is the optimal policy. In queueing modeling, the N -policy concept is mainly incorporated to maintain the techno-economic constraints more effectively. Yadin and Naor (1963) introduced the idea of N -policy for vacation strategy. Henceforth, the N -policy is applied by many researchers in queueing problems of a variety of scenarios for providing better cost-effective service to arrivals. The N -policy utilizes the server's utility properly with no wastage of available resources (or servers) (Grey et al., 2000; Zhang and Tian, 2004; Parthasarathy and Sudhesh, 2008; Jain et al., 2016a; Shekhar et al., 2017d; Shekhar et al., 2020c). On looking at the benefits of N -policy in the socio-economic-techno environment, many theorists applied it in the machining system (Gupta, 1999; Jain and Upadhyaya, 2009; Sharma, 2012; Jain et al., 2016b; Shekhar et al., 2020d). The server's vacation and breakdown are highly correlated or sometimes contrary to each other and directly affect the service system (Wang et al., 1999; Wang et al., 2004; Jain and Bhargava, 2009).

Before starting the service and repair of failed machines and server, the respective startup time is required, as preparatory in many real-time systems, is referred to as organizational delay. It is also a significant factor as it also affects the overall cost as well as time taken in the service (Baker, 1973; Borthakur et al., 1987; Lee, 1990; Medhi and Templeton, 1992; Choudhury, 1997; Ke, 2004; Wang et al., 2007; Kumar and Jain, 2014). The repair facility seeks minimum setup time to initiate the service (repair) to failed machines (server). In machining systems and service systems, repair in phases is necessary to maintain the services' standards. At the end of one phase, the repairman can examine the process and results to start the next phase, or sometimes different expertise is

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required in different phases. The duration and sequence of the repair in phases are critical in repairing the failed machines (Neuts and Chakravarthy, 1981; Alfa and Frigui, 1996; Van Houdt and Alfa, 2005; Tadj et al., 2012).

The cost factors are prevalent from failure to repair through downtime and directly correlated to the system's running cost and affect revenue. To find the maximum profit for different machining systems, many of the researchers developed a profit function. They employ different numerical techniques like the Newton-quasi method, nature-inspired techniques, direct-search method, etc. to determine maximum benefit (Ke and Lin, 2008; Yang et al., 2013; Ke et al., 2015; Shekhar et al., 2017c; Pant et al., 2019; Shekhar et al., 2020e). The optimal analysis is useful to determine the optimal value of decision parameters at a maximum profit under some parametric or performance constraints.

3. Model Description

Notations

M	:	Number of operating machines	ε	:	Mean rate of vacationing for the server
S	:	Number of standby machines	a_0	:	Mean breakdown rate
λ	:	Mean failure rate of an operating machine	δ	:	Mean rate for organizational delay
α	:	Mean failure rate of a standby machine	k	:	Number of phases for the repair of server
λ_d	:	Mean failure rate of an operating machine when the system is in a degraded state	b_i	:	Mean repair rate in i^{th} phase $i = 1, 2, \ldots, k$
μ	:	Mean service rate of a server	L	:	Total number of machines
τ	:	Setup time			

In this article, we study a machine repair problem (MRP) comprising M identical operating machines working in parallel and the provisioning of S warm standby machines under the care of an unreliable server. The following are some assumptions and notations which we use to describe the present mathematical model:

- The time-to-failure of an operating machine is exponentially distributed with a mean rate λ.
- The time-to-failure of warm standby machines in spare follows the Poisson process with parameters α $(0 < \alpha < \lambda)$.
- As an operating machine fails, it is immediately replaced with the available warm standby machine. The switching is perfect, and switchover time is instantaneous. The functioning and failure characteristics of the substituted machine are as same as the operating machine. On the exhaust of all warm standbys, the system undergoes in degradation wherein the operating machine' time-to-failure follows an exponential distribution with an increased rate λ_d $(\lambda_d > \lambda)$

  Hence, the state-dependent failure rate of the machines in the system is as follows

$$\lambda_i = \begin{cases} M\lambda + (S-i)\alpha & ; \quad 0 \le i \le S-1 \\ (M+S-i)\lambda_d & ; \quad S \le i \le M+S-1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

- Failed machines are serviced in the order of failure, i.e., follows $FCFS$ discipline, by the working server. The server follows the threshold vacation policy, i.e., the server turns on when all standbys machines are exhausted and continues till there is no failed machine in waiting.

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The time-to-service of the failed machine is independent and identically distributed exponential variate with parameter μ.

- The setup time for initiating service taken by the server of random duration is exponentially distributed with a mean rate τ.
- On finding the system empty from the waiting failed machine, the time-to-go back to vacation for the server is exponentially distributed with a rate ε.
- The unreliable server is also subjected to breakdown, and the time-to-breakdown is exponentially distributed with mean a_0.
- The breakdown server is brought back to a normal working state with k-phase repair by a reliable repairman. For repairing the failed server, the setup time taken by the repairman or organizational delay is exponentially distributed with a rate of δ. The time-to-repair for the server in the ith phase $(i = 1, 2, ..., k)$ is exponentially distributed with parameter b_i.

The lifetime of the operating machine, warm standby machine, or server, service time, repair time, setup time, and organizational delay is independent of the state of the others. The system availability is analyzed based on the assumption that the system fails as soon as all the machines $L = M + S$ fail.

It is assumed that the system initially starts with no failed machine, and thus the server is in vacation state. A birth and death stochastic process can be used to analyze the system performance under the assumption of the exponential distributions for the following periods considered: failure time, setup time, service time, breakdown time, organizational delay, and repair time. The state of the system at a time t can be described by the Markov process $\Omega(t) = \{I(t), J(t), t \geq 0\}$ where $I(t), t \geq 0$ represents the number of failed machines in the system at the time t and $J(t)$ represents states of the server at the time t such that

$$J(t) = \begin{cases} 0 & ; \quad \text{the server is on vacation at the time } t \\ 1 & ; \quad \text{the server is busy at the time } t \\ 2 & ; \quad \text{the server is in breakdown state at the time } t \\ 3 & ; \quad \text{the setup completed for repair at the time } t \\ l+2 & ; \quad l^{th}; 2 \leq l \leq k \text{ phase of repair completed at the time } t \end{cases}$$

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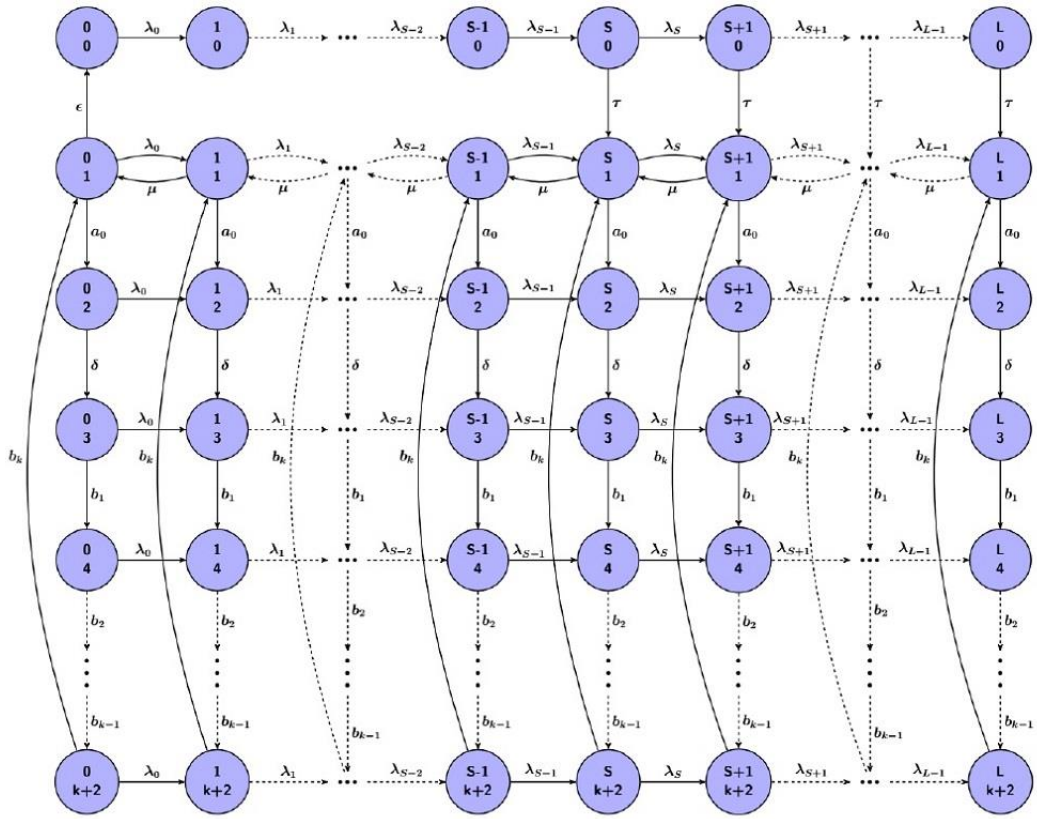

Figure 1. Transition state diagram.

The steady-state probability is denoted by $P_{i,j} = \lim_{t\to 0} P(I(t)=i, J(t)=j), 0 \le i \le M+S,$ $0 \le j \le k+2.$ The state transition of the present stochastic model is described in the transition state diagram in Figure 1.

4. Matrix Representation

Because the continuous-time Markov chain (CTMC) shown in Figure 1 is irreducible, aperiodic, and positive recurrent, the system has a unique steady-state probability distribution. We implement the matrix-analytic method to obtain the stationary probabilities. To solve the balance equations for the stationary distribution, we construct the following transition rate matrix **Q** in the block partition form

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$$\mathbf{Q}=\begin{bmatrix} \mathbf{A}_0 & \mathbf{C}_0 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{A}_1 & \mathbf{C}_1 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} & \mathbf{A}_2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{L-2} & \mathbf{C}_{L-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B} & \mathbf{A}_{L-1} & \mathbf{C}_{L-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B} & \mathbf{A}_L \end{bmatrix}$$

where all the sub-matrices are the square matrices of order $(k+3)$. We define each matrix element of the rate matrix **Q** as follows

$$\mathbf{A}_0 =$$

$$\begin{bmatrix} -\lambda_0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \varepsilon & -(\lambda_0+\varepsilon+a_0) & a_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -(\lambda_0+\delta) & \delta & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -(\lambda_0+b_1) & b_1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\lambda_0+b_2) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -(\lambda_0+b_{k-1}) & b_{k-1} \\ 0 & b_k & 0 & 0 & 0 & \cdots & 0 & -(\lambda_0+b_k) \end{bmatrix}$$

$$\mathbf{A}_i =$$

$$\begin{bmatrix} -\lambda_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -(\lambda_i+\mu+a_0) & a_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -(\lambda_i+\delta) & \delta & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -(\lambda_i+b_1) & b_1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\lambda_i+b_2) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -(\lambda_i+b_{k-1}) & b_{k-1} \\ 0 & b_k & 0 & 0 & 0 & \cdots & 0 & -(\lambda_i+b_k) \end{bmatrix}$$

for $1 \le i \le S-1$

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$$\mathbf{A}_i = \begin{bmatrix} -(\lambda_i+\tau) & \tau & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -(\lambda_i+\mu+a_0) & a_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -(\lambda_i+\delta) & \delta & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -(\lambda_i+b_1) & b_1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & -(\lambda_i+b_2) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -(\lambda_i+b_{k-1}) & b_{k-1} \\ 0 & b_k & 0 & 0 & 0 & \cdots & 0 & -(\lambda_i+b_k) \end{bmatrix}$$

for $S \le i \le L-1$

$$\mathbf{A}_L = \begin{bmatrix} -\tau & \tau & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -(\mu+a_0) & a_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -\delta & \delta & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -b_1 & b_1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & -b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -b_{k-1} & b_{k-1} \\ 0 & b_k & 0 & 0 & 0 & \cdots & 0 & -b_k \end{bmatrix}$$

$\mathbf{B} = diag[0, \mu, 0, ..., 0]$ and $\mathbf{C}_i = diag[\lambda_i, \lambda_i, \lambda_i, ..., \lambda_i]$; $0 \le i \le L-1$

4.1 Recursive Approach

We define $\mathbf{\Pi} = [\mathbf{\Pi}_0, \mathbf{\Pi}_1, \mathbf{\Pi}_2, ..., \mathbf{\Pi}_L]$ as the steady-state probability vector corresponding to rate matrix **Q**, such that,

$$\mathbf{\Pi Q} = \mathbf{0} \tag{1}$$

where $\mathbf{\Pi}_i = [P_{i,0}, P_{i,1}, P_{i,2}, ..., P_{i,k+2}]$; $i = 0,1,2,...,L$ is a $1 \times (k+3)$ dimensional vector. Then, with the help of partition of the steady-state probability vector $\mathbf{\Pi}$, we get

$$[\mathbf{\Pi}_0 \mathbf{A}_0 + \mathbf{\Pi}_1 \mathbf{B}] = \mathbf{0} \tag{2}$$

$$[\mathbf{\Pi}_{i-1} \mathbf{C}_{i-1} + \mathbf{\Pi}_i \mathbf{A}_i + \mathbf{\Pi}_{i+1} \mathbf{B}] = \mathbf{0}; 1 \le i \le L-1 \tag{3}$$

$$[\mathbf{\Pi}_{L-1} \mathbf{C}_{L-1} + \mathbf{\Pi}_L \mathbf{A}_L] = \mathbf{0} \tag{4}$$

After performing some routine substitution, we obtain the following results

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$$\mathbf{\Pi}_L = -\mathbf{\Pi}_{L-1}\mathbf{C}_{L-1}\mathbf{A}_L^{-1} \tag{5}$$

$$\mathbf{\Pi}_i = \mathbf{\Pi}_{i-1}\mathbf{X}_i;\ \ 1 \le i \le L-1 \tag{6}$$

$$\mathbf{\Pi}_0\left[\mathbf{A}_0 + \mathbf{X}_1\mathbf{B}\right] = \mathbf{0}, \tag{7}$$

where,

$$\mathbf{X}_i = -\mathbf{C}_{i-1}\left[\mathbf{A}_i + \mathbf{X}_{i+1}\mathbf{B}\right]^{-1}\ ;1 \le i \le L-2 \tag{8}$$

$$\mathbf{X}_{L-1} = -\mathbf{C}_{L-2}\left[\mathbf{A}_{L-1} - \mathbf{C}_{L-1}\mathbf{A}_L^{-1}\mathbf{B}\right]^{-1} \tag{9}$$

Here, with the help of (5) and (6), one can find easily $\mathbf{\Pi}_L, \mathbf{\Pi}_{L-1}, \mathbf{\Pi}_{L-2}, \ldots, \mathbf{\Pi}_1$ recursively, and $\mathbf{\Pi}_0$ can be determined by (7) with the following normalization condition,

$$\sum_{i=0}^{L} \mathbf{\Pi}_i \boldsymbol{e} = 1 \tag{10}$$

where $\boldsymbol{e}$ is a column vector with a suitable size, and each element equals to one.

5. Performance Measures

For validation purposes, we propose some system performance measures of the machine repair problem in this section.

5.1 Queueing Characteristics

We obtain the following performance measures in terms of the steady-state probabilities $\mathbf{\Pi}_i = \left[P_{i,0}, P_{i,1}, P_{i,2}, \ldots, P_{i,k+2}\right]$; for $i = 0, 1, 2, \ldots, L$ obtained in the previous section.

The expected number of failed machines in the system

$$E(N) = \sum_{j=0}^{L} j\mathbf{\Pi}_j \boldsymbol{e} \tag{11}$$

The throughput of the system is defined as the expected number of the serviced machine in the system. It can be represented as

$$\tau_p = \sum_{j=1}^{L} \mu \mathbf{\Pi}_j \boldsymbol{u}_2 \tag{12}$$

The expected number of operating machines in the system

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$$E(O) = M\sum_{j=0}^{S}\mathbf{\Pi}_j\boldsymbol{e} + \sum_{j=S+1}^{L}(M+S-j)\mathbf{\Pi}_j\boldsymbol{e} \tag{13}$$

The expected number of standby machines in the system

$$E(S) = \sum_{j=0}^{S}(S-j)\mathbf{\Pi}_j\boldsymbol{e} \tag{14}$$

The effective failure rate of the machines in the system

$$\lambda_{eff} = \sum_{j=0}^{L-1}\lambda_j\mathbf{\Pi}_j\boldsymbol{e} \tag{15}$$

The expected waiting time of the failed machines in the system

$$E(W) = \frac{E(N)}{\lambda_{eff}} \tag{16}$$

The availability of the system

$$A_\upsilon = (1-\mathbf{\Pi}_L\boldsymbol{e}) \tag{17}$$

The expected delay time

$$E(D) = \frac{E(N)}{\tau_p} \tag{18}$$

The failure frequency of the system

$$FF = \lambda_{L-1}\mathbf{\Pi}_{L-1}\boldsymbol{e} \tag{19}$$

The probability that the server is in the busy state

$$P(B) = \sum_{j=1}^{L}\mathbf{\Pi}_j\boldsymbol{u}_2 \tag{20}$$

The probability that the server is in the breakdown state

$$P(D) = \sum_{j=0}^{L}\mathbf{\Pi}_j\boldsymbol{u}_3 \tag{21}$$

The probability that the lth $(l = 1,2,\ldots,k-1)$ phase repairing of the server is completed

$$P_l(R) = \sum_{j=0}^{L}\mathbf{\Pi}_j\boldsymbol{u}_{l+4} \tag{22}$$

The probability that the server is in the accumulation state

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$$P(A)=\sum_{j=0}^{S-1}\mathbf{\Pi}_j\boldsymbol{u}_1 \tag{23}$$

where $\boldsymbol{u}_i$ $(i=1,2,\ldots,k+3)$ is the column vector of dimension $(k+3)\times 1$ with i^{th} entry equals to 1 and 0 elsewhere.

5.2 Expected Profit Function

We numerically analyze the worth of the studied machining system under random failure and systematic service or repair, so we construct an expected total profit function to quantify the total profit incurred in operating the system. We develop the expected total profit function in terms of critical system performance measures defined in the previous section.

The governing revenue and cost elements related to different states of the Markovian model of machine repair problem with an unreliable server, repair in phases and setup time are defined as follows

$R_\upsilon \equiv$ Revenue when one operating machine is operating.

$C_O \equiv$ Cost for each of the operating machines present in the system.

$C_H \equiv$ Cost for each failed machine present in the system.

$C_S \equiv$ Cost for each standby machine present in the system.

$C_M \equiv$ Fixed cost of offering service with rate μ to the failed machine.

$C_B \equiv$ Cost whenever the server is in the first phase of service.

$C_T \equiv$ Fixed cost for setup of the server to repair the failed machine.

Based on the definition of each element which is listed above and its corresponding system performance, which is linearly proportional to revenue/cost element, the expected total profit function is given by:

$$E(TP(S,\mu,\tau))=(R_\upsilon-C_O)E(O)-C_H E(N)-C_S E(S)-C_M\mu-C_B b_1-C_T\tau \tag{24}$$

where $E(O)$, $E(N)$, $E(S)$, μ, b_1, τ are defined in the previous section.

The expected total profit function $E(TP(S,\mu,\tau))$ in equation (24) is nonlinear in the expected number of respective quantity and rates. Due to the highly nonlinear and complex nature of the above optimization problem, it is a complicated task to develop an analytic result for the optimum solution, say, (S^*,μ^*,τ^*). For obtaining the optimal value of the decision parameters (S^*,μ^*,τ^*) numerically, the direct search method (DSM) and the Newton-quasi method (NQM) are employed as the optimization techniques, respectively, for discrete and continuous variables in nature. In the next section, we first use a direct search method to determine the optimal number of warm standbys

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machines (S) say S^*, when μ and τ are fixed. Subsequently, we set S^* and apply the Newton-quasi method to determine the optimal solution of (μ, τ), say (μ^*, τ^*).

6. Profit Maximization

This section aims to determine the optimal values of decision parameters (S, μ, τ) for the studied machine repair problem. For that purpose, we develop the expected total profit function in terms of three decision variables $S, \mu,$ and τ in equation (24) where the discrete variable S is necessary to be a positive integer and the continuous variables μ and τ are positive real numbers. We aim to find the optimal value of $S, \mu,$ and τ say, S^*, μ^* and τ^* to maximize the expected total profit value of the studied system. In the following subsections, we employ the direct search method (DSM) for a discrete variable and the Newton-quasi method (NQM) for continuous variables to maximize the profit function. Some numerical results are also provided for illustrative purposes.

6.1 Direct Search Method

On substituting the equations (11), (13), and (14) for $E(N)$, $E(O)$, and $E(S)$ in equation (24), the expected total profit function $E(TP(S, \mu, \tau))$ is too difficult to express explicitly. It is too challenging to develop an analytic procedure for determining the optimal value also. Additionally, it is complicated to show the concavity of this function analytically. We illustrate graphically in the next section for some specified parameters. To determine the optimum solution in the desired region, we employ the direct search method to find an optimal value S^*.

The objective is to find the optimal value S^* to maximize the expected total profit function while maintaining the minimum availability requirement A_0. We consider the following optimization problem

$$TP(S^*, \mu, \tau) = \max_S TP(S, \mu, \tau) \text{ Subject to: } A_\upsilon \geq A_0 .$$

In the direct search method, we check all possible value of S such that

$$TP(S^* - 1, \mu, \tau) \leq TP(S^*, \mu, \tau) \text{ and } TP(S^* + 1, \mu, \tau) \leq TP(S^*, \mu, \tau).$$

6.2 Newton-Quasi Method

After determining the optimal value S^* through the direct search method, we fix S^* and employ the Newton-quasi method to search μ and τ globally until the maximum value $TP(S^*, \mu, \tau)$ is achieved. The profit maximization problem can be illustrated mathematically as:

$$TP(S^*, \mu^*, \tau^*) = \max_{\mu, \tau} TP(S^*, \mu, \tau) \text{ Subject to : } A_\upsilon \geq A_0 .$$

The Newton-quasi method is a good and reliable technique for finding a maximum of the nonlinear function. The significance of NQM is to find a search direction with an iterative procedure and then trying different step size along that direction for a better solution until the tolerance is achieved.

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We designate the vector $\mathbf{\Theta}$ consisting of μ and τ, and establish the respective gradient $\vec{\nabla} TP(\mathbf{\Theta})$, which includes $\frac{\partial(TP)}{\partial\mu}$ and $\frac{\partial(TP)}{\partial\tau}$. Let the initial solution corresponding to (μ,τ) be (μ_0,τ_0). The step-by-step procedure of the Newton-quasi method is performed as follows:

1. Let $\mathbf{\Theta}_{\mathrm{n}=0} = [\mu_0, \tau_0]^T$.
2. When the parameter S^* is fixed through the DSM, set the initial trial solution for $\mathbf{\Theta}_{\mathrm{n}=0}$ and compute the value of profit function $TP(S^*, \mu_0, \tau_0)$, i.e.,

$$TP(\mathbf{\Theta}_{\mathrm{n}=0}) = (R_\upsilon - C_O)E(O) - C_H E(N) - C_S E(S) - C_M \mu - C_B b_1 - C_T \tau.$$

3. Compute the profit function $\vec{\nabla} TP(\mathbf{\Theta}_{\mathrm{n}=0}) = \left[\frac{\partial(TP)}{\partial\mu}, \frac{\partial(TP)}{\partial\tau}\right]^T \Bigg|_{\mathbf{\Theta}_{\mathrm{n}=0}}$ and the Hessian profit matrix

$$\mathbf{H}(\mathbf{\Theta}_{\mathrm{n}=0}) = \begin{bmatrix} \frac{\partial^2(TP)}{\partial\mu^2} & \frac{\partial^2(TP)}{\partial\mu\partial\tau} \\ \frac{\partial^2(TP)}{\partial\mu\partial\tau} & \frac{\partial^2(TP)}{\partial\tau^2} \end{bmatrix}_{\mathbf{\Theta}_{\mathrm{n}=0}}.$$

4. Compute the new trial solution $\mathbf{\Theta}_{\mathrm{n}+1} = \mathbf{\Theta}_{\mathrm{n}} - [\mathbf{H}(\mathbf{\Theta}_{\mathrm{n}})]^{-1} \vec{\nabla} TP(\mathbf{\Theta}_{\mathrm{n}})$.
5. Set $n = n+1$ and repeat steps 2 to 4 until $\frac{\partial(TP)}{\partial\mu} < \Delta_1$ and $\frac{\partial(TP)}{\partial\tau} < \Delta_2$, where $\Delta_1 = \Delta_2 = 10^{-6}$ are the tolerances.
6. Find the globally maximum value $TP(\mu^*, \tau^*)$.

7. Numerical Results

To establish the given model's quality, the analytical discussion of the performance measures is not sufficient. Numerous numerical experiments are provided and performed in this section. Section 5.1 presents queueing characteristics to visualize the efficiency of the studied machining model. The profit function is then provided in section 5.2 to determine the machining system's optimal governing parameter values.

The base case for the setting of system parameters and unit cost/revenue values is set below:

$$M = 30,\ m = 5,\ k = 2,\ \lambda = 0.05,\ \lambda_d = 0.09,\ \alpha = 0.01,\ a_0 = 0.001,\ \varepsilon = 0.2,\ \tau = 1,\ \delta = 2,$$
$$\mu = 3,\ b_i = 25,\ C_H = 30,\ C_O = 25,\ C_S = 20,\ C_M = 2,\ C_B = 5,\ R_\upsilon = 150,\ C_T = 5.$$

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This section first examines how each parameter affects the system characteristics by varying each governing system parameter value. Each system parameter takes turns to vary in a specific range while keeping other system parameters fixed at the level of the base case; the results are summarized in Tables 1-7 and Figures 2-8.

Figures 2 and 3 depict the line plot for the expected number of failed machines in the system $E(N)$ wrt failure rate of operating machines λ and service rate μ, respectively. It is observed that $E(N)$ is increased with the increment of λ, and it is more prevalent for the higher value of the number of operating machines M and standby machines S. The reverse trend is observed with an increased value of service rate μ. The results prompt system designers to manage preventive measures properly to avoid the operating machines' failure and to employ corrective actions timely to provide the service to failed machines on demand.

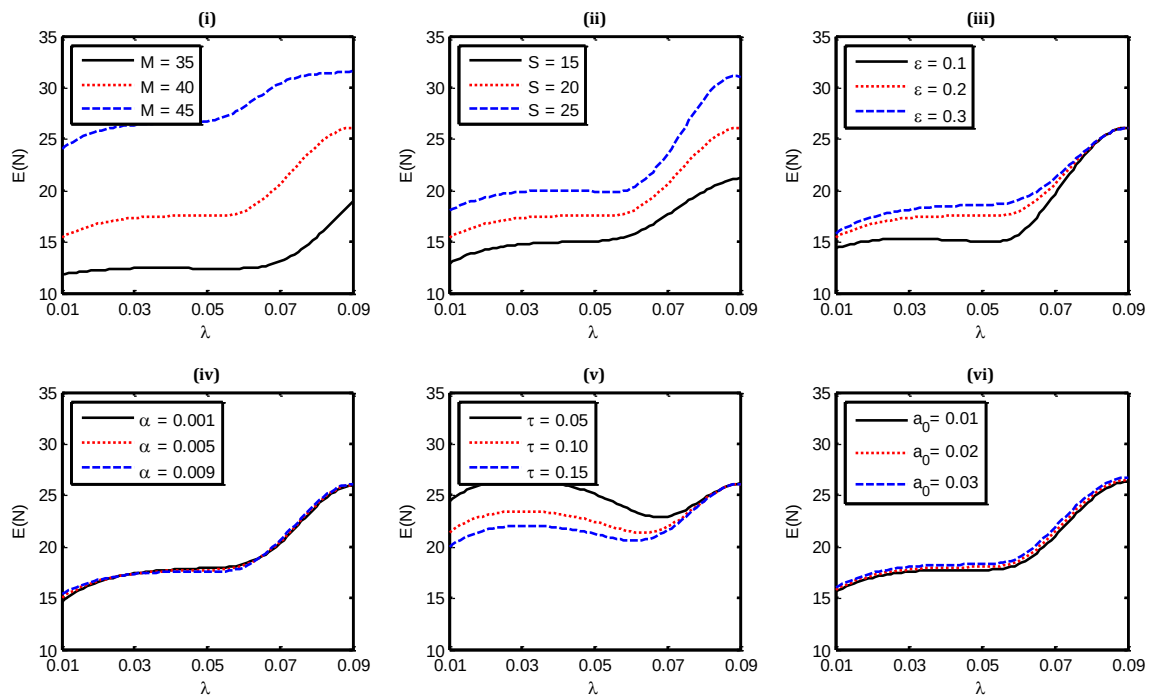

Figure 2. The expected number of failed machines in the system $E(N)$ wrt failure rate of the operating machines (λ).

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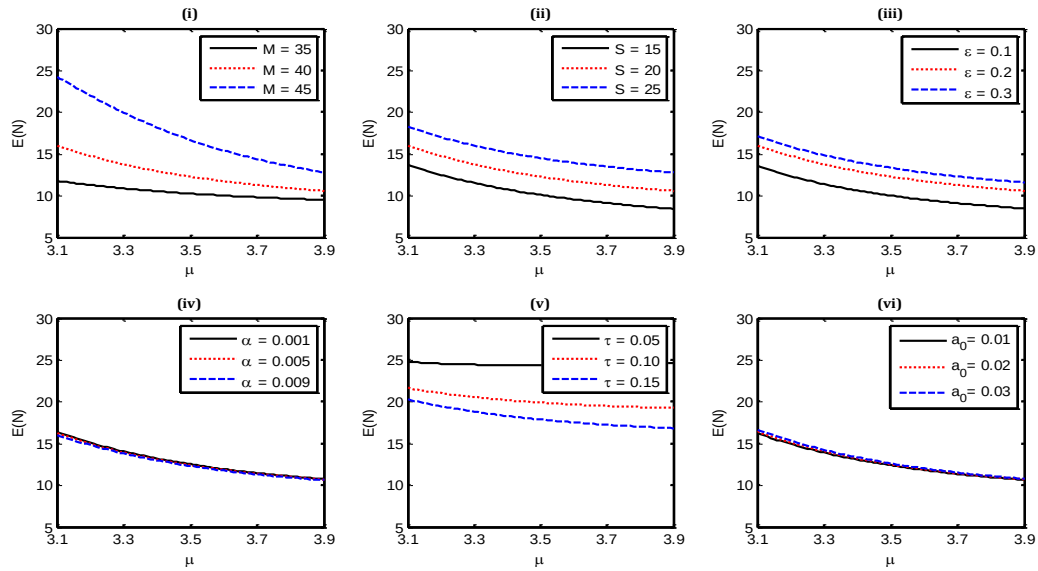

Figure 3. The expected number of failed machines in the system $E(N)$ wrt service rate of the failed machines (μ).

The throughput of the system τ_p variation is depicted in Figures 4 and 5 as a bar graph with respect to the failure rate of the operating machines λ and service rate μ, respectively. Besides failure rate and service rate, the number of operating machines *M*, setup time τ are critical parameters. In contrast, the number of standby machines *S*, standby failure rate α, the time before going for vacation ε have moderate effects. These results also recommend the same to follow preventive-corrective measures efficiently and timely.

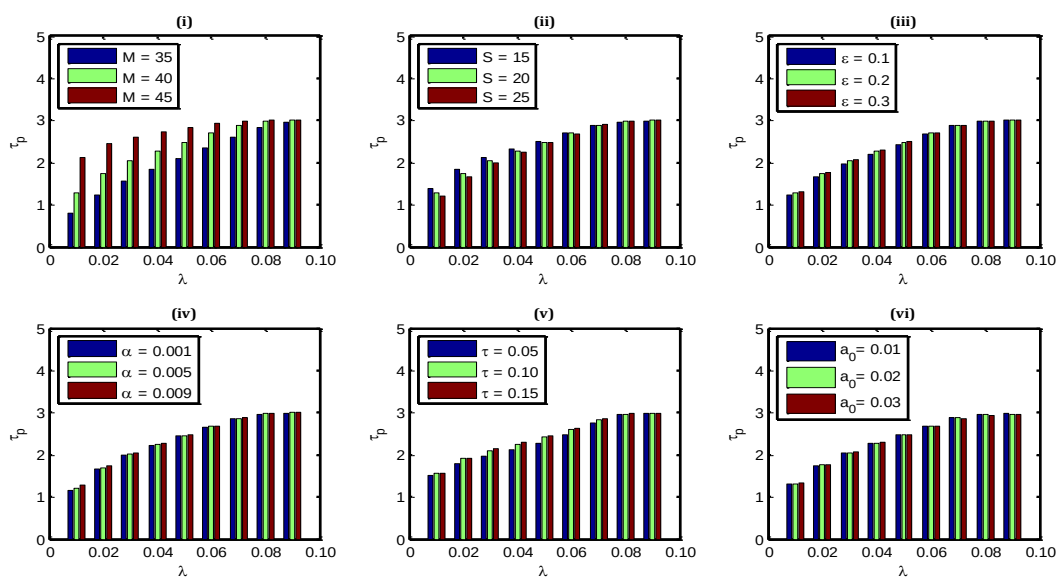

Figure 4. The throughput of the system (τ_p) wrt the failure rate of the operating machines (λ).

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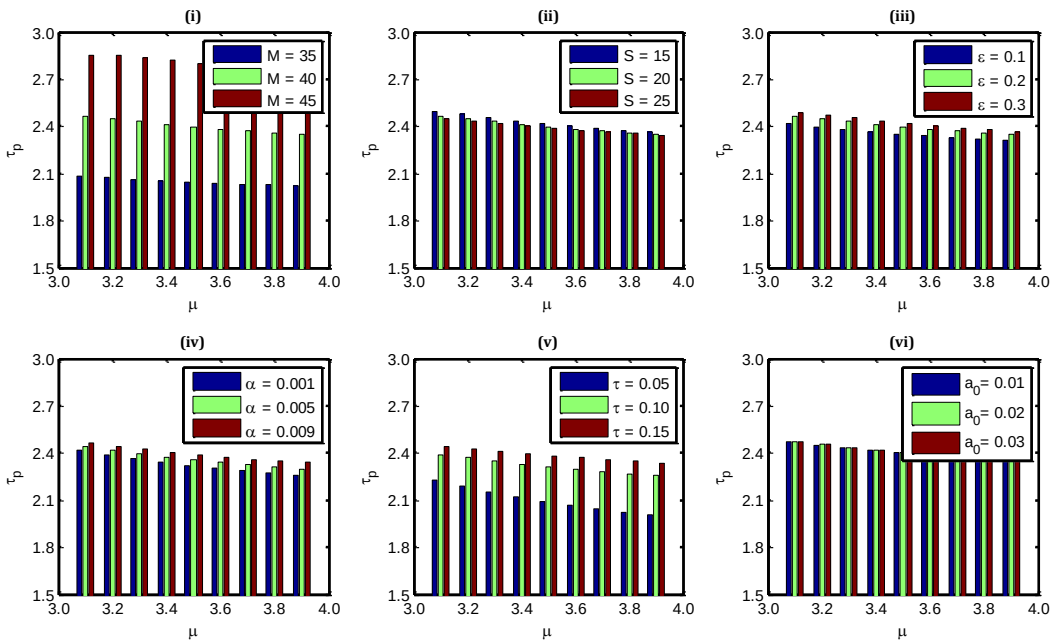

Figure 5. The throughput of the system (τ_p) wrt service rate of the failed machines (μ).

In Tables 1-7, numerical results for $E(O)$, $E(S)$, $E(O)$, λ_{eff}, $E(W)$, $E(D)$, FF, $P(B)$, $P(D)$, and $P(A)$ are summarized for varying values of various parameters. These tables give a concise framework for the machining system's running/downtime period as per requirements.

Table 1. Performance measures by varying the number of operating machine (M) and the failure rate of operating machine (λ).

M	λ	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
35	0.02	33.7061	9.0735	1.2277	9.9536	9.9536	3.88E-09	4.09E-01	2.52E-04	4.76E-01
35	0.05	33.4394	9.1725	2.0957	5.9111	5.9111	3.70E-09	6.99E-01	3.94E-04	1.93E-01
35	0.08	33.1877	6.3922	2.8258	5.4569	5.4569	1.16E-09	9.42E-01	4.82E-04	3.07E-02
40	0.02	36.4270	6.8276	1.7404	9.6218	9.6218	1.07E-09	5.80E-01	3.27E-04	3.30E-01
40	0.05	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
40	0.08	33.9901	1.7443	2.9790	8.1457	8.1457	4.95E-10	9.93E-01	4.98E-04	3.23E-03
45	0.02	36.0548	3.1914	2.4434	10.5403	10.5403	5.40E-10	8.14E-01	4.25E-04	1.42E-01
45	0.05	35.5853	2.7279	2.8321	9.4228	9.4228	5.05E-10	9.44E-01	4.82E-04	3.25E-02
45	0.08	33.4896	0.2062	2.9975	10.4433	10.4433	5.38E-10	9.99E-01	5.00E-04	1.15E-04

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Table 2. Performance measures by varying the number of operating machine (M) and the degraded failure rate of operating machine (λ_d).

M	λ_d	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
35	0.05	34.8185	11.5993	1.8569	4.6217	4.6217	8.38E-13	6.19E-01	3.66E-04	2.45E-01
35	0.09	33.4394	9.1725	2.0957	5.9111	5.9111	3.70E-09	6.99E-01	3.94E-04	1.93E-01
35	0.13	24.5168	1.7041	2.8306	10.1672	10.1672	3.09E-06	9.44E-01	4.80E-04	3.59E-02
40	0.05	39.7503	11.3482	2.1010	4.2368	4.2368	8.59E-14	7.00E-01	3.98E-04	1.83E-01
40	0.09	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
40	0.13	23.2717	0.1658	2.9852	12.2481	12.2481	3.42E-06	9.95E-01	4.98E-04	2.68E-03
45	0.05	44.6471	10.8026	2.3404	4.0807	4.0807	9.98E-15	7.80E-01	4.28E-04	1.29E-01
45	0.09	35.5853	2.7279	2.8321	9.4228	9.4228	5.05E-10	9.44E-01	4.82E-04	3.25E-02
45	0.13	23.0759	0.0072	2.9978	13.9824	13.9824	3.46E-06	9.99E-01	5.00E-04	8.61E-05

Table 3. Performance measures by varying the number of standby machines (S) and the failure rate of standby machine (α).

S	α	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
15	0.01	35.5224	4.4266	2.5123	5.9909	5.9909	1.00E-09	8.37E-01	4.49E-04	8.85E-02
15	0.02	35.6398	4.5348	2.5473	5.8200	5.8200	9.60E-10	8.49E-01	4.54E-04	8.07E-02
15	0.03	35.7663	4.6383	2.5835	5.6494	5.6494	9.11E-10	8.61E-01	4.58E-04	7.31E-02
20	0.01	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
20	0.02	36.1884	6.8061	2.5350	6.7082	6.7082	8.44E-10	8.45E-01	4.48E-04	9.30E-02
20	0.03	36.4215	7.1232	2.5890	6.3557	6.3557	7.75E-10	8.63E-01	4.55E-04	8.07E-02
25	0.01	36.3782	8.7322	2.4657	8.0666	8.0666	8.17E-10	8.22E-01	4.36E-04	1.18E-01
25	0.02	36.6584	9.3293	2.5363	7.4962	7.4962	7.42E-10	8.45E-01	4.45E-04	1.00E-01
25	0.03	37.0153	9.9954	2.6129	6.8849	6.8849	6.47E-10	8.71E-01	4.55E-04	8.19E-02

Table 4. Performance measures by varying the number of standby machines (S) and the setup time taken by the server (τ).

S	τ	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
15	0.50	34.9430	4.0064	2.5252	6.3563	6.3563	7.29E-07	8.42E-01	4.49E-04	8.01E-02
15	1.00	35.5224	4.4266	2.5123	5.9909	5.9909	1.00E-09	8.37E-01	4.49E-04	8.85E-02
15	1.50	35.7811	4.6579	2.4998	5.8248	5.8248	3.34E-10	8.33E-01	4.49E-04	9.31E-02
20	0.50	35.4230	5.9431	2.4981	7.4593	7.4593	6.64E-07	8.33E-01	4.42E-04	9.61E-02
20	1.00	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
20	1.50	36.2450	6.8104	2.4714	6.8564	6.8564	2.97E-10	8.24E-01	4.41E-04	1.10E-01
25	0.50	35.8296	8.0437	2.4800	8.5187	8.5187	6.05E-07	8.27E-01	4.36E-04	1.08E-01
25	1.00	36.3782	8.7322	2.4657	8.0666	8.0666	8.17E-10	8.22E-01	4.36E-04	1.18E-01
25	1.50	36.6234	9.1009	2.4536	7.8560	7.8560	2.67E-10	8.18E-01	4.35E-04	1.23E-01

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Table 5. Performance measures by varying the number of phases for repair (k) and the repair rate (b_1).

k	b_1	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
2	20.00	35.9905	6.5049	2.4839	7.0472	7.0472	9.04E-10	8.28E-01	4.42E-04	1.05E-01
2	25.00	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
2	30.00	35.9912	6.5056	2.4839	7.0466	7.0466	9.03E-10	8.28E-01	4.42E-04	1.05E-01
4	20.00	35.9886	6.5032	2.4839	7.0486	7.0486	9.08E-10	8.28E-01	4.42E-04	1.05E-01
4	25.00	35.9894	6.5040	2.4839	7.0480	7.0480	9.06E-10	8.28E-01	4.42E-04	1.05E-01
4	30.00	35.9900	6.5045	2.4839	7.0476	7.0476	9.05E-10	8.28E-01	4.42E-04	1.05E-01
6	20.00	35.9870	6.5018	2.4839	7.0498	7.0498	9.11E-10	8.28E-01	4.42E-04	1.05E-01
6	25.00	35.9881	6.5028	2.4839	7.0489	7.0489	9.08E-10	8.28E-01	4.42E-04	1.05E-01
6	30.00	35.9889	6.5035	2.4839	7.0484	7.0484	9.07E-10	8.28E-01	4.42E-04	1.05E-01

Table 6. Performance measures by varying the number of phases (k) and the setup time taken by repairmen (δ).

k	δ	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
2	1.00	35.9790	6.4950	2.4839	7.0559	7.0559	2.08E-09	8.28E-01	8.83E-04	1.05E-01
2	2.00	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
2	3.00	35.9948	6.5088	2.4839	7.0439	7.0439	8.90E-10	8.28E-01	2.94E-04	1.05E-01
4	1.00	35.9775	6.4937	2.4839	7.0571	7.0571	2.16E-09	8.28E-01	8.83E-04	1.05E-01
4	2.00	35.9894	6.5040	2.4839	7.0480	7.0480	9.06E-10	8.28E-01	4.42E-04	1.05E-01
4	3.00	35.9933	6.5074	2.4839	7.0450	7.0450	8.92E-10	8.28E-01	2.94E-04	1.05E-01
6	1.00	35.9762	6.4925	2.4839	7.0581	7.0581	2.24E-09	8.28E-01	8.83E-04	1.05E-01
6	2.00	35.9881	6.5028	2.4839	7.0489	7.0489	9.08E-10	8.28E-01	4.42E-04	1.05E-01
6	3.00	35.9920	6.5062	2.4839	7.0460	7.0460	8.93E-10	8.28E-01	2.94E-04	1.05E-01

Table 7. Performance measures by varying the rate by which the server goes back to vacation (ε) and the server breakdown rate (a_0).

ε	a_0	$E(O)$	$E(S)$	λ_{eff}	$E(W)$	$E(D)$	FF	$P(B)$	$P(D)$	$P(A)$
0.1	0.001	36.6579	8.3342	2.4326	6.1695	6.1695	7.52E-10	8.11E-01	4.51E-04	8.75E-02
0.1	0.003	36.6338	8.3075	2.4328	6.1898	6.1898	7.86E-10	8.11E-01	1.35E-03	8.69E-02
0.1	0.005	36.6096	8.2808	2.4331	6.2101	6.2101	8.22E-10	8.11E-01	2.25E-03	8.64E-02
0.2	0.001	35.9909	6.5053	2.4839	7.0469	7.0469	9.03E-10	8.28E-01	4.42E-04	1.05E-01
0.2	0.003	35.9638	6.4806	2.4841	7.0671	7.0671	9.44E-10	8.28E-01	1.32E-03	1.04E-01
0.2	0.005	35.9366	6.4559	2.4843	7.0875	7.0875	9.87E-10	8.28E-01	2.21E-03	1.04E-01
0.3	0.001	35.7042	5.7191	2.5060	7.4129	7.4129	9.68E-10	8.35E-01	4.37E-04	1.13E-01
0.3	0.003	35.6760	5.6957	2.5061	7.4331	7.4331	1.01E-09	8.35E-01	1.31E-03	1.12E-01
0.3	0.005	35.6476	5.6722	2.5063	7.4533	7.4533	1.06E-09	8.35E-01	2.19E-03	1.11E-01

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From the profit function, the variation of expected total profit $E(TP)$ for the varying values of system parameters is illustrated in Figures 6 and 7 as a surface plot. The trend shows how and which parameters are critical, and which parameters have strong, moderate, and weak effects on expected profit.

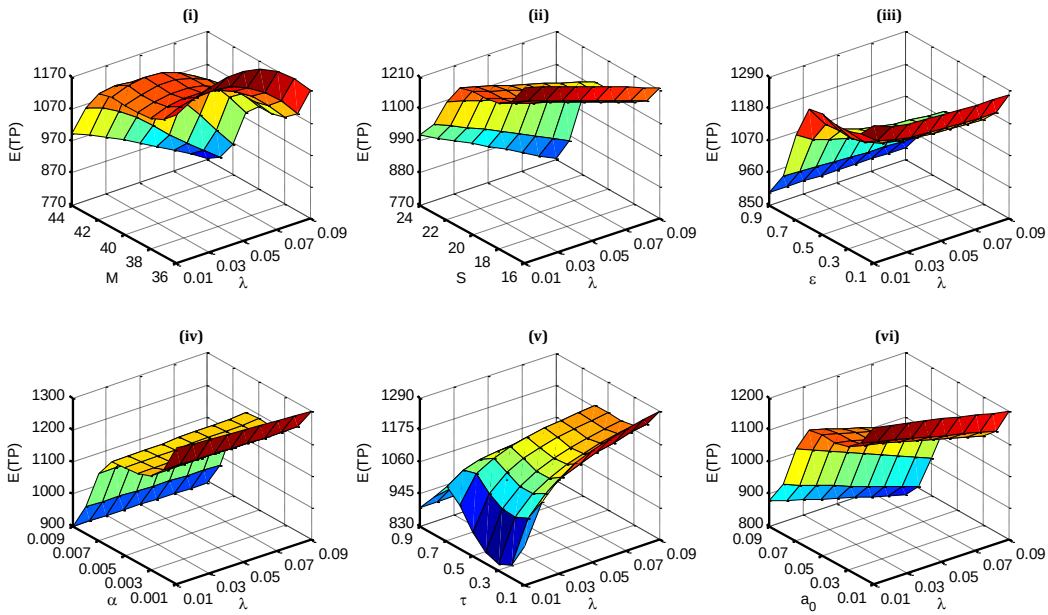

Figure 6. The expected total profit of the system $E(TP)$ wrt the failure rate of the operating machines (λ).

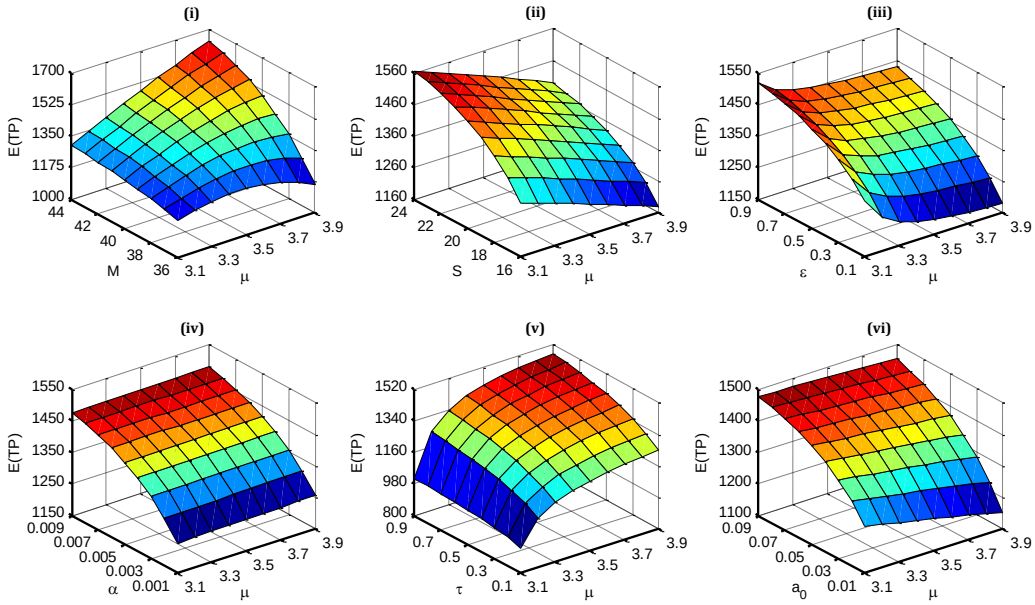

Figure 7. The expected total profit of the system $E(TP)$ wrt service rate of the failed machines (μ).

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For the optimal analysis, we identify the number of the parameters, namely, the number of standby machines S, service rate μ, repair rate b_1, and setup time τ as decision variables. The concavity trends of these decision variables are portrayed in Figure 8.

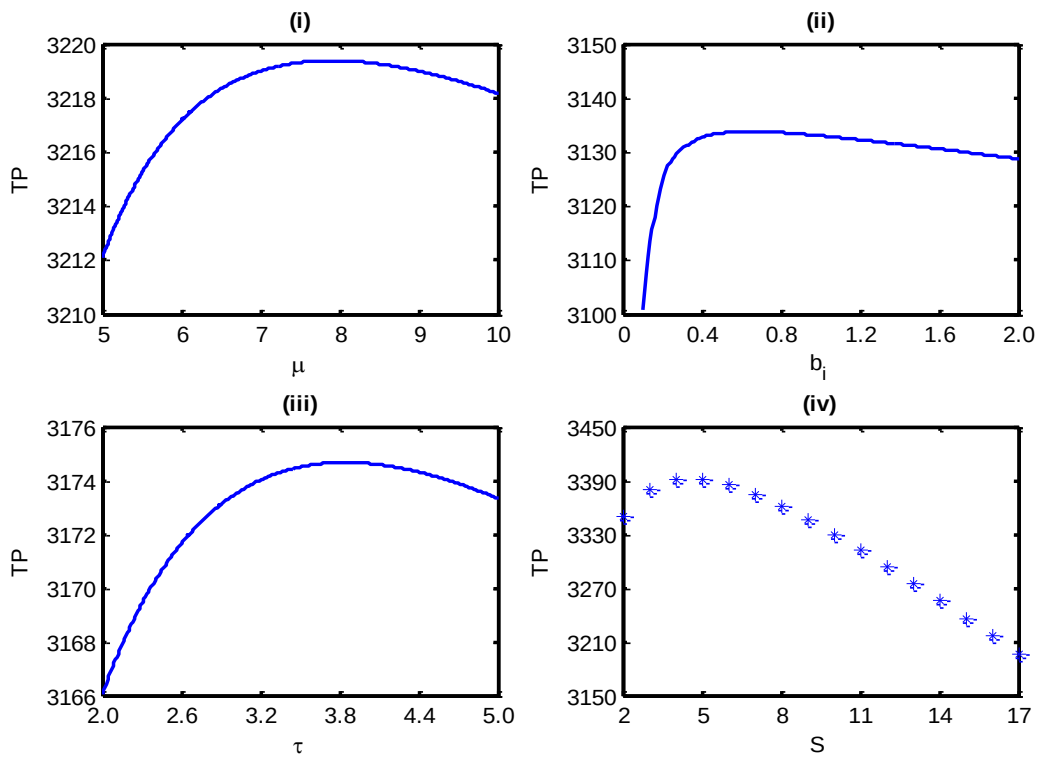

Figure 8. The concavity nature of the expected total profit function $E(TP)$ wrt decision variables.

The results prove that the expected total profit function $E(TP)$ is concave wrt decision variables since the analytical proof is not possible due to the highly implicit and complex nature of $E(TP)$. It is crucial to determine the optimal values of these decision parameters to maximize the expected total profit.

We employ the direct search method given in section 6.1, and the Newton-quasi method explained in section 6.2 to determine the optimal values of the discrete decision variable, S, and continuous decision variables, μ, τ. For the illustrative purpose of the employed method, we conduct the three experiments, and results are depicted in Tables 8-10 for different mentioned values of design parameters.

For Table 8 $M = 30,\ m = 5,\ k = 2,\ \lambda = 0.05,\ \alpha = 0.01,\ a_0 = 0.001,\ C_H = 30,\ C_O = 25,\ C_S = 20,\ C_M = 2,\ C_B = 5,\ R_\upsilon = 150,\ C_T = 5$

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Table 8. An illustrative example of the Newton-quasi method.

S^*	μ	τ	$TP(S^*,\mu,\tau)$	$\left\lvert\frac{\partial(TP)}{\partial\mu}\right\rvert$	$\left\lvert\frac{\partial(TP)}{\partial\tau}\right\rvert$	**Tolerance**
4	3.0000	1.0000	3277.18	2.86E+02	1.07E+02	2.86E+02
4	3.4850	1.5132	3400.73	1.09E+02	4.15E+01	1.09E+02
4	4.0206	2.0821	3453.54	4.35E+01	1.57E+01	4.35E+01
4	4.6594	2.6596	3477.17	1.74E+01	5.41E+00	1.74E+01
4	5.4388	3.0917	3487.31	6.84E+00	1.45E+00	6.84E+00
4	6.3486	3.2257	3491.35	2.49E+00	2.57E-01	2.49E+00
4	7.2271	3.1856	3492.68	7.39E-01	6.92E-02	7.39E-01
4	7.7624	3.1588	3492.90	1.25E-01	1.64E-02	1.25E-01
4	7.8933	3.1540	3492.90	5.26E-03	7.91E-04	5.26E-03
4	7.8993	3.1538	3492.90	7.72E-06	2.02E-06	7.72E-06
4	7.8993	3.1538	3492.90	0.00E+00	1.36E-08	1.36E-08

For Table 9 $M=40,\ m=5,\ k=2,\ \lambda=0.05,\ \alpha=0.01,\ a_0=0.001,\ C_H=30,\ C_O=25,\ C_S=20,\ C_M=2,\ C_B=5,\ R_\upsilon=150,\ C_T=5$

Table 9. An illustrative example of the Newton-quasi method.

S^*	μ	τ	$TP(S^*,\mu,\tau)$	$\left\lvert\frac{\partial(TP)}{\partial\mu}\right\rvert$	$\left\lvert\frac{\partial(TP)}{\partial\tau}\right\rvert$	Tolerance
9	3.0000	1.0000	3797.71	1.50E+03	1.01E+02	1.50E+03
9	3.6477	2.3239	4417.23	3.66E+02	2.60E+01	3.66E+02
9	4.0747	3.1602	4530.81	1.42E+02	8.95E+00	1.42E+02
9	4.5694	3.7813	4579.77	5.74E+01	2.65E+00	5.74E+01
9	5.1903	3.9442	4603.22	2.35E+01	6.27E-01	2.35E+01
9	5.9869	3.7363	4615.33	9.62E+00	3.02E-01	9.62E+00
9	6.9613	3.5416	4621.30	3.73E+00	2.27E-01	3.73E+00
9	7.9936	3.4313	4623.67	1.24E+00	1.26E-01	1.24E+00
9	8.7753	3.3822	4624.23	2.79E-01	4.09E-02	2.79E-01
9	9.0694	3.3697	4624.27	2.31E-02	4.17E-03	2.31E-02
9	9.0982	3.3687	4624.27	1.77E-04	3.44E-05	1.77E-04
9	9.09844	3.3687	4624.27	4.27E-07	5.46E-08	4.27E-07

For Table 10 $M=30,\ m=5,\ k=2,\ \lambda=0.05,\ \alpha=0.01,\ a_0=0.001,\ C_H=30,\ C_O=25,\ C_S=20,\ C_M=6,\ C_B=5,\ R_\upsilon=150,\ C_T=5$

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Table 10. An illustrative example of the Newton-quasi method.

S^*	μ	τ	$TP(S^*,\mu,\tau)$	$\left\lvert\frac{\partial(TP)}{\partial\mu}\right\rvert$	$\left\lvert\frac{\partial(TP)}{\partial\tau}\right\rvert$	**Tolerance**
4	3.0000	1.0000	3265.18	2.82E+02	1.07E+02	2.82E+02
4	3.4784	1.5128	3386.07	1.07E+02	4.16E+01	1.07E+02
4	3.9925	2.0846	3436.36	4.13E+01	1.58E+01	4.13E+01
4	4.5653	2.6751	3457.26	1.56E+01	5.44E+00	1.56E+01
4	5.1604	3.1433	3464.52	5.23E+00	1.44E+00	5.23E+00
4	5.6471	3.3392	3466.17	1.30E+00	2.01E-01	1.30E+00
4	5.8715	3.3538	3466.33	1.46E-01	1.02E-02	1.46E-01
4	5.9041	3.3516	3466.33	2.43E-03	1.50E-04	2.43E-03
4	5.9047	3.3515	3466.33	5.41E-07	2.27E-08	5.41E-07

The last row represents the optimal value, and predecessor rows show how this result is approaching from the initial value in the first row beside the number of iteration, tolerance, and other required values. These tables are essential to understand the iterative and converging pattern of the Newton-quasi method for optimization problems.

Tables 11-13 tabulate the optimal values of the decision variables (S^*,μ^*,τ^*) along with the optimal value of the expected total profit $TP^*(S^*,\mu^*,\tau^*)$ for a different combination of system parameters, design parameters, and unit cost/revenue elements, respectively. These tables also comprise the iteration size, system characteristics at optimal values. These results are critical to understanding the trends of various system and design parameters on system efficiency and associated costs.

For Table 11 $M=30,\ m=5,\ k=2,\ \lambda_d=0.09,\ \varepsilon=0.2,\ \tau=1,\ \delta=2,\ \mu=3,\ b_i=25,\ C_H=30,\ C_O=25,$ $C_S=20,\ C_M=2,\ C_B=5,\ R_\upsilon=150,\ C_T=5$

Table 11. Optimal profit analysis wrt failure rates $\lambda,\ \alpha,$ and a_0 .

$(\lambda,\ \alpha,\ a_0)$	S^*	μ^*	τ^*	$TP^*(S^*,\mu^*,\tau^*)$	$E(O)$	τ_p	**Iterations**
(0.05,0.01,0.001)	4	7.8993	3.1538	3492.90	39.8714	2.1632	11
(0.10,0.01,0.001)	4	8.7570	3.0559	3491.36	39.8843	2.1545	11
(0.15,0.01,0.001)	2	11.5145	2.9499	3526.54	39.8806	2.1804	11
(0.05,0.02,0.001)	5	7.7480	3.0976	3470.82	39.8754	2.1613	11
(0.05,0.03,0.001)	5	7.7493	3.0983	3470.80	39.8754	2.1612	11
(0.05,0.01,0.010)	5	7.7970	3.0932	3470.70	39.8763	2.1607	11
(0.05,0.01,0.100)	5	8.2890	3.0621	3469.22	39.8852	2.1557	11

For Table 12 $\lambda=0.05,\ \alpha=0.01,\ a_0=0.001,\ \lambda_d=0.09,\ \varepsilon=0.2,\ \tau=1,\ \delta=2,\ \mu=3,\ b_i=25,\ C_H=30,$ $C_O=25,\ C_S=20,\ C_M=2,\ C_B=5,\ R_\upsilon=150,\ C_T=5$

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Table 12. Optimal profit analysis wrt design parameters M, m, and k.

(M,m,k)	S^*	μ^*	τ^*	$TP^*(S^*,\mu^*,\tau^*)$	$E(O)$	τ_p	**Iterations**
(30,5,2)	4	7.8993	3.1538	3492.90	39.8714	2.1632	11
(35,5,2)	7	8.4169	3.2335	4047.68	39.9069	2.1516	11
(40,5,2)	9	9.0984	3.3687	4624.27	39.9278	2.1486	12
(30,7,2)	4	7.8993	3.1538	3492.90	39.8714	2.1632	11
(30,9,2)	4	7.8993	3.1538	3492.90	39.8714	2.1632	11
(30,5,3)	4	7.8997	3.1538	3492.90	39.8714	2.1632	11
(30,5,4)	4	7.9001	3.1537	3492.90	39.8714	2.1632	11

For Table 13 $M = 30,\ m = 5,\ k = 2,\ \lambda = 0.05,\ \lambda_d = 0.09,\ \alpha = 0.01,\ a_0 = 0.001,\ \varepsilon = 0.2,\ \tau = 1,\ \delta = 2,$ $\mu = 3,\ b_i = 25$

Table 13. Optimal profit analysis wrt unit cost elements C_H, C_O, C_S, C_M, C_B, R_υ and C_T.

$(C_H,C_O,C_S,C_M,C_B,R_\upsilon,C_T)$	S^*	μ^*	τ^*	$TP^*(S^*,\mu^*,\tau^*)$	$E(O)$	τ_p	**Iterations**
(20,25,20,2,5,150,5)	5	7.1430	2.9336	3484.10	39.8487	2.1738	10
(30,25,20,2,5,150,5)	4	7.8993	3.1538	3492.90	39.8714	2.1632	11
(40,25,20,2,5,150,5)	4	8.4472	3.3003	3483.16	39.8896	2.1531	11
(30,15,20,2,5,150,5)	5	7.8366	3.1590	3770.16	39.8803	2.1589	11
(30,35,20,2,5,150,5)	4	7.8020	3.0863	3193.65	39.8656	2.1661	11
(30,25,15,2,5,150,5)	5	7.9941	3.1457	3489.73	39.8832	2.1573	11
(30,25,25,2,5,150,5)	4	7.6666	3.1132	3477.54	39.8631	2.1678	11
(30,25,20,6,5,150,5)	4	5.9047	3.3515	3466.33	39.7731	2.2128	9
(30,25,20,10,5,150,5)	4	5.2592	3.4692	3444.16	39.6763	2.2497	9
(30,25,20,2,3,150,5)	4	7.8993	3.1538	3542.90	39.8714	2.1632	11
(30,25,20,2,7,150,5)	4	7.8993	3.1538	3442.90	39.8714	2.1632	11
(30,25,20,2,5,100,5)	3	7.6341	2.8207	2018.53	39.8246	2.1966	10
(30,25,20,2,5,200,5)	5	8.1656	3.3872	4967.63	39.8962	2.1510	11
(30,25,20,2,5,150,3)	4	7.7241	3.8685	3499.84	39.8929	2.1536	11
(30,25,20,2,5,150,7)	4	8.0317	2.7596	3487.02	39.8529	2.1704	11

In a nutshell, we have the following important observation for the decision-makers

- The operating machines' failure rate λ significantly affects system performance, and hence, preventive measures and regular examination should be taken frequently.
- The repair/service facilities incur running costs and directly affect the expected profit function. There is a need for timeliness corrective measures optimally to diminish the downtime of the machining systems.
- The setup time and the number of phases for repair are also critical; and need to reduce as much as possible.

8. Conclusion and Future Scope

This article studies the queueing analysis and optimal analysis of a machine repair problem (MRP) under the provisioning of warm standby machines and a single unreliable server with N -policy, setup time, and repair in phases. All random times deliberated in this article are assumed to follow exponential distributions and be independent. The state transition diagram is used to develop the governing forward Chapman-Kolmogorov equations and corresponding matrix representations. State probabilities and performance characteristics are next derived from these matrices. Finally, the optimal analysis for profit function is done employing a direct search method and Newton-quasi method to identify optimal parametric and design specifications. The wide-ranging numerical experimentation is premeditated and accomplished for decision-makers. The results are highlighted herewith as the state-of-the-art of the present study. Operating/standby machine failure and server breakdown need to be prolonged with suitable preventive measures. Naturally, interruption is unavoidable due to machines and servers' wear and tear, prompt corrective measures are necessary. Less organizational delay/setup time requirement is preferable. Repair in phases is reasonable corrective measures, but the number of phases must not be too less or more. This article is beneficial for the machining system to maintain quality of service (QoS) and Just-in-time (JIT) policy of service.

For the future, some possible research topics are planned as (i) some random time with more general distributions, (ii) transient and reliability characteristics analysis of the system, and (iii) the sensitivity analysis of the parameters.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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